The Color Glass Condensate
and
The High Energy Limit of QCD

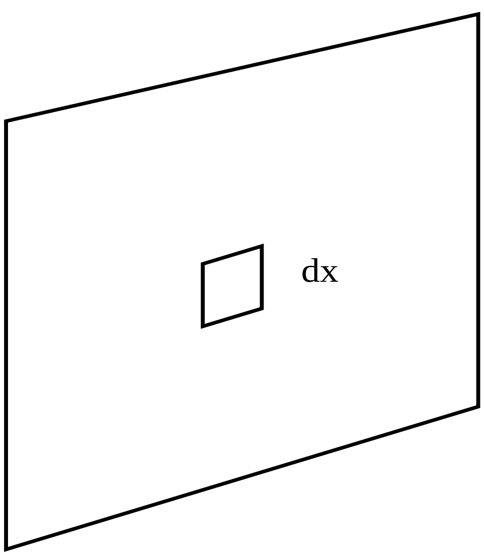
A Very High Energy Hadron

In the limit that $E_{had} \to \infty$ for $x \to 0$

$$\Lambda^2 = \frac{1}{\pi R^2}\frac{dN}{dy} >> \Lambda^2_{QCD}$$

implies that $\alpha_s << 1$ The typical transverse momentum scale

$$p_T^2 \sim \Lambda^2 >> \frac{1}{R^2_{had}}$$



Very thin sheet because of Lorentz contraction

Very large sheet because $p_T >> 1/R_{had}$

Resolution scale Δx is $\Delta x << 1/R_{had}$ and $1/\Lambda_{QCD}$

Different Rapidity Definitions:

Momentum Space Rapidity:

$$y = \frac{1}{2} ln \left(\frac{p^+}{p^-} \right) = ln \left(\frac{2p^+}{M_T} \right)$$

$$= ln \left(\frac{2p^+_{had}}{M_T} \right) + ln \left(\frac{p^+}{p^+_{had}} \right) = y_{had} - ln \left(\frac{1}{x} \right)$$

Coordinate Space Rapidity:

$$y = \frac{1}{2} ln \left(\frac{x^+}{x^-} \right) = ln \left(\frac{2\tau}{x^-} \right)$$

where $\tau = \sqrt{t^2 - z^2}$

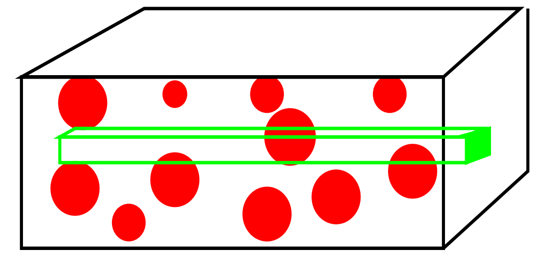
Using the uncertainty principle $x^{\pm} \sim 1/p^{\mp}$

$$y_{particles} \sim y_{constituent} \sim y_{Bjorken} \sim y_{space-time}$$

All rapidities the same up to $\Delta y \sim 1$

Can map momentum space into coordinate space!

Distributions of Particles:



y_{max} $\qquad$ y_{min}

largest p_z $\qquad$ smallest p_z

smallest Δz $\qquad$ largest Δz

$\Delta x << 1\ Fm$

Longitudinal separation
between quarks and gluons
in tube is big as
$\Delta x \to 0$
Color source is random

Particles with $y_{max} > y > y_{min}$ act
as sources for fields with $y_{min} > y$

If the density of sources is big,
$\Delta x^2 \rho >> 1$
the sources become **classical**:
$[Q^a, Q^b] = i f^{abc} Q_c << Q^2$

The current associated with this source of
color is
$J^\mu_a = \delta^{\mu +} \delta(x^-) \rho_a(x_T)$

The $\delta^{\mu +}$ is because p^+ is big.

The $\delta(x^-)$ is approximate and localizes
the fields on the sheet $t = z$.

The source $\rho_a(x_T)$ is random in color
on 2 dimensional sheet.

