Testing fluid dynamics with the relationship between v_2 and v_4

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Testing fluid dynamics with the relationship between v_2 and v_4

v_2, v_4: different anisotropic flow harmonics...

- which can be computed within ideal fluid dynamics
  - analytically: difference between slow and fast particles;
  - numerically;

- whose behavior in a non-equilibrium scenario can be qualitatively predicted;

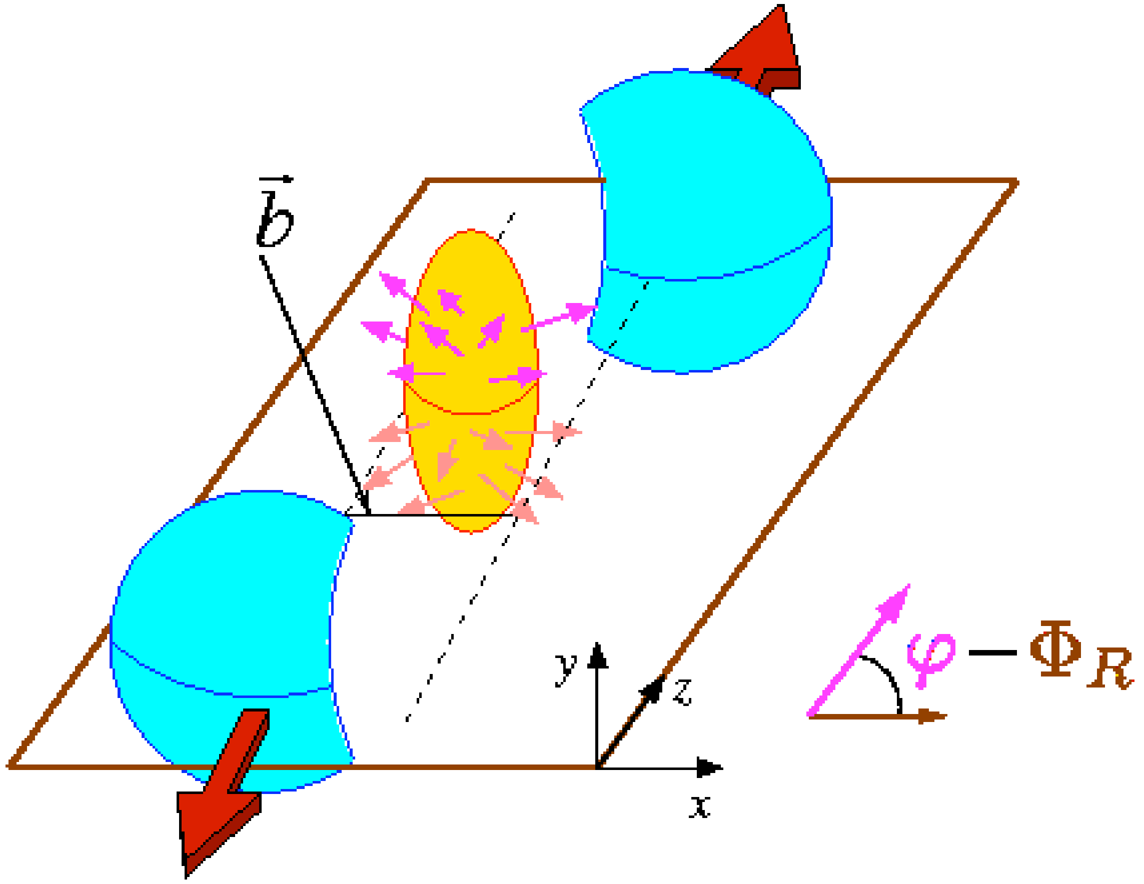
and which can be measured, testing models.

