High energy hadronic interactions in QCD and applications to heavy ion collisions

I – Introduction and phenomenology

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General outline

- Lecture I : Introduction and phenomenology
- Lecture II : Lessons from Deep Inelastic Scattering
- Lecture III : QCD on the light-cone
- Lecture IV : Saturation and the Color Glass Condensate
- Lecture V : Calculating observables in the CGC

Lecture I : Introduction

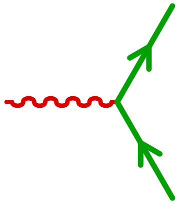
- Basic features of QCD
- Deconfinement phase transition
- Heavy ion collisions
- Parton model
- Saturation of parton distributions

Prerequisites

- Things you should have heard of before :
  - A bit of quantum field theory
  - Perturbation theory
  - Renormalization group
  - Gauge theories
  - Protons, neutrons, nuclei...
- Tools that will be introduced as needed :
  - Operator product expansion
  - KLN theorem
  - Cutting rules
- Stuff that I may take for granted, and that may not be obvious : do not hesitate to ask for it !!

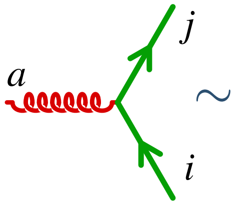
Quarks and gluons

- Electromagnetic interaction : Quantum electrodynamics
  - Matter : electron , interaction carrier : photon
  - Interaction :

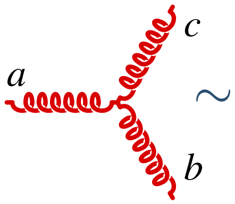


$\sim \; e$ (electric charge of the electron)

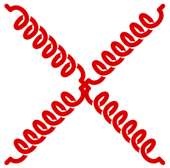
- Strong interaction : Quantum chromodynamics
  - Matter : quarks , interaction carriers : gluons
  - Interactions (Note: the gluons are charged):



$\sim \; g\,(t^a)_{ij}$



$\sim \; g\,(T^a)_{bc}$



  - i,j : colors of the quarks (3 possible values)
  - a,b,c : colors of the gluons (8 possible values)
  - $(t^a)_{ij}$: 3×3 matrix , $(T^a)_{bc}$: 8×8 matrix

QCD Lagrangian

- QCD Lagrangian :

$$\mathcal{L} = -\frac{1}{2}\mathrm{tr}\,(F_{\mu\nu}F^{\mu\nu}) + \overline{\psi}(i\not{D} - m)\psi$$

  - the gauge field A^μ belongs to $SU(3)$
  - $D^\mu \equiv \partial^\mu - igA^\mu$ is the covariant derivative
  - $F^{\mu\nu} \equiv i[D^\mu, D^\nu]/g = \partial^\mu A^\nu - \partial^\nu A^\mu - ig[A^\mu, A^\nu]$

- The Lagrangian is invariant under gauge transformations :

$$A^\mu(x) \to \Omega(x)A^\mu(x)\Omega^{-1}(x) + \frac{i}{g}\Omega(x)\partial^\mu\Omega^{-1}(x)$$

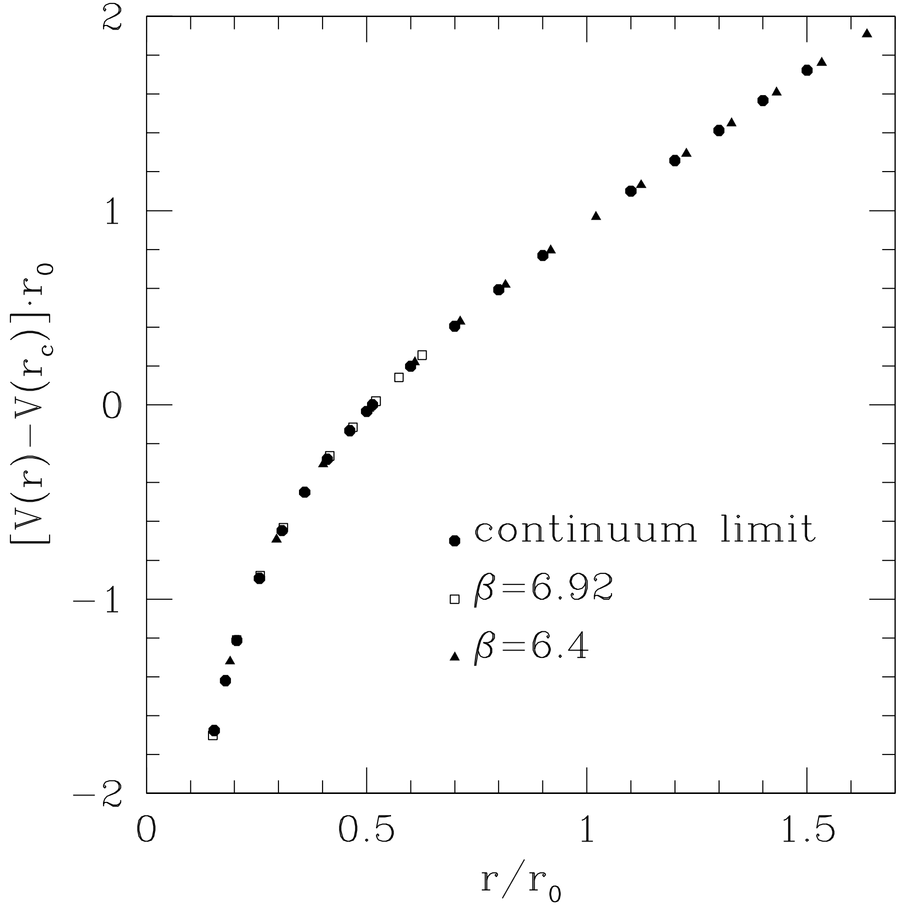
$$\psi(x) \to \Omega(x)\psi(x)$$

where $\Omega(x) \in SU(3)$

  - Note: the field strength is not invariant but transforms as :

$$F^{\mu\nu}(x) \to \Omega(x)F^{\mu\nu}(x)\Omega^{-1}(x)$$

Quark confinement

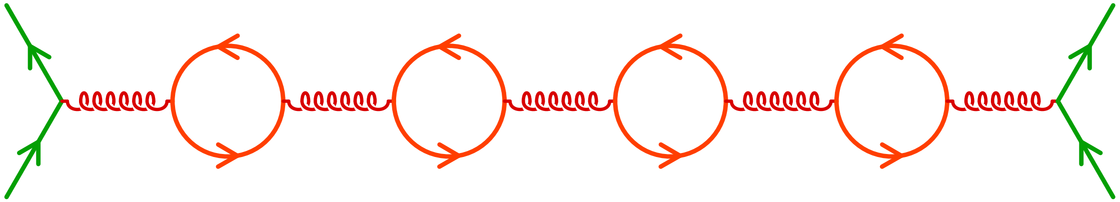


- The quark potential increases linearly with distance
- Quarks are confined into color singlet hadrons

Asymptotic freedom

- Running coupling : $\alpha_s = g^2/4\pi$

$$\alpha_s(r) = \frac{2\pi N_c}{(11N_c - 2N_f)\log(1/r\Lambda_{QCD})}$$



- The effective charge seen at large distance is screened by fermionic vacuum fluctuations (as in QED)

Asymptotic freedom

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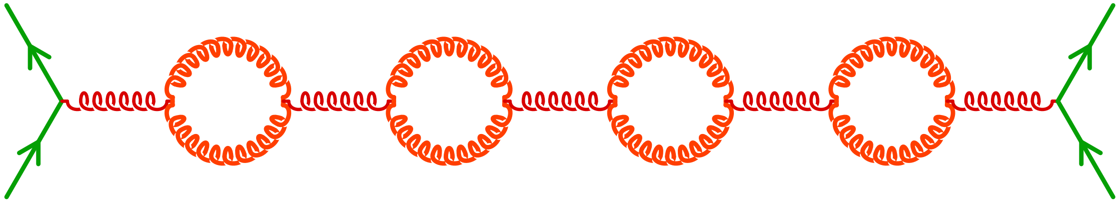
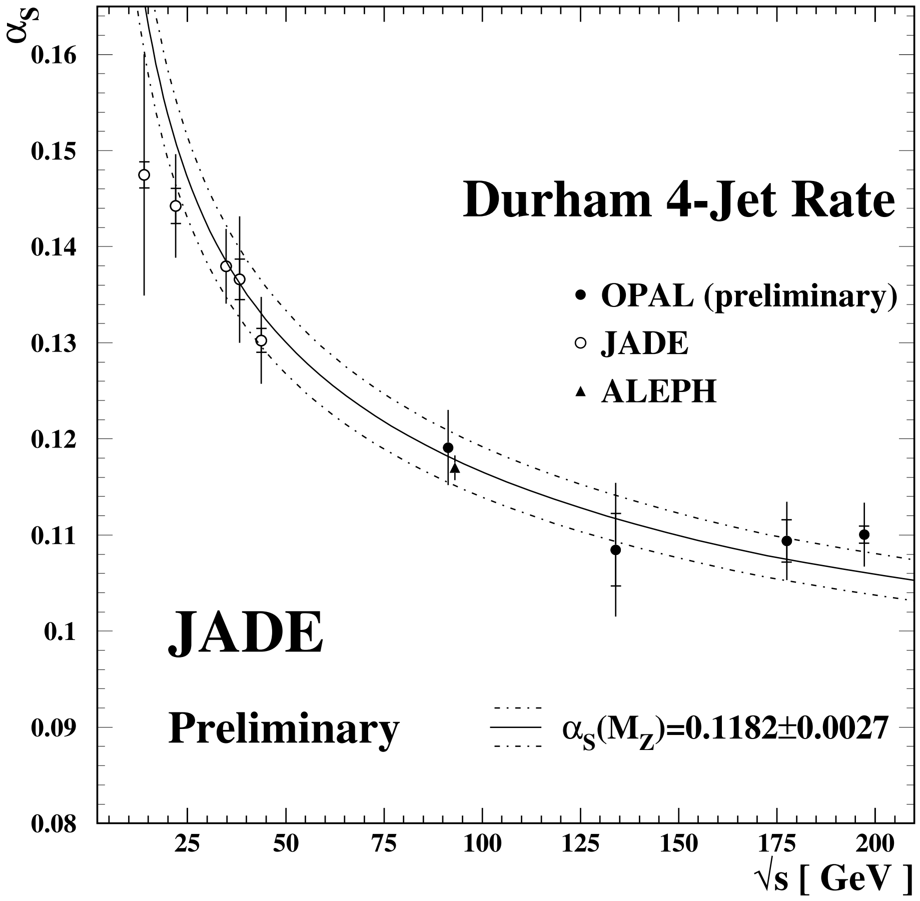