Sources:

The distribution of sources is in reality:

$\rho(x^-, x_T) \sim \delta(x^-)$

where

$\rho(x_T) = \int dx^- \; \rho(x^-, x_T) = \int dy \; \rho(y, x_T)$ The width $\Delta x^- = 1/p^+_{min}$.

Note that ρ is time, x^+ independent:

Glass

Color Glass Condensate

Yang Mill's theory in presence of random source:

$$\int [dA][d\rho] exp\left(iS[A + iJ^+A^- - \frac{1}{2}\int dyd^2x_T \frac{\rho^2(y,x_T)}{\mu^2(y)} \right)$$

The Gaussian ansatz is McLerran-Venugopalan model:

$$< \rho_a(y, x_T)\rho_b(y' y_T) >= \delta^{ab}\delta(y - y')\delta^{(2)}(x_T - y_T)\mu^2(y)$$

$\mu^2(y)$ is the color charge squared per unit $area \times y \times Nc^2 - 1$

Random Source $< - >$ **Color Glass** $\sim$ **Spin Glass**

Incoherent sum $<=>$ Glass

Some Comments on CGC:

Theory is defined by a cutoff p^+_{min}
The sources arose from fields with $p^+ > p^+_{min}$
The dynamical fields exist for $p^+ < p^+_{min}$

Cutoff p^+_{min} is arbitrary
Can be changed => Renormalization Group

So long as p^+/p^+_{min} is not too small
the solution is classical field in presence of ρ
Find solution to

$$D_\mu F^{\mu\nu} = J^\nu$$

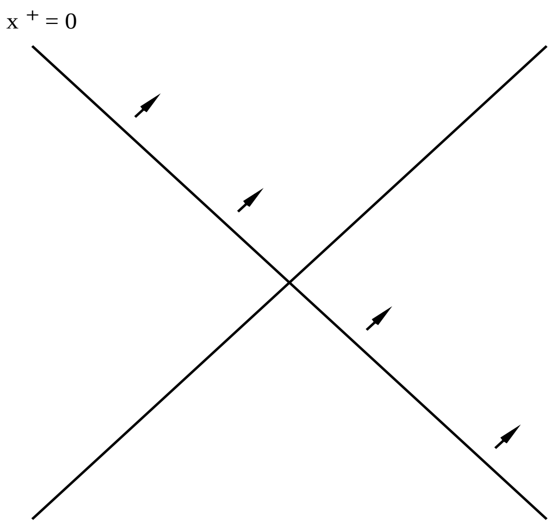
and average physical $F[A]$ over ρ
Big corrections $\sim \alpha_s ln(p^+/p^+_{min})$
if p^+/p^+_{min} is too small
=> **Renormalization Group**

Solution to classical equation has $A \sim 1/g$.
The phase space density:

$$\frac{dN}{d^2x_T dy d^2p_T} \sim < AA > \sim 1/\alpha_s$$

Condensate

Review of Light Cone Quantization:



Light cone Hamiltonian: P^-
Light cone time: X^+
Light cone momentum: P^+
Light cone coordinate: X^-

The Klein-Gordon Field:

$$\left(p^2 - M^2\right)\phi = 0$$

$$p^-\phi = \frac{p_T^2 + M^2}{2p^+}\phi$$

The Hamiltonian is $\frac{p_T^2 + M^2}{2p^+}$

Second Quantization:

$$S = \int d^4x \ \left\{\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}M^2\phi^2\right\}$$

The canonical momentum is:

$$\Pi(x^-, x_T) = \frac{\delta S}{\delta \partial_+ \phi} = \partial^+\phi = \frac{\partial}{\partial x^-}\phi$$

Π is on equal time x^+ surface!