N.Borghini & J.-Y.Ollitrault, Phys. Lett. B **642** (2006) 227

R.S.Bhalerao, J.-P.Blaizot, N.B., J.-Y.O., Phys. Lett. B **627** (2005) 49

Anisotropic (collective) flow

Consider a non-central collision:



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⇒ anisotropic pressure gradients (larger along the impact parameter)

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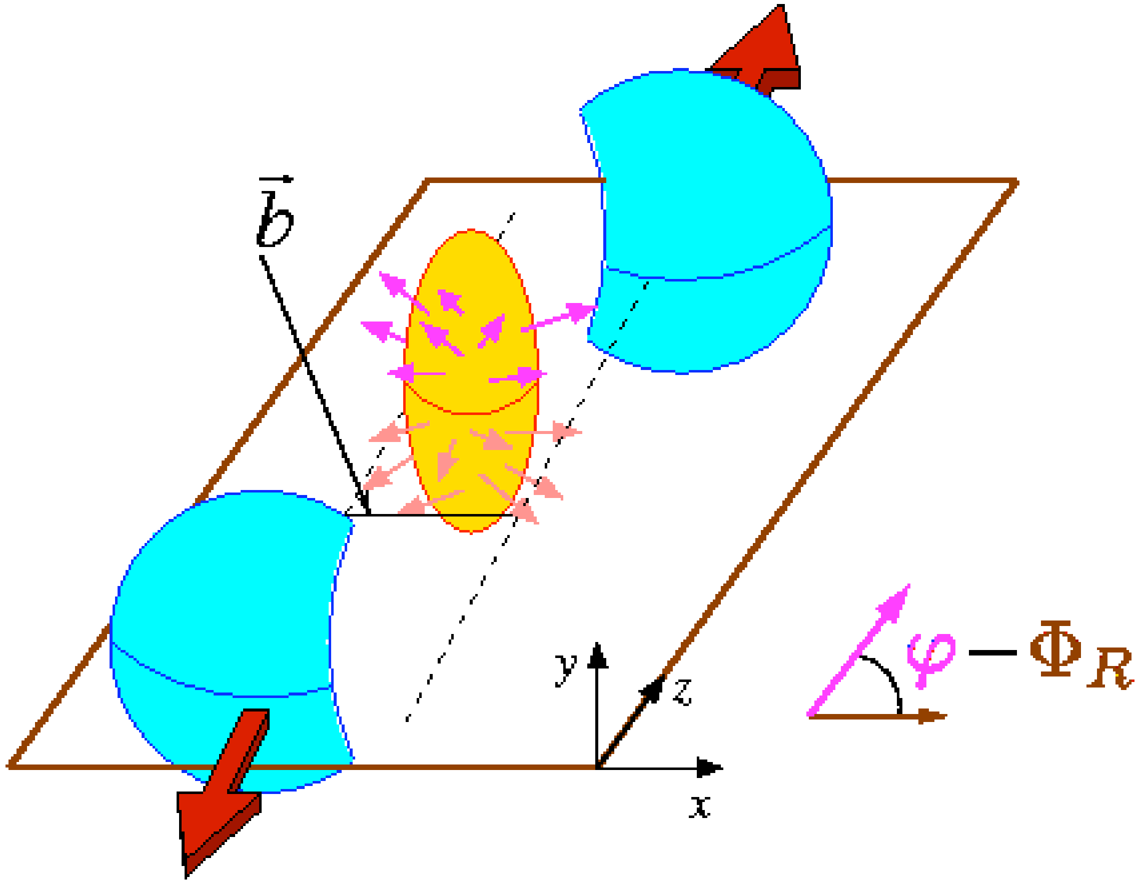

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"anisotropic collective flow"

$$E\frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{p}} \propto \frac{\mathrm{d}N}{p_T\,\mathrm{d}p_T\,\mathrm{d}y}\left[1+2\,v_1\cos\left(\varphi-\Phi_R\right)+2\,v_2\cos 2(\varphi-\Phi_R)+\cdots\right]$$