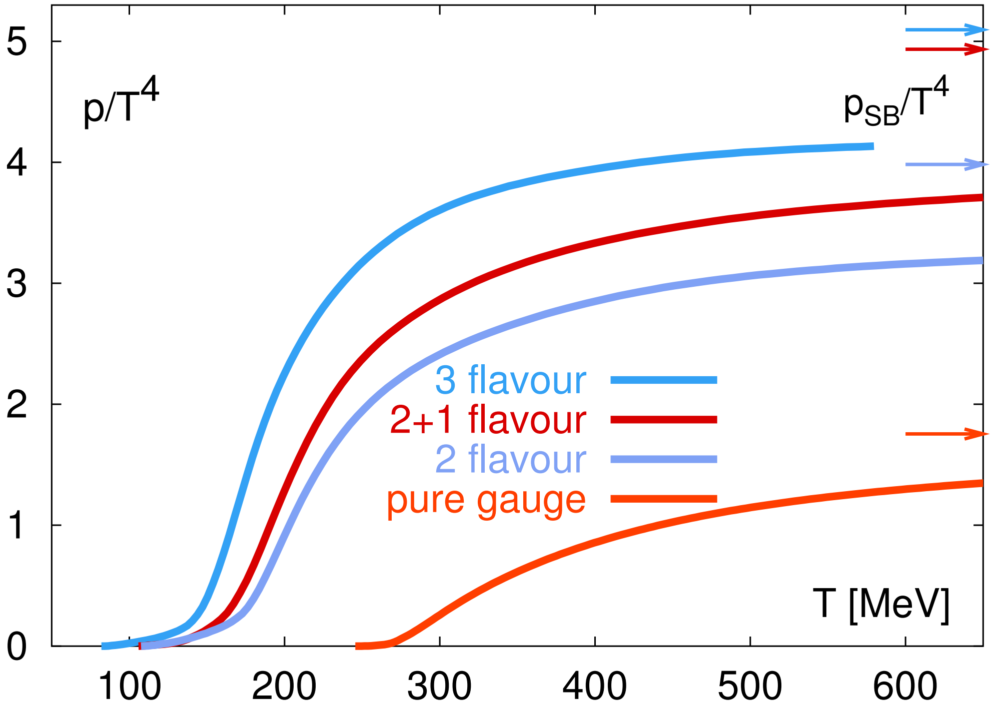
- The effective charge seen at large distance is screened by fermionic vacuum fluctuations (as in QED)
- But gluonic vacuum fluctuations produce an anti-screening (because of the non-abelian nature of their interactions)
- As long as $N_f < 11N_c/2 = 16.5$, the gluons win. In nature, there are 6 flavors of quarks

Asymptotic freedom



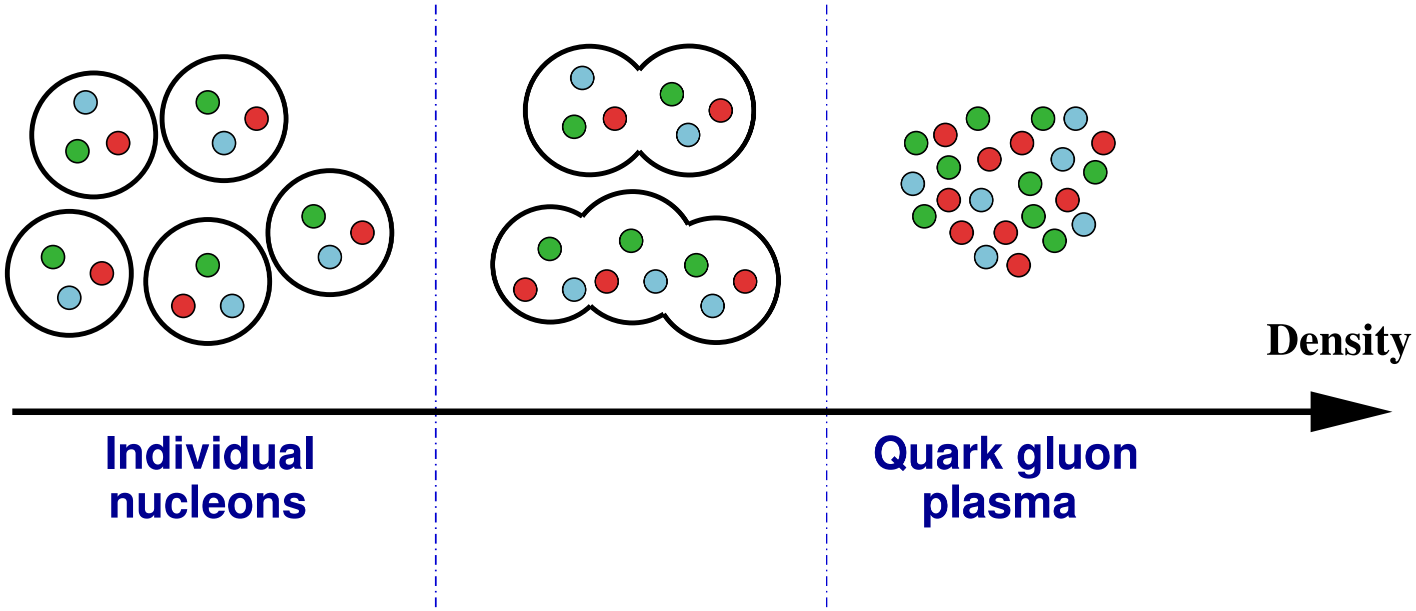
- The coupling constant is small at short distances
- At high density, a hadron gas may undergo deconfinement
  ▷ quark gluon “plasma”

Deconfinement



- Fast increase of the pressure :
  - at $T \sim 270$ MeV, if there are only gluons
  - at $T \sim 150\text{–}170$ MeV, depending on the number of light quarks

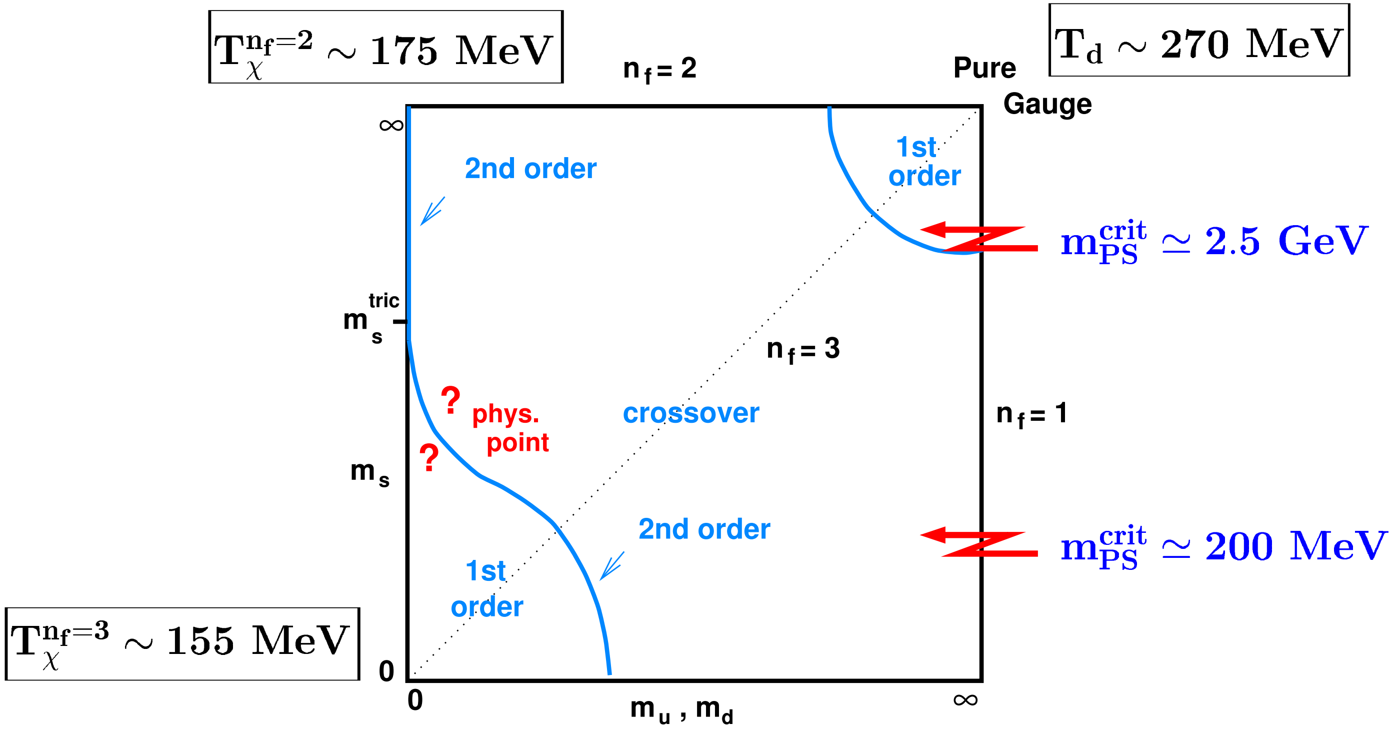
Deconfinement



- When the nucleon density increases, they merge, enabling quarks and gluons to hop freely from a nucleon to its neighbors
- This phenomenon extends to the whole volume when the phase transition ends
- Note: if the transition is first order, it goes through a mixed phase containing a mixture of nucleons and plasma