Not independent of ϕ

Equal Time Commutation Relations:

Postulate:

$$[\Pi(x^-, x_T), \phi(y^-, y_T)] = \frac{-i}{2}\delta^{(3)}(x-y)$$

so that

$$\partial_-^x[\phi(x), \phi(y)] = \frac{-i}{2}\delta^{(3)}(x-y)$$

or

$$[\phi(x), \phi(y)] = \frac{-i}{2}\epsilon(x^- - y^-)\delta^{(2)}x_T - y_T)$$

These commutation relations are realized by

$$\phi(x) = \int \frac{d^3p}{(2\pi)^3 2p^+}\theta(p^+)\left\{e^{-ipx}a(p) + e^{ipx}a^\dagger(p)\right\}$$

where

$$[a_i^a(p), a_j^b(q)] = 2p^+\delta^{ab}\delta_{ij}\delta^{(3)}(p-q)$$

Only $p^+ > 0$ so only postive momentum particles => vacuum trivial

---

QCD on the Light Cone:

Light Cone Gauge: $A_a^+ = 0$

$$D_\mu F^{\mu\nu} = -D_i F^{i+} + D^+ F^{-+} = 0$$

so that

$$A^- = \frac{1}{\partial^{+2}} D^i \partial^+ A^i$$

The transverse fields are dynamical degrees of freedom.

$$A_a^i = \int \frac{d^3p}{(2\pi)^3 2p^+}\left\{e^{-ipx}a_i^a(p) + e^{ipx}a_i^{a\dagger}(p)\right\}$$

The Gluon Content of a Hadron:

$$\frac{2p^+}{(2\pi)^3}\frac{dN}{d^3p} = < h \mid a^\dagger(p) a(p) \mid h >$$

which is the same as

$$\frac{dN}{d^3p} = \frac{2p^+}{(2\pi)^3} < h \mid A^{ia}(p) A_{ia}(-p) \mid h > =$$
$$\frac{2p^+}{(2\pi)^3} G^{ii}_{aa}(p, p; x^+ - y^+ \to 0 \mid h >$$

Phase space distribution known in terms of propogators.

Solving the MV Model:

Strategy: Solve in simple gauge and rotate to light cone gauge.

Simple gauge $A^- = 0$, since

$$D_\mu F^{\mu\nu} = \delta^{\nu +} \rho(x^-, x_T)$$

is solved by

$A^i = 0$ and

$$-\nabla_T^2 \overline{A}^+ = \overline{\rho}$$

The overline means quantities in $A^- = 0$ gauge.

Note that

$$\overline{\rho} = U^\dagger(x) \rho U(x)$$

Solving the MV Model

Note that because the integration measure is gauge invariant (we will have to fix up the action a little!), the gauge rotation does not affect physical gauge invariant quantitities.

Explicitly:

$$\overline{A}^\mu = U^\dagger A^\mu U + \frac{i}{g} U^\dagger \partial^\mu U$$

so that

$$\overline{A}^+ = \frac{i}{g} U^\dagger (\partial^+ U)$$

Or defining

$$\alpha = \overline{A}^+ = \frac{1}{-\nabla_T^2} \overline{\rho}$$

Therefore:

$$U^\dagger(x) = Pexp\left\{ ig \int_{x^0}^{x^-} dz^- \alpha(z^-, x_T) \right\}$$

We will choose a retarded boundary condition $x^0 \to -\infty$.

The fields in $A^+ = 0$ gauge are therefore:

$$A^+ = A^- = 0$$

$$A^i = \frac{i}{g} U \nabla^i U^\dagger$$

For x^- outside the range where the source sits:

$$A^i = \theta(x^-) V \nabla^i V^\dagger$$

where

$$V^\dagger(x) = Pexp\left\{ ig \int_{-\infty}^{\infty} dz^- \alpha(z^-, x_T) \right\}$$

The Gluon Distribution and Saturation

Recall that:

$$\frac{dN}{d^3k} = \frac{2k^+}{(2\pi)^3} A^i_a(k, x^+) A^i_a(-k, x^+)$$

where

$$A^i_a(x, x^+) = \frac{i}{g} U(x) \nabla^i U^\dagger(x)$$

You can compute:

$$< A^i_a(x, x^+) A^i_a(y, x^+) >= \frac{N_c^2 - 1}{\pi \alpha_s N_c} \frac{1 - e^{x_T^2 Q_s^2 ln(x_T^2 \Lambda^2_{QCD})/4}}{x_T^2}$$

In this equation, both x^- and y^- are outside the range where the source sits. The saturation momentum is:

$$Q_s^2 = 2\pi N_c \alpha_s^2 \int dx^- \mu^2(x^-) \sim \alpha_s^2 \frac{charge^2}{area \times (N_c^2 - 1)}$$

Formula true only for $x_T << 1/\Lambda_{QCD}$

Also

$$\int dx^- \mu^2(x^-) = \int_{y_{min}}^{y_{hadron}} dy \mu^2(y)$$

so it is the total charge at all rapidities greater than where we measure. This can related to the gluon density by DGLAP and that charge density, up to Casimir is gluon density.