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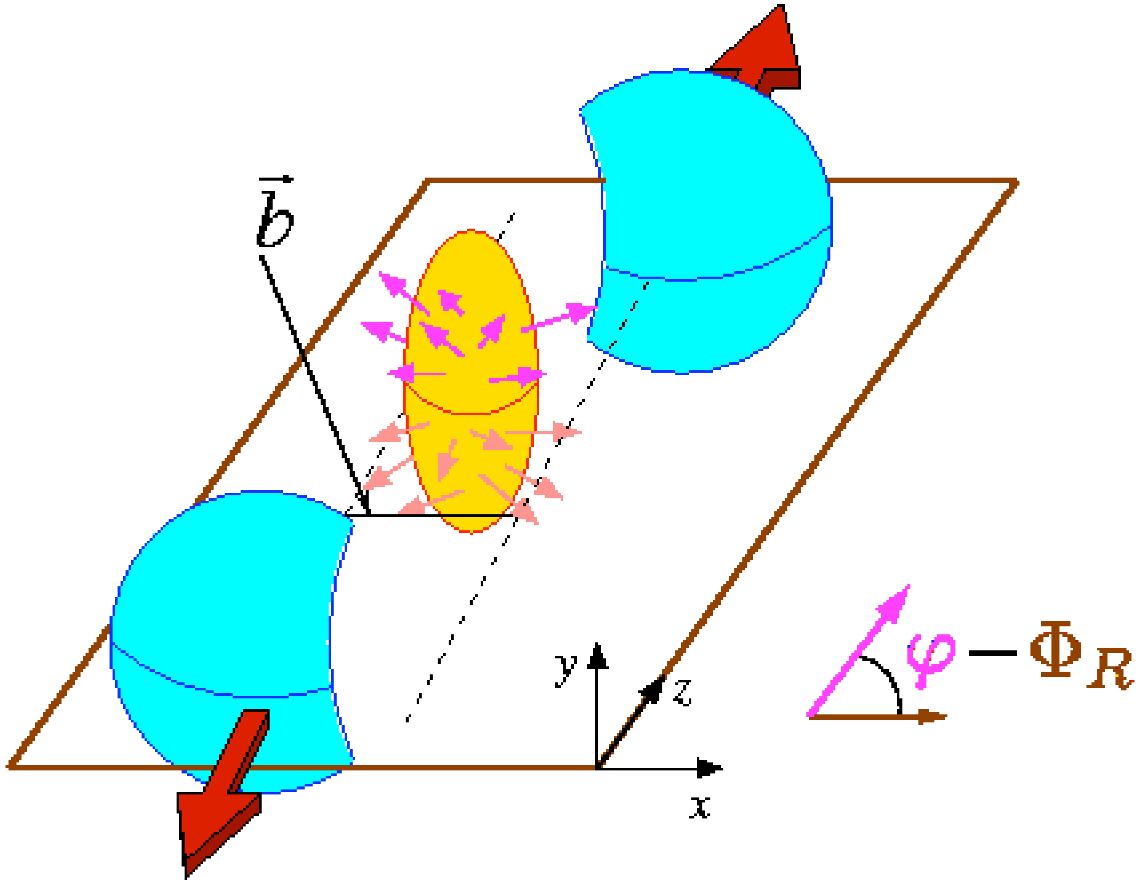

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More particles along the impact parameter ($\varphi - \Phi_R = 0$ or 180°) than perpendicular to it ☞ "elliptic flow" $v_2 \equiv \langle \cos 2(\varphi - \Phi_R) \rangle > 0$.

average over particles

Anisotropic (collective) flow

In the collision of identical nuclei (Au-Au, Cu-Cu, Pb-Pb...), the odd Fourier harmonics vanish at midrapidity by symmetry:

☞ $$E\frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{p}} \propto \frac{\mathrm{d}N}{p_T\,\mathrm{d}p_T\,\mathrm{d}y}\left[1+2\,v_2\cos 2(\varphi-\Phi_R)+2\,v_4\cos 4(\varphi-\Phi_R)+\cdots\right]$$

RHIC has confirmed that we should not care too much about v_6, v_8...

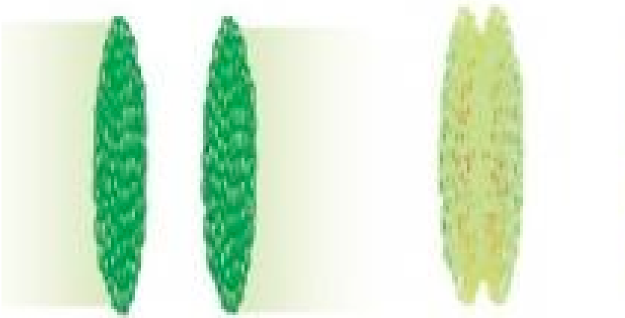
However, v_4 measurements are now common!

To be of any relevance for phenomenology, any computation of elliptic flow v_2 should be accompanied by a simultaneous computation of v_4!