Deconfinement

3-flavour phase diagram


$T_\chi^{n_f=2} \sim 175$ MeV

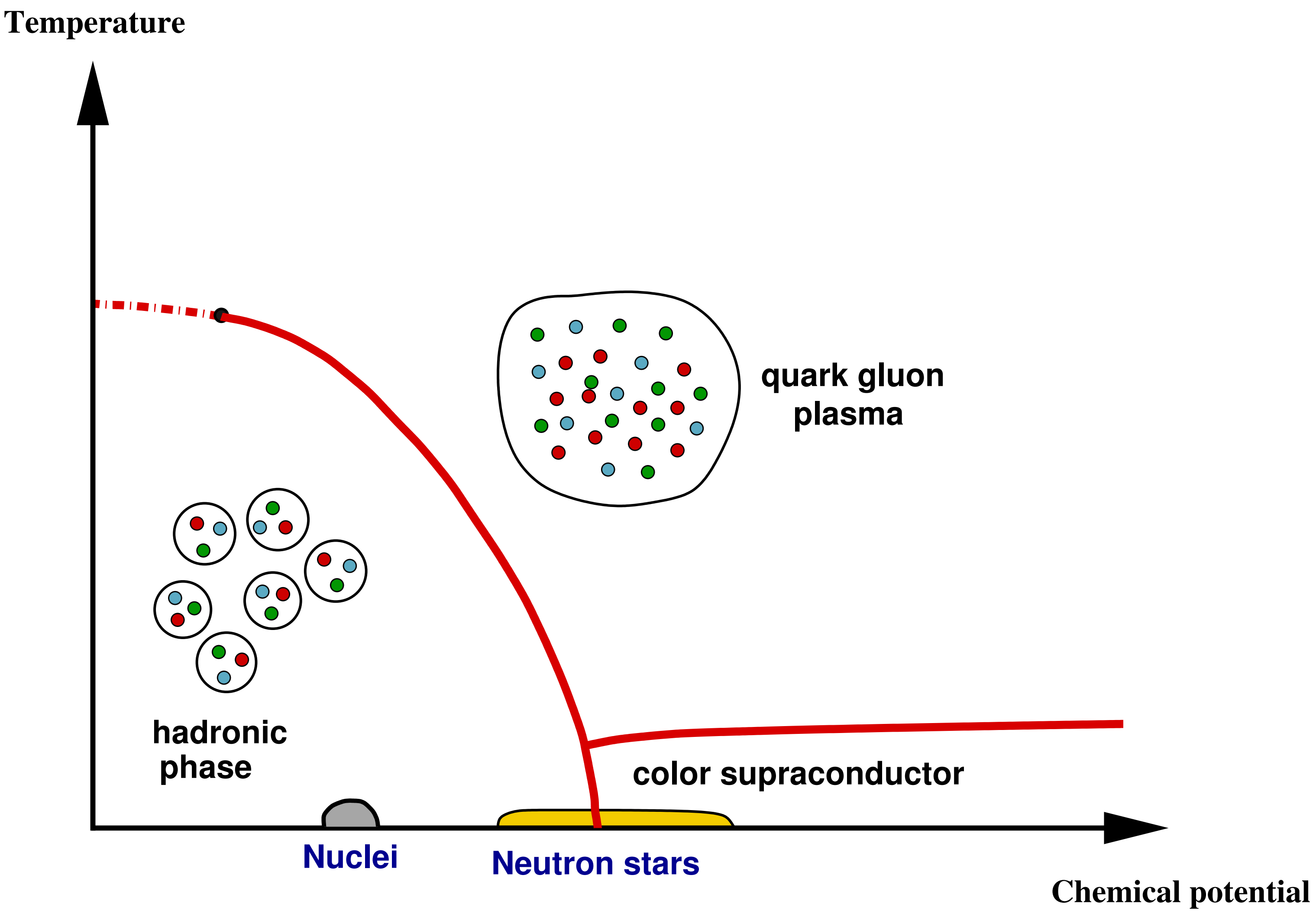
$T_d \sim 270$ MeV

$m_{PS}^{crit} \simeq 2.5$ GeV

$m_{PS}^{crit} \simeq 200$ MeV

$T_\chi^{n_f=3} \sim 155$ MeV

QCD phase diagram



Temperature

quark gluon plasma

hadronic phase

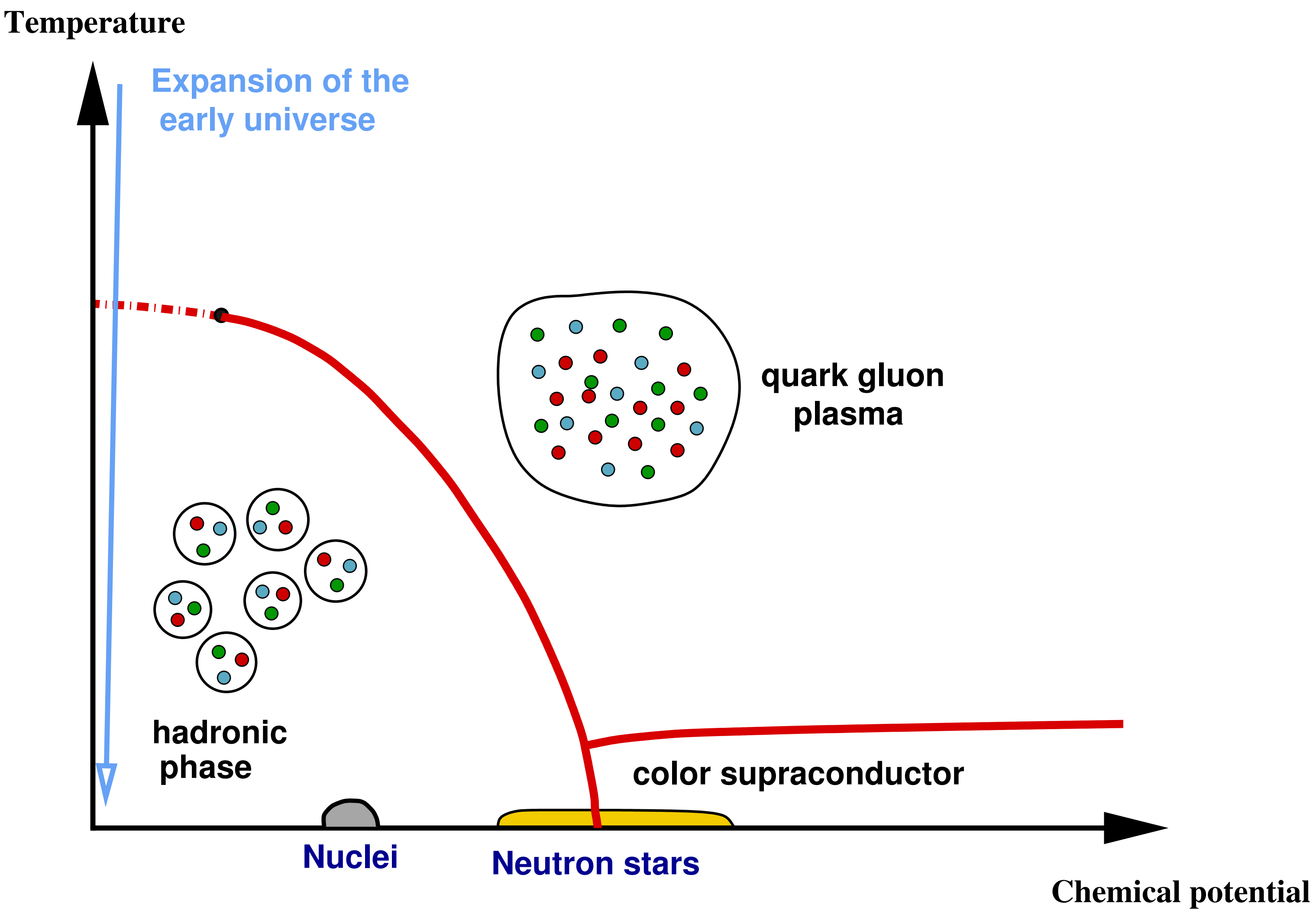
color supraconductor

Nuclei

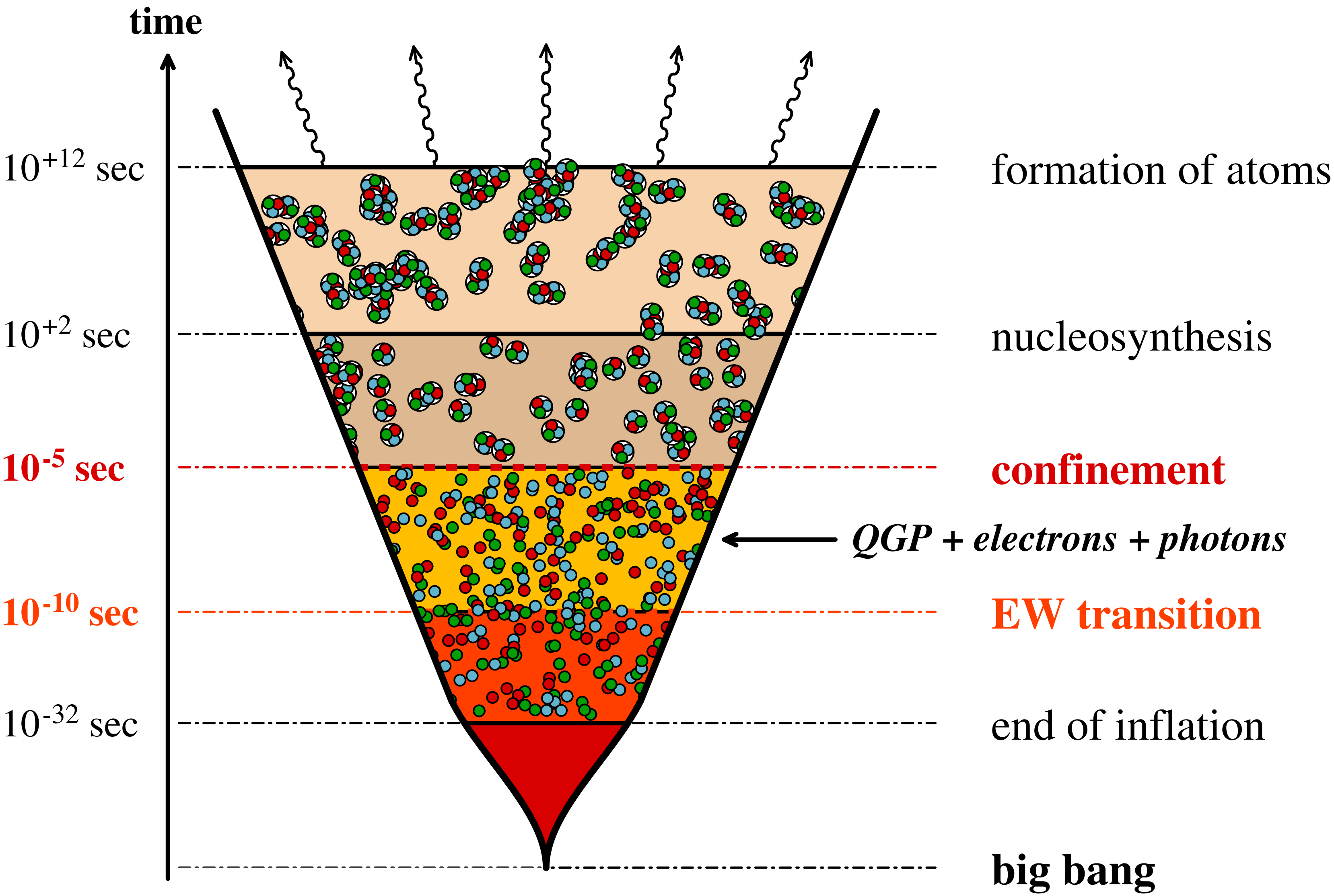
Neutron stars

Chemical potential

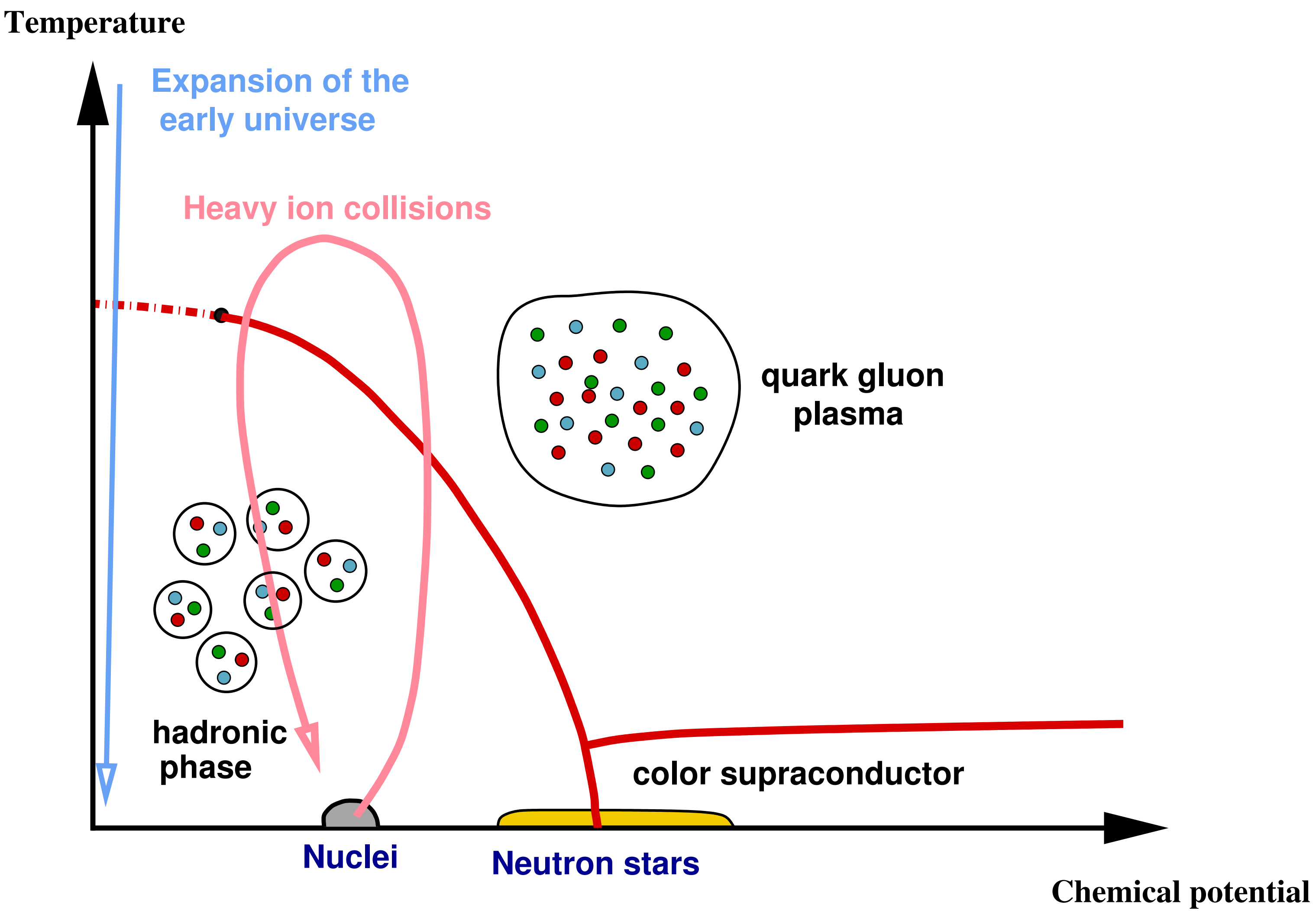