The Gluon Distribution and Saturation

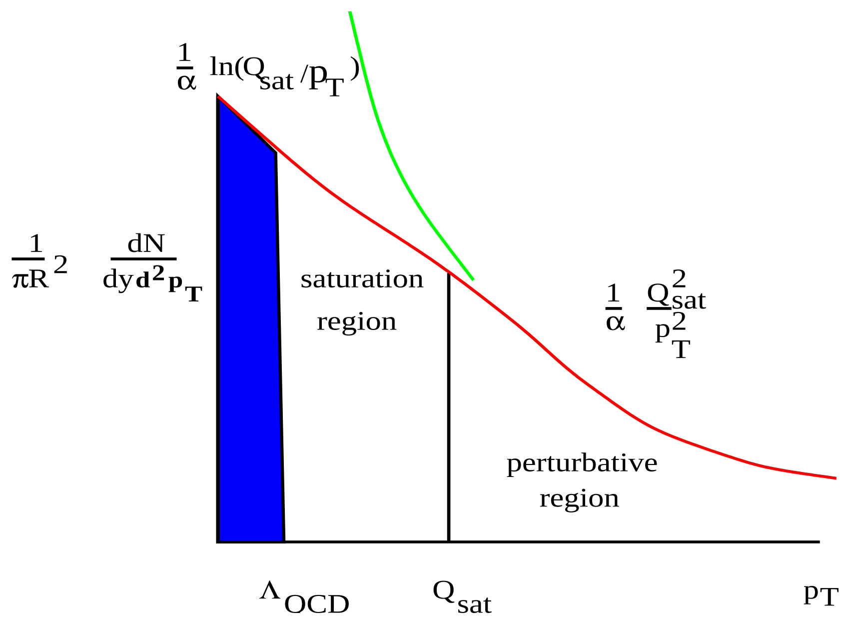
$$\frac{1}{\pi R^2}\frac{dN}{d^2p_T dy} = \frac{2p^+}{(2\pi)^3} \times$$
$$\int d^3x e^{-ipx} < A_a^i(0,x^+) A_a^i(x,x^+) >$$

At small x_T correlation function goes as

$$\frac{ln(x_T^2)Q_s^2}{\alpha_s} \to \frac{Q_s^2}{\alpha_s p_T^2}$$

At large x_T, we have

$$\frac{1}{\alpha_s x_T^2} \to \frac{ln(Q_s^2/p_T^2)}{\alpha_s}$$



$1/\alpha_s$ is condensation

Gluon Distribution and Saturation

$Q_s^2/p_T^2 \rightarrow$ perturbative bremstrahlung
$ln(Q_s^2/p_T^2) \rightarrow$ saturation
The fields are Lorentz boosted Coulomb fields:
$(\delta x_T)^2 >> 1/\rho$, the fields cancel.
$ln(x_T^2) \rightarrow 1/x_T$
Two powers of x_T softer becomes two of p_T.

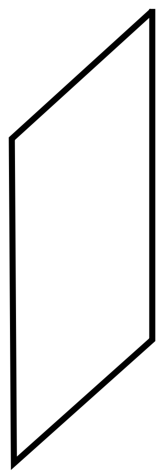
Q_s^2 **can grow rapidly with energy!**
In saturated region $\sim ln(Q_s^2)$
In perturbative tail, fast growth.
Repulsive gluon interactions $=>$
Growth of intrinsic p_T
but new gluons are small.

$xG(x,Q^2) \sim \int_0^{Q^2} d^2p_T \frac{dN}{d^2p_T dy}$
$\sim \pi R^2 Q^2$ saturation region
$\sim \pi R^2 Q_s^2$ large Q
$Q_s^2 \sim charge^2/area \sim R$
$xG \sim$ surface in saturation region
$xG \sim$ volume in perturbative region

No Problem with Unitarity!
Cross section at fixed $Q^2 \sim xG$.