This requires zero additional computing time, since both can be expressed in terms of the same quantities:

$$v_2 = \frac{\langle p_x^2 - p_y^2\rangle}{\langle p_x^2 + p_y^2\rangle} \qquad v_4 = \frac{\langle\, p_x^4 - 6p_x^2p_y^2 + p_y^4\,\rangle}{\langle\, p_x^4 + 2p_x^2p_y^2 + p_y^4\,\rangle}$$

Heavy-ion collisions: fluid-dynamics description



stolen from Steffen Bass

⓪ Creation of a dense “collection” of particles.

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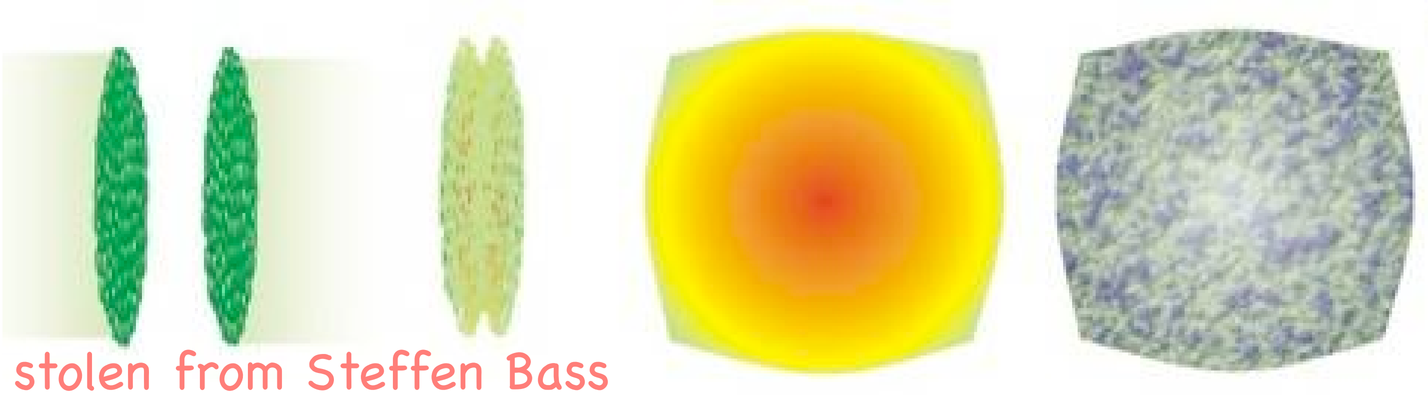

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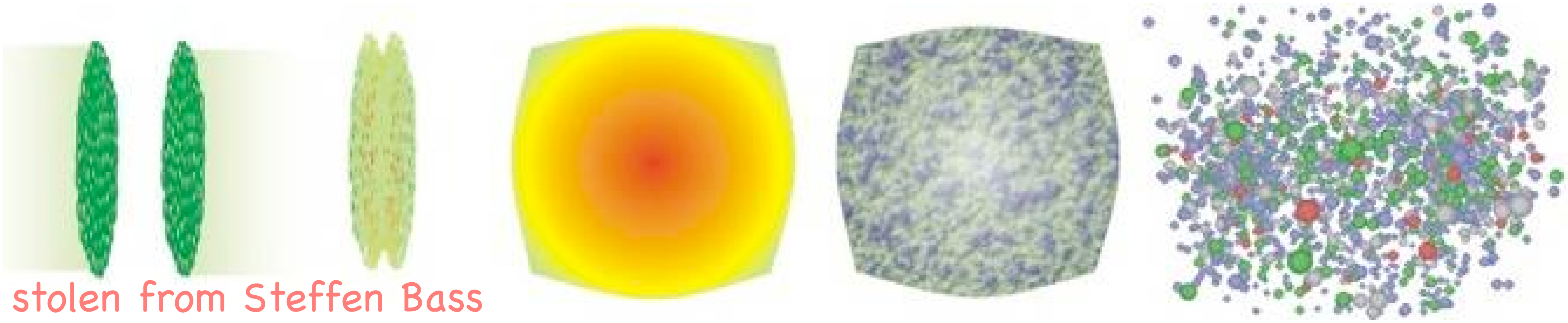

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② The fluid expands: density decreases, λ increases (system size also).

③ At some time, the mean free path is of the same order as the system size: fluid dynamics is no longer a valid description:

(kinetic) "freeze-out"

usually parameterized in terms of a temperature $T_{\rm f.o.}$.