The QGP in the early universe



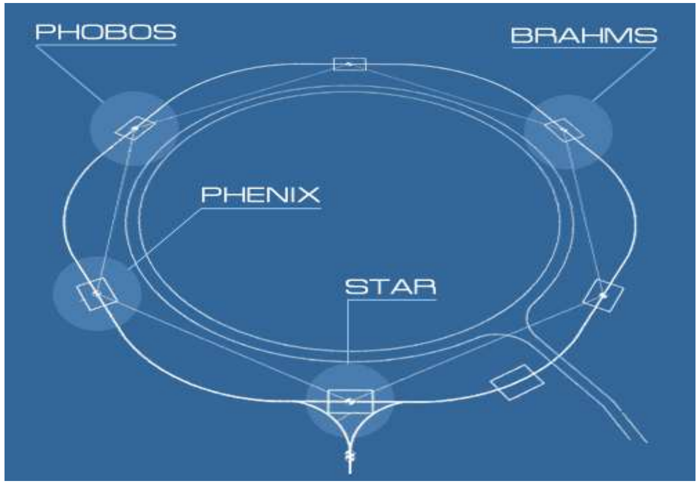
The QGP in the early universe



Heavy ion collisions



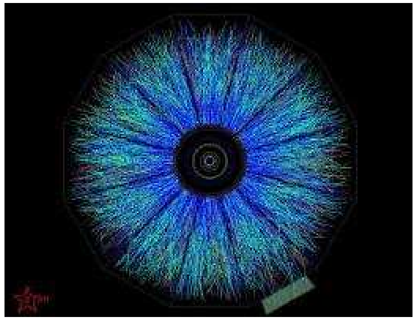
Since 2000 : RHIC



Since 2000 : RHIC

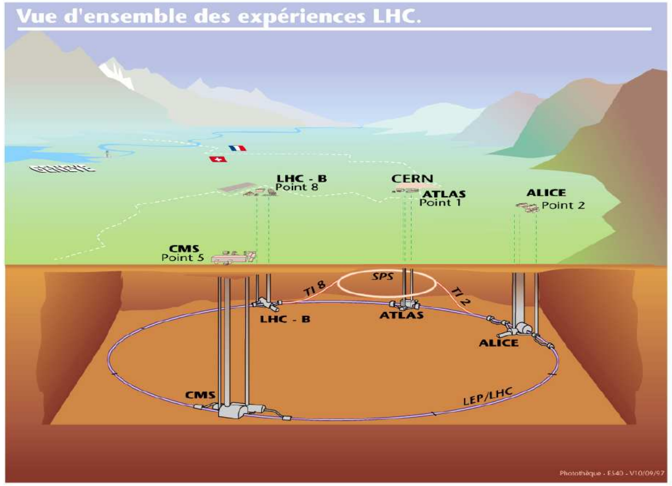
- Collision of two gold ions at RHIC :

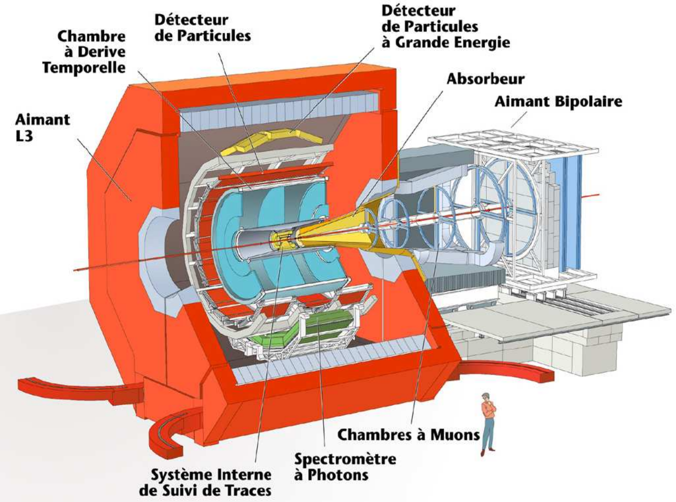
- Nucleus-nucleus collision event in the STAR detector :



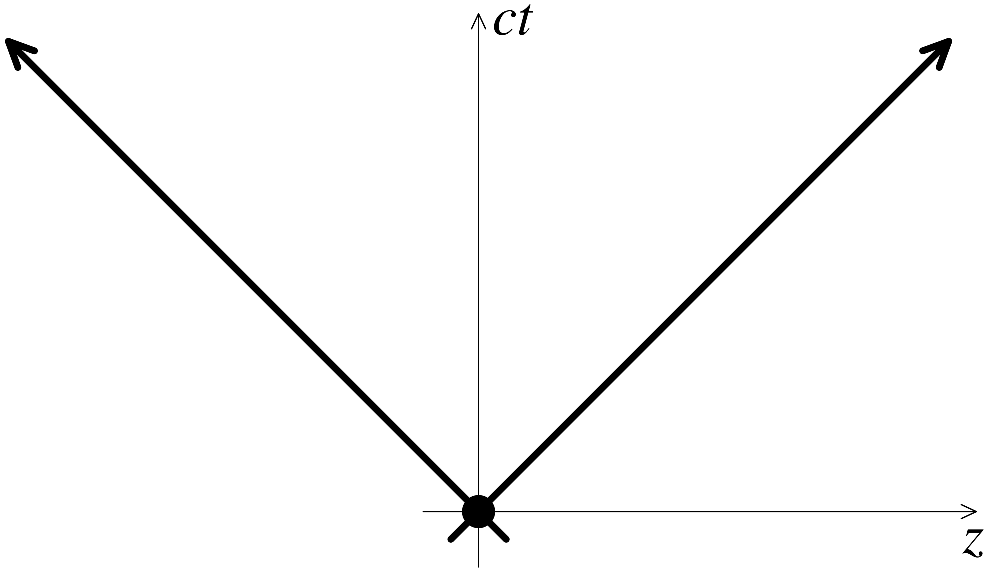
(animations courtesy of Brookhaven National Laboratory)