The Color Glass Fields

$U\nabla^i U^\dagger$

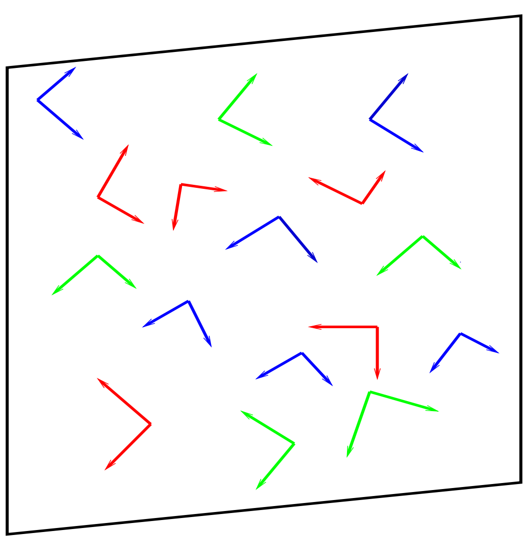


$V\nabla^i V^\dagger$

$$F^{i+} \sim \delta(x^-)$$
$$F^{i-} \sim 0$$
$$F^{ij} \sim 0$$

Note that $F^{i\pm} = F^{i0} \pm F^{iz}$:

$$\vec{E} \perp \vec{B} \perp \vec{z}$$



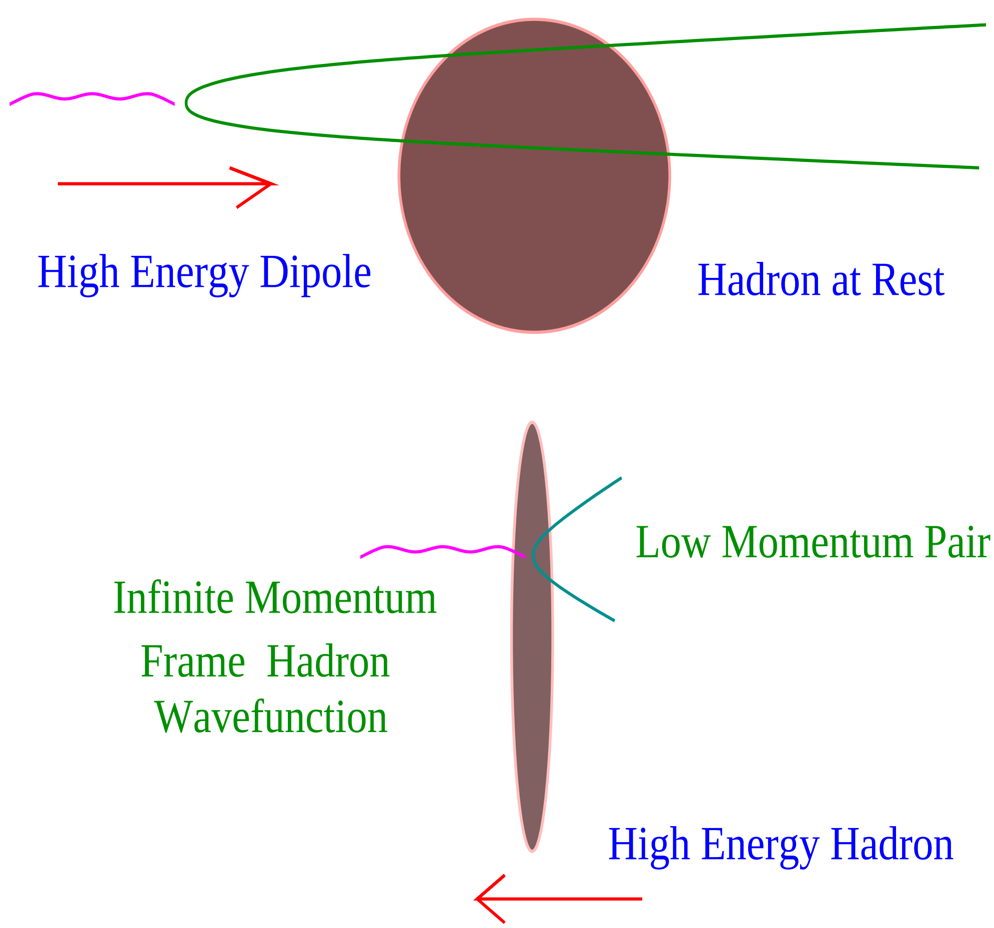
density $\sim 1/\alpha_s$

fields random

fields frozen in time
0.5in

fields not stringy

Deep Inelastic Scattering:



For deep inelastic scattering:

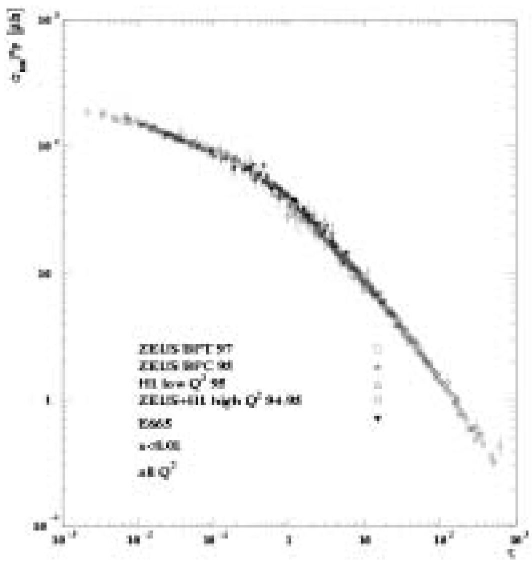
$< J^\mu(x) J^\nu(0) >$

Can be used to compute structure functions:

Can also compute diffractive structure functions.

Results in agreement with dipole picture.

Geometric Scaling:



$$\sigma_{\gamma^* p} \sim F_2(x, Q^2)/Q^2$$

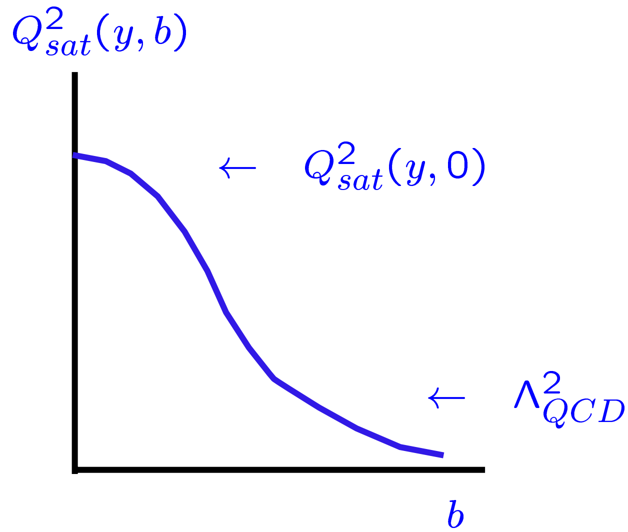
$$\sim G(Q^2/Q^2_{sat})$$

Obvious: $Q^2 << Q^2_{sat}$
True up to:
$Q^2 \leq Q^4_{sat}/\Lambda^2$

Obsereved scaling is consequence of BFKL evolution and the Color Glass Condensate. The two conspire together to generate geometric scaling in an extended region.

Schildnecht and Surrow; Stasto, Goloec-Biernat, and Kwieczinski; Iancu, Itakura and McLerran; Mueller and Triantafyllopoulos

Froissart Bound and Saturation



$\sigma \sim ln^2(E)$?

Near the edge of the hadron:

$$Q^2_{sat}(y,b) = Q^2_{sat}(y,0)F(b)$$

True for large Q and large b.
At large b, $F(b) \sim e^{-\mu b}$
so that a fixed Q^2 cross section solves

$$Q^2 \sim Q^2_{sat}(y)exp^{-\mu b}$$

This Q_{sat} solves fixed Q BFKL
and is slightly different from the
Q_{sat} in the center of the hadron.
Requires $b \sim \kappa y$.

Saturation of Froissart!

Kovner, Weidemann; Iance, Itakura and McLerran