Heavy-ion collisions: fluid-dynamics description

At freeze-out, each fluid cell emits particles according to thermal distributions (Bose–Einstein, Fermi–Dirac):

$$E\frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{p}} = C\int_\Sigma \exp\left(-\frac{p^\mu u_\mu(x)}{T_{\mathrm{f.o.}}}\right) p^\mu\,\mathrm{d}\sigma_\mu$$

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A consistent ideal-hydrodynamics picture requires that $T_{\rm f.o.} \ll T_{\rm in.}$

$$\Leftrightarrow$$

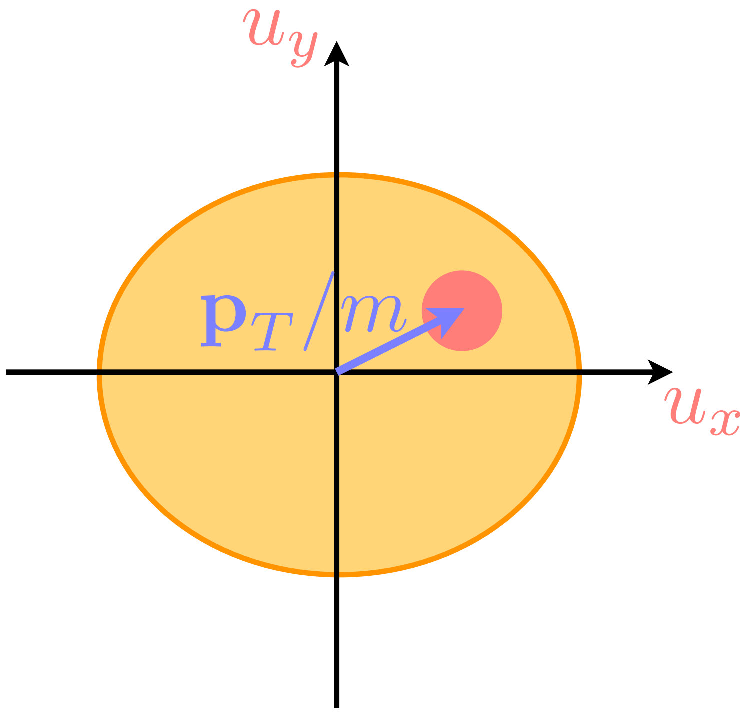
ideal-fluid limit = small-$T_{\rm f.o.}$ limit

☞ one can compute the particle distribution in a model-independent, analytic way (using a saddle-point approximation).

N.Borghini, J.-Y.Ollitrault PLB **642** (2006) 227

Ideal fluid-dynamics: analytical results

Slow particles $(p_T/m < u_{\max}(\frac{\pi}{2}))$ move together with the fluid.



There exists a fluid cell whose velocity equals the particle velocity: minimizes $p^\mu u_\mu$.

☞ Integrand in the Cooper-Frye spectrum is Gaussian, with width ∝ $1/\min(\sqrt{p^\mu u_\mu}) = 1/\sqrt{m}$.

→ saddle-point approximation!

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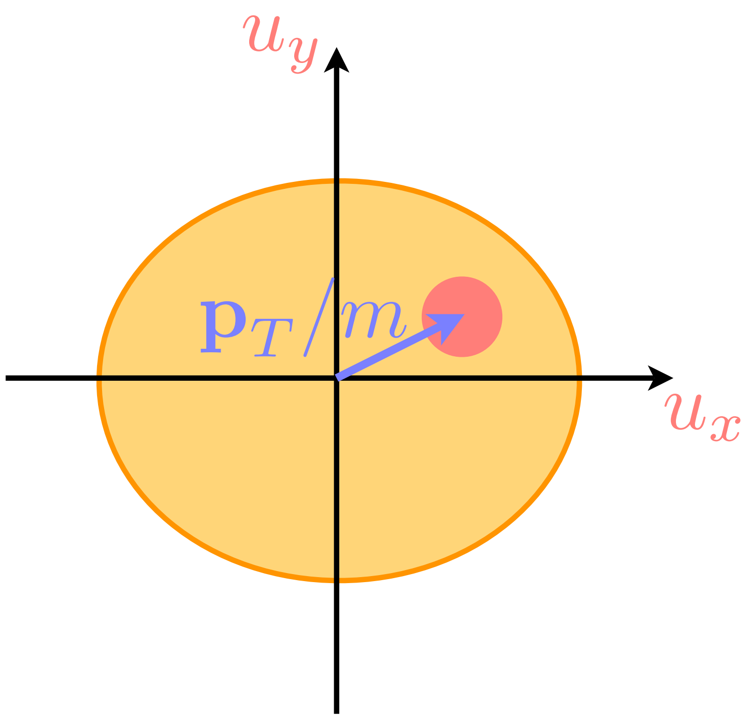