Starting in 2007 : LHC / ALICE



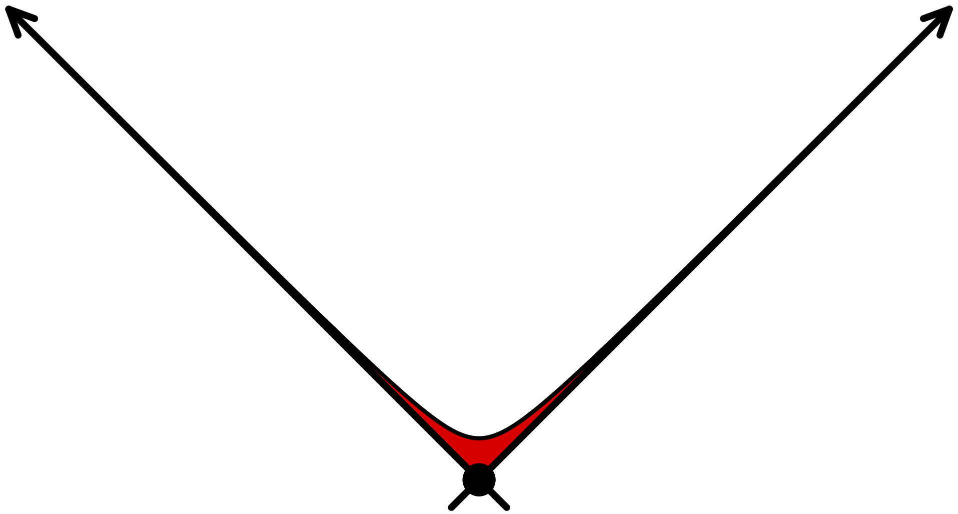


Heavy ion collisions



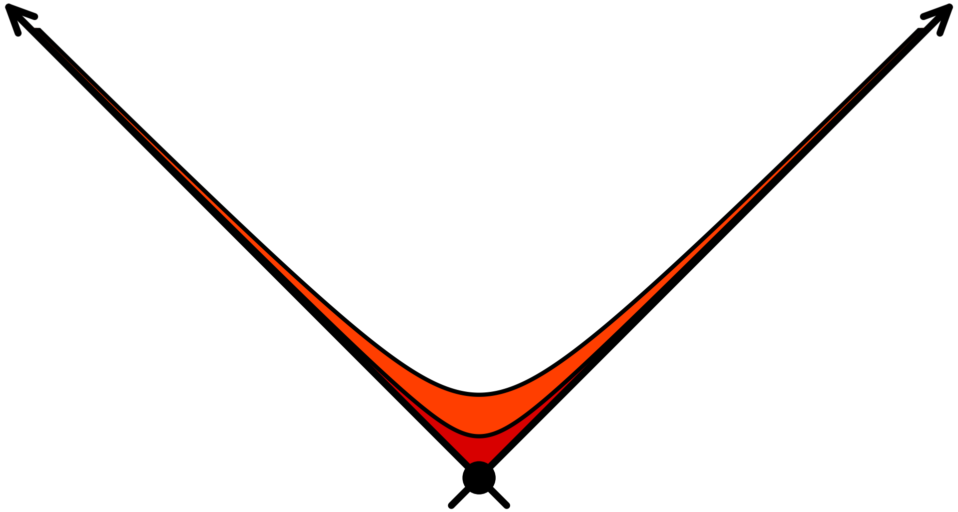
- $\tau \sim 0$ fm/c
- Production of hard particles :
  - jets
  - heavy quarks
  - direct photons
- calculable with the tools of perturbative QCD

Heavy ion collisions



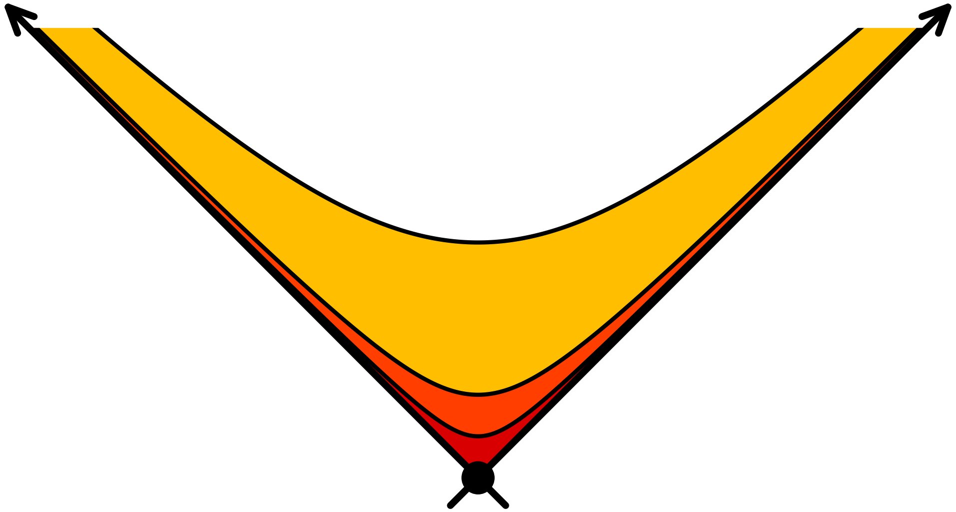
- $\tau \sim 0.2$ fm/c
- Production of semi-hard particles :
  - gluons, light quarks
- relatively small momentum : $p_\perp \lesssim 1$–2 GeV
- make up for most of the multiplicity
- sensitive to the physics of saturation (CGC)

Heavy ion collisions



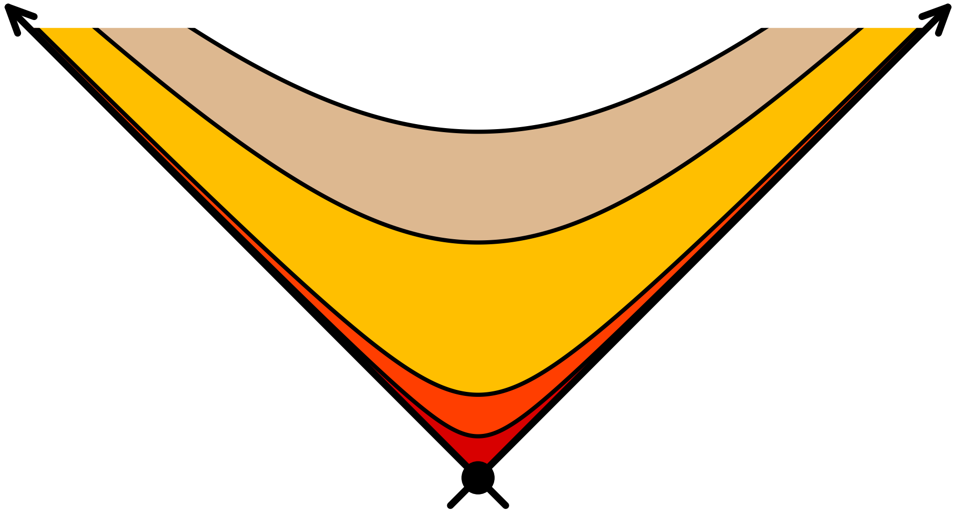
- $\tau \sim 1$–2 fm/c
- Thermalization
  - experiments suggest a fast thermalization
  - but this is still not understood from QCD

Heavy ion collisions



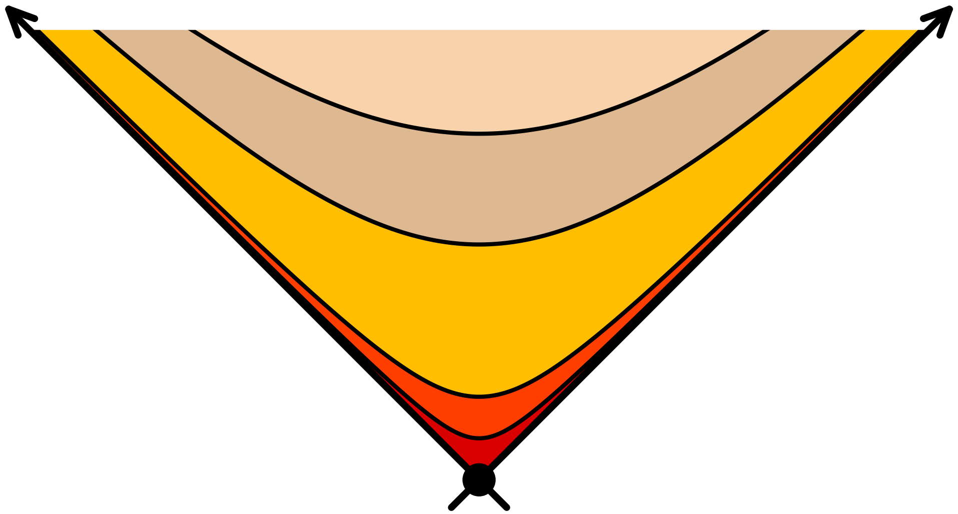
- $2 \leq \tau \lesssim 10$ fm/c
- Quark gluon plasma