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- Similar momentum distributions for different particles

$$E\frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{p}} = c^h(m)\, f\left(\frac{p_T}{m}, y, \varphi\right)$$

- $v_n\left(\frac{p_T}{m}, y\right)$ identical for all particles!

$\Rightarrow$ mass-ordering of $v_2(p_T, y)$, $\frac{v_4}{v_2^2}\left(\frac{p_T}{m}, y\right)$ universal

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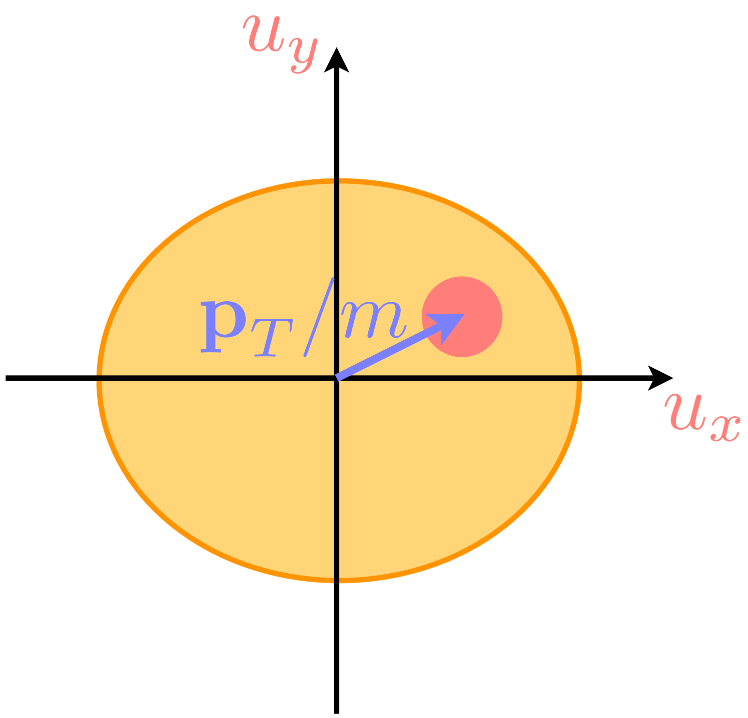

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$m \gg T_{\mathrm{f.o.}}$

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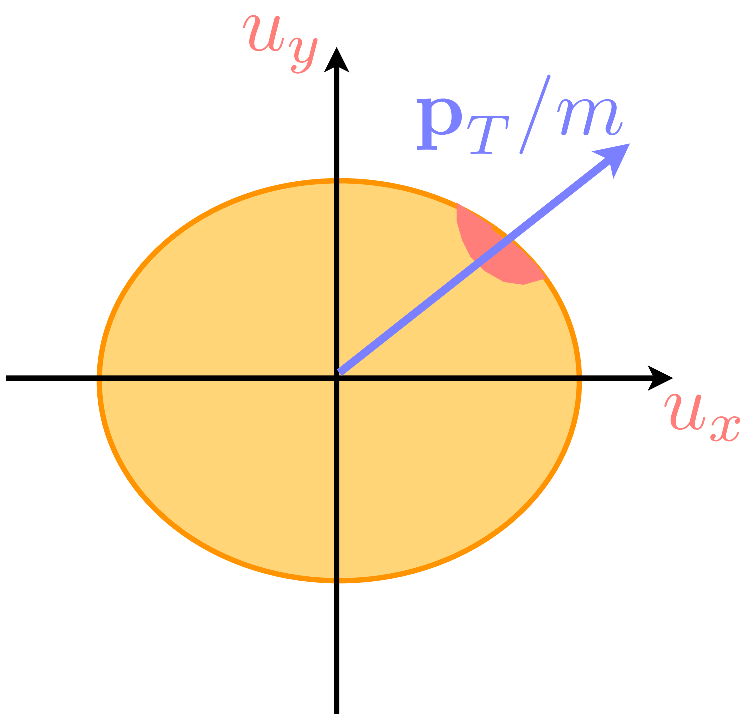
Fast particles ($p_T/m > u_{\max}(0)$) move faster than the fluid.