Heavy ion collisions



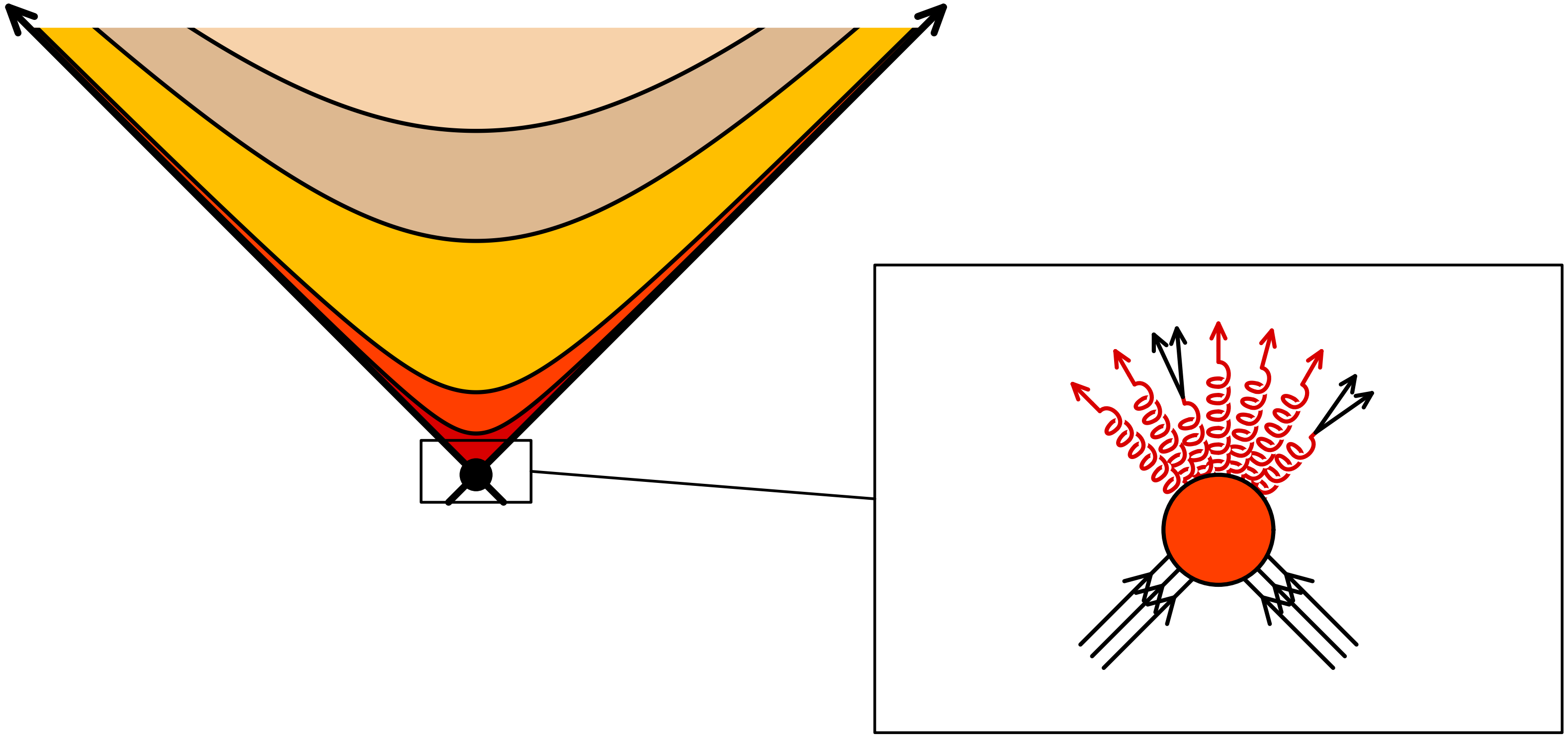
- $10 \lesssim \tau \lesssim 20$ fm/c
- Hot hadron gas

Heavy ion collisions



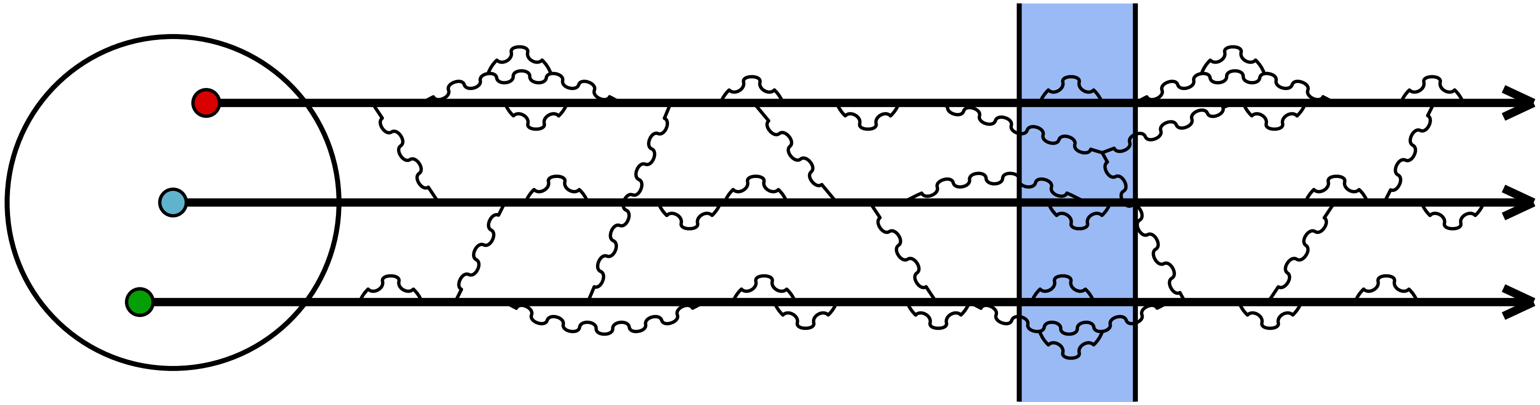
- $\tau \longrightarrow +\infty$
- Chemical freeze-out :
  density too small to have inelastic interactions
- Kinetic freeze-out :
  no more elastic interactions

Goals of these lectures



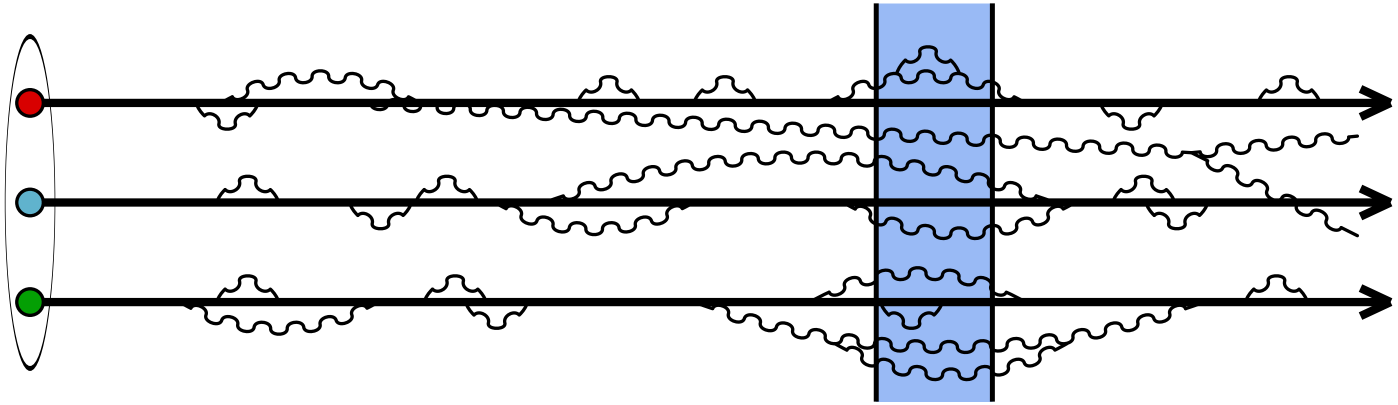
- describe the semi-hard content of nucleons and nuclei
- calculate the production of semi-hard particles

Nucleon at rest



- Very complicated non-perturbative object...
- Contains fluctuations at all space-time scales smaller than its own size
- Only the fluctuations that are longer lived than the external probe participate in the interaction process
- The only role of short lived fluctuations is to renormalize the masses and couplings
- Interactions are very complicated if the constituents of the nucleon have a non trivial dynamics over time-scales comparable to those of the probe

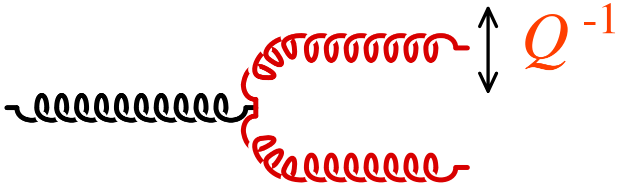
Nucleon at high energy



- Dilation of all internal time-scales of the nucleon
- Interactions among constituents now take place over time-scales that are longer than the characteristic time-scale of the probe
  ▷ the constituents behave as if they were free
- Many fluctuations live long enough to be seen by the probe. The nucleon appears denser at high energy (it contains more gluons)
- Pre-existing fluctuations are totally frozen over the time-scale of the probe, and act as static sources of new partons

Parton model

- At the time of the interaction, the nucleon can be seen as a collection of free constituents, called partons
- The nucleon content is described by parton distributions, that depend on the momentum fraction x of the parton
- One needs only to calculate the cross-section between the probe and the partons. If the parton density is low, only one parton interacts
- One can separate the hard diffusion, perturbative, from the non-perturbative parton distributions, because the strong interactions responsible for these non-perturbative effects act on much longer time-scales ("factorization")
- Note: parton distributions also depend on a "transverse resolution scale", Q :



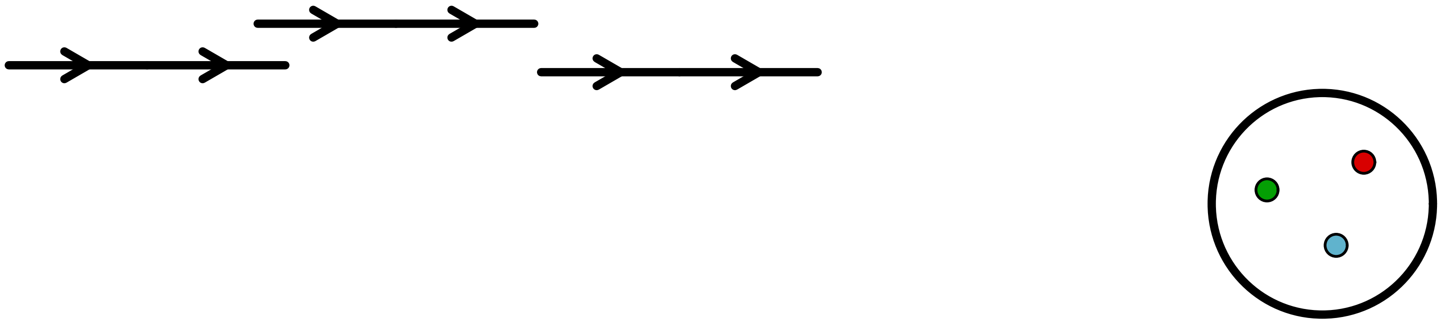
Lecture II : Deep Inelastic Scattering

- Experimental facts :
  - $\sigma_L^{\gamma^* p}$ is much smaller than $\sigma_{\rm total}^{\gamma^* p}$
  - $Q^2\sigma_{\rm total}^{\gamma^* p}$ has a very weak dependence on Q^2 (Bjorken scaling)
- Naive parton model : these experimental observations can be reproduced in a model which assumes that the proton is made of free point-like fermionic constituents
- Operator Product Expansion in a free field theory : using the OPE, one can show that these results emerge naturally in a field theory of free fermions. The “parton distributions” appear as expectation values of some operators in the state of the proton
- Scaling violations from the OPE : turning on the interactions, and computing the Renormalization Group corrections to the OPE, we can predict from QCD the deviations from Bjorken scaling
- Factorization : qualitative discussion of why the non-perturbative physics can be factored out in universal parton distributions in the study of reactions involving hadrons

Lecture III : QCD on the light-cone

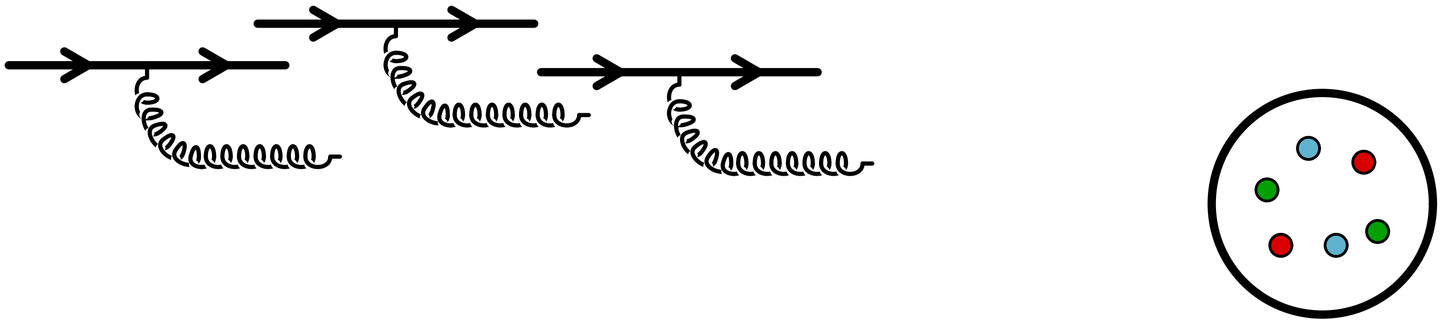
- **Light-cone coordinates :** light-cone coordinates make the kinematics of high-energy scattering more transparent. We will also see that the Poincaré algebra exhibits a Galilean sub-algebra when expressed in terms of these coordinates
- **Quantum field theory on the light-cone :** these ideas are formalized by introducing “light-cone”, or “equal-x^+”, quantization. This is first done for a scalar field theory, and then for QCD
- **Eikonal approximation :** the scattering of particles in an external potential takes a particularly simple form in the high-energy limit. We prove the “eikonal” approximation using the tools of light-cone quantum field theory. Factorization between long distance (wave function) and short distance physics (hard scattering), i.e. the parton model, becomes manisfest in this language

Parton saturation



▷ at low energy, only valence quarks are present in the hadron wave function

Parton saturation

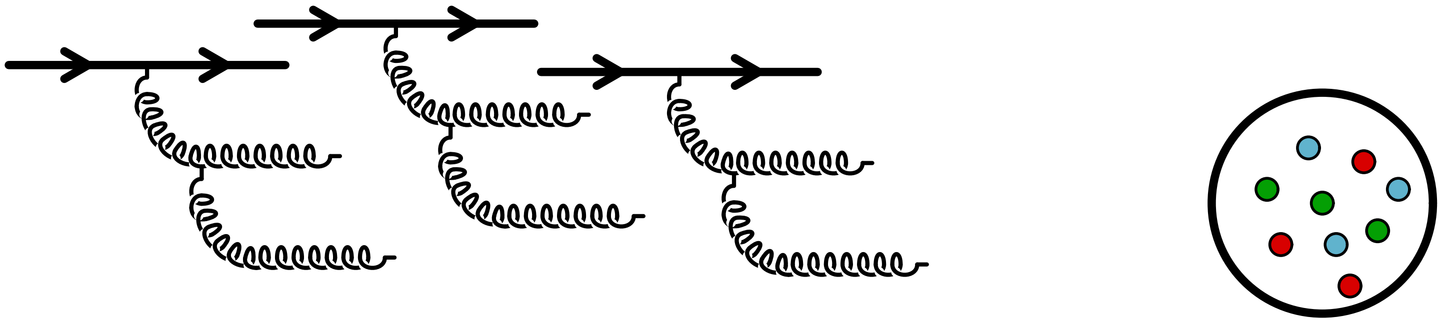


▷ when energy increases, new partons are emitted

▷ the emission probability is $\alpha_s \int \frac{dx}{x} \sim \alpha_s \ln(\frac{1}{x})$, with x the longitudinal momentum fraction of the gluon

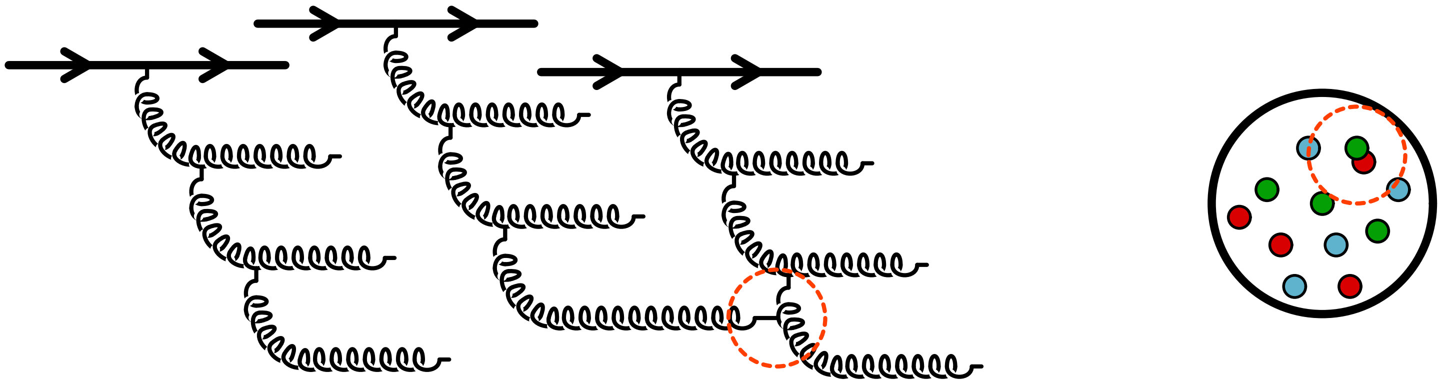
▷ at small-x (i.e. high energy), these logs need to be resummed

Parton saturation



▷ as long as the density of constituents remains small, the evolution is linear: the number of partons produced at a given step is proportional to the number of partons at the previous step (BFKL)

Parton saturation



▷ eventually, the partons start overlapping in phase-space

▷ parton recombination becomes favorable

▷ after this point, the evolution is non-linear:
the number of partons created at a given step depends non-linearly on the number of partons present previously

Saturation criterion

Gribov, Levin, Ryskin (1983)

- Number of gluons per unit area:

$$\rho \sim \frac{xG(x,Q^2)}{\pi R^2}$$

- Recombination cross-section:

$$\sigma_{gg\to g} \sim \frac{\alpha_s}{Q^2}$$

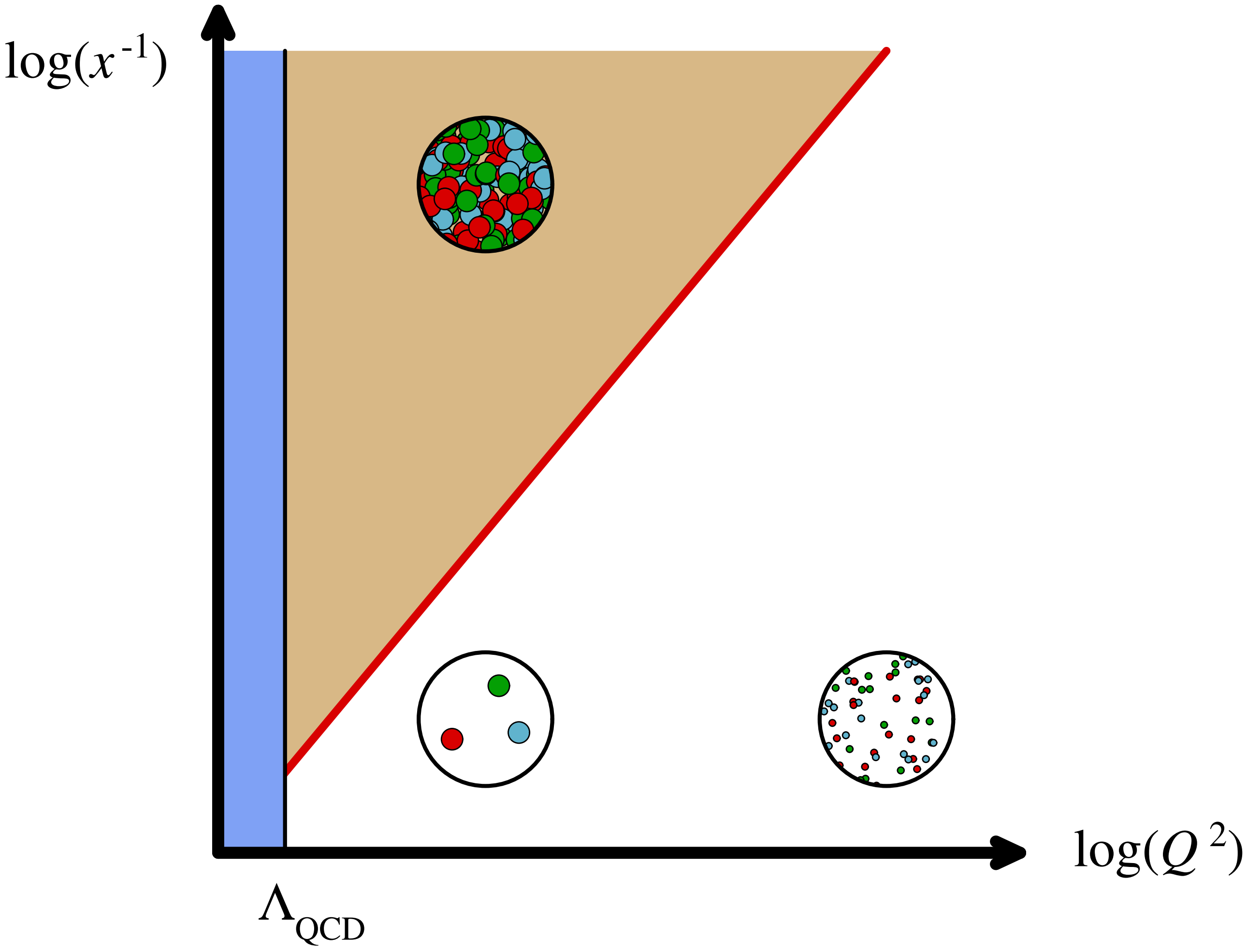
- Recombination happens if $\rho\sigma_{gg\to g} \gtrsim 1$, i.e. $Q^2 \lesssim Q_s^2$, with:

$$Q_s^2 \sim \frac{\alpha_s xG(x,Q_s^2)}{\pi R^2} \qquad \sim A^{1/3}\frac{1}{x^{0.3}}$$

- At saturation, the phase-space density is:

$$\frac{dN_g}{d^2\vec{\boldsymbol{x}}_\perp d^2\vec{\boldsymbol{p}}_\perp} \sim \frac{\rho}{Q^2} \sim \frac{1}{\alpha_s}$$

Saturation domain



- Boundary defined by $Q^2 = Q_s^2(x)$

Degrees of freedom

McLerran, Venugopalan (1994)
Iancu, Leonidov, McLerran (2001)

- Small x modes have a large occupation number
  - ▷ they can be described by a classical color field A^μ
- Large x modes, slowed down by time dilation, are described as static color sources ρ
- The classical field obeys Yang-Mills equations :

$$D_\nu F^{\nu\mu} = J^\mu = \delta^{\mu+}\delta(x^-)\rho(\vec{\boldsymbol{x}}_\perp)$$

- The color sources ρ are random, and described by a statistical distribution $W_{x_0}[\rho]$, where x_0 is the separation between “small x” and “large x”
- An evolution equation (JIMWLK) controls the changes of $W_{x_0}[\rho]$ with x_0 (generalizes BFKL to the saturated regime)

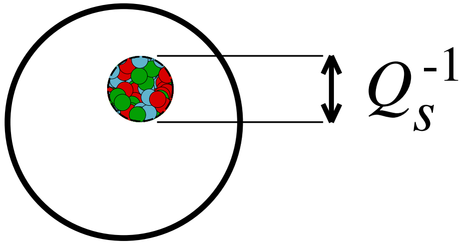
A brief lesson of semantics...

McLerran (mid 2000)

- Color : more or less obvious...
- Glass : the system has degrees of freedom whose time-scale is much larger than the typical time-scales for interaction processes. Moreover, these degrees of freedom are stochastic variables, like in “spin glasses” for instance
- Condensate : the soft degrees of freedom are as densely packed as they can (the density remains finite, of order α_s^{-1}, due to repulsive interactions between gluons)

Correlation length

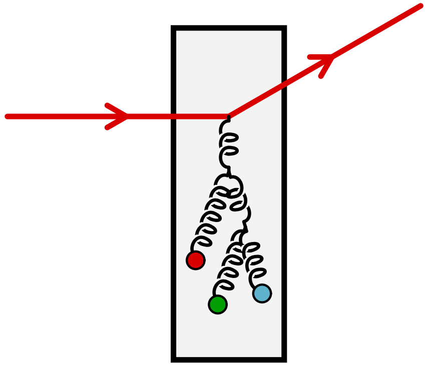
- In a nucleon at low energy, the typical correlation length among color charges is of the order of the nucleon size, i.e. $\Lambda_{QCD}^{-1} \sim 1\ \mathrm{fm}$. Indeed, at low energy, color screening is due to confinement, controlled by the non-perturbative scale Λ_{QCD}
- At high energy (small x), partons are much more densely packed, and it can be shown that color neutralization occurs in fact over distances of the order of $Q_s^{-1} \ll \Lambda_{QCD}^{-1}$



- This implies that all hadrons and nuclei behave in the same way at high energy. In this sense, the small x regime described by the CGC is universal

Leading twist shadowing

- Interactions between the partons of the target :



- At small x, the wave function of a parton “spreads” outside of the nucleon it belongs to, so that it can interact with partons from other nucleons. This implies :

$$xG_{\rm nucleus}(x,Q^2) \quad < \quad A\; xG_{\rm nucleon}(x,Q^2)$$

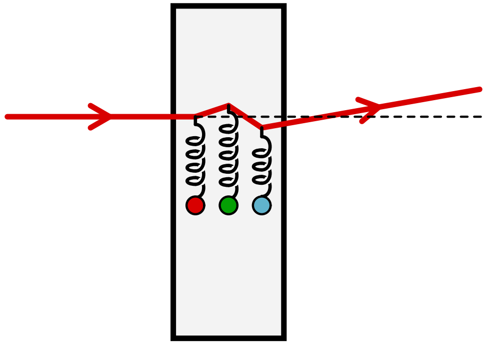
- At small x, one has a suppression of nuclear cross-sections :

$$d\sigma_{pA}/d^2\vec{\boldsymbol{p}}_\perp \sim A^\alpha \quad \text{with} \quad \alpha < 1$$

- Note: these interactions are the same as those involved in the phenomenon of saturation

Multiple scatterings

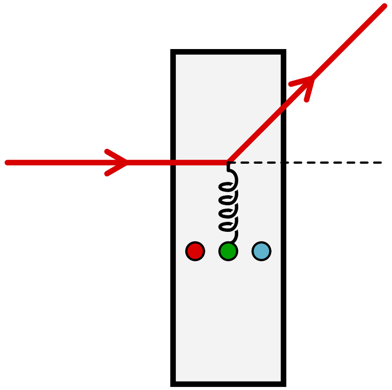
- Because of the large parton density at small x in the target, the external probe can interact several times :



- One of the scatterings "produces" the final state, and the others merely change its momentum ("higher twist" shadowing)
- Each additional scattering brings a correction $\alpha_s A^{1/3}\mu^2/p_\perp^2$
  ▷ important effect at small $p_\perp$, despite the α_s suppression
- At leading order, multiple scattering only affect the momentum distribution of the final particles, but not their total number. The suppression at small $p_\perp$ is compensated by an increase at larger $p_\perp$ (Cronin effect)

Multiple scatterings

- At high $p_\perp$, a single scattering dominates :



  - Standard result for a random walk in an external potential, when the potential does not decrease fast at large momentum ("intermittency")
  - Differential cross-sections scale like the atomic number A at high $p_\perp$

Lecture IV : Saturation and CGC

- **BFKL equation :** evolution with x of parton distributions, in the linear regime
- **McLerran-Venugopalan model :** a model in which the degrees of freedom are separated in fields (small-x partons) and color sources (large-x partons). This model assumes a fixed, gaussian, distribution of the color sources
- **CGC and non-linear evolution :** from renormalization group arguments, one derives the non-linear evolution equation for the distribution of color sources. A mean field approximation (Balitsky-Kovchegov) of this equation is also discussed
- **Analogies with reaction-diffusion processes :** some analogies exist between high-energy scattering in QCD and the physics of reaction-diffusion processes. In particular, the BK equation can be remapped into the FKPP equation
- **Pomeron loops :** a (non-exhaustive) presentation of very recent developments, related to “Pomeron loops”

Lecture V : Calculating observables

- QFT in an external field : in practical applications, the CGC can be seen as a field theory coupled to a strong external source. We first discuss general properties, including the diagrammatic expansion of the solution of the classical equation of motion
- Calculation of multiplicities : when the external source is strong, many particles are produced in each collision. We derive here techniques to calculate the average number of produced particles
- Less inclusive quantities : we show that the distribution of produced particles is non-Poissonian, even at the classical level. We explain why average multiplicities are relatively easy to calculate, compared to the probabilities of producing a fixed number of particles. We show how to calculate higher moments
- Numerical methods : in the case of collisions between two high-energy projectiles both described by strong external sources, one cannot go very far analytically. We explain how to set up the problem in order to solve it numerically

Lecture II : Lessons from DIS

- Kinematics of Deep Inelastic Scattering
- Structure functions
- Experimental facts
- Naive parton model
- Light-cone behavior of a free field theory
- Scaling violations
- Factorization

Lecture III : QCD on the light-cone

- Light-cone coordinates - Infinite Momentum Frame
- Poincaré algebra on the light-cone - Galilean sub-algebra
- Canonical quantization on the light-cone
- Scattering by an external potential
- Light-cone QCD

Lecture IV : Saturation and CGC

- BFKL equation
- Saturation of parton distributions
- Balitsky-Kovchegov equation
- Color Glass Condensate - JIMWLK
- Analogies with reaction-diffusion processes
- Pomeron loops

Lecture V : Calculating observables

- Field theory coupled to time-dependent sources
- Generating function for the probabilities
- Average particle multiplicity
- Numerical methods for nucleus-nucleus collisions
    - Gluon production
    - Quark production