Such a particle was emitted by a cell along the direction of its velocity where the fluid is fastest (often, close to the edge of the fluid).

Saddle-point expansion of the Cooper-Frye formula around the minimum of $p^\mu u_\mu$.

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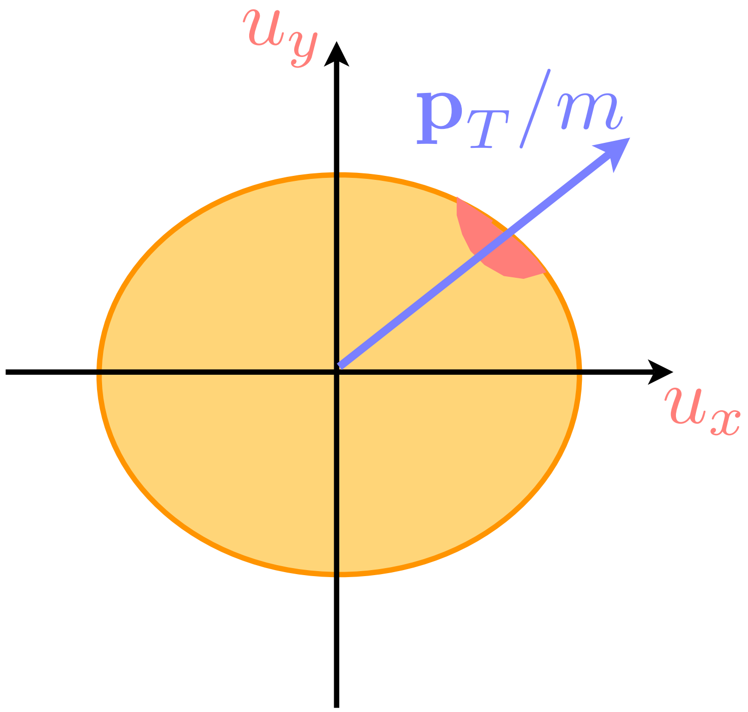

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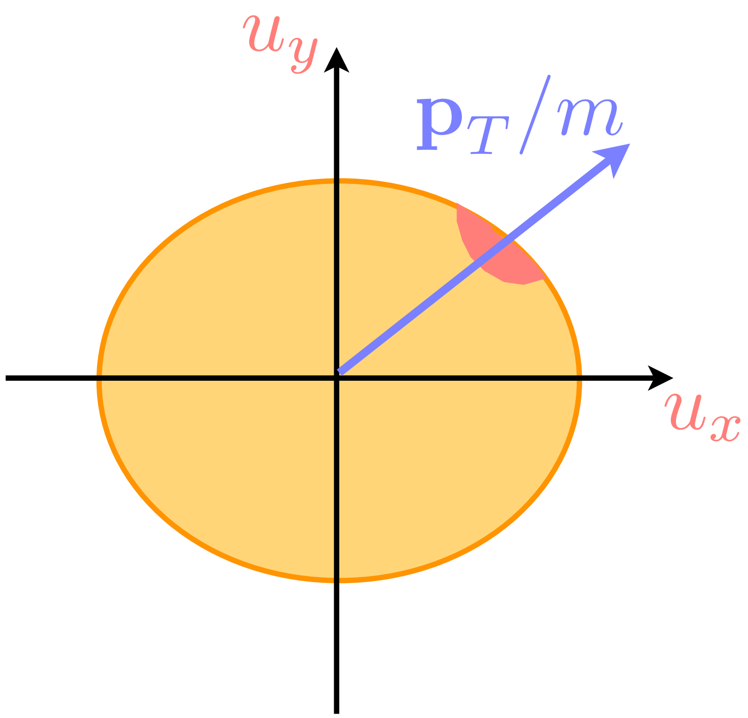

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- Momentum distribution $\propto \exp\left(\dfrac{p_T u_{\max} - m_T u^0_{\max}}{T_{\rm f.o.}}\right)$
- To leading order in the fluid-velocity anisotropies V_n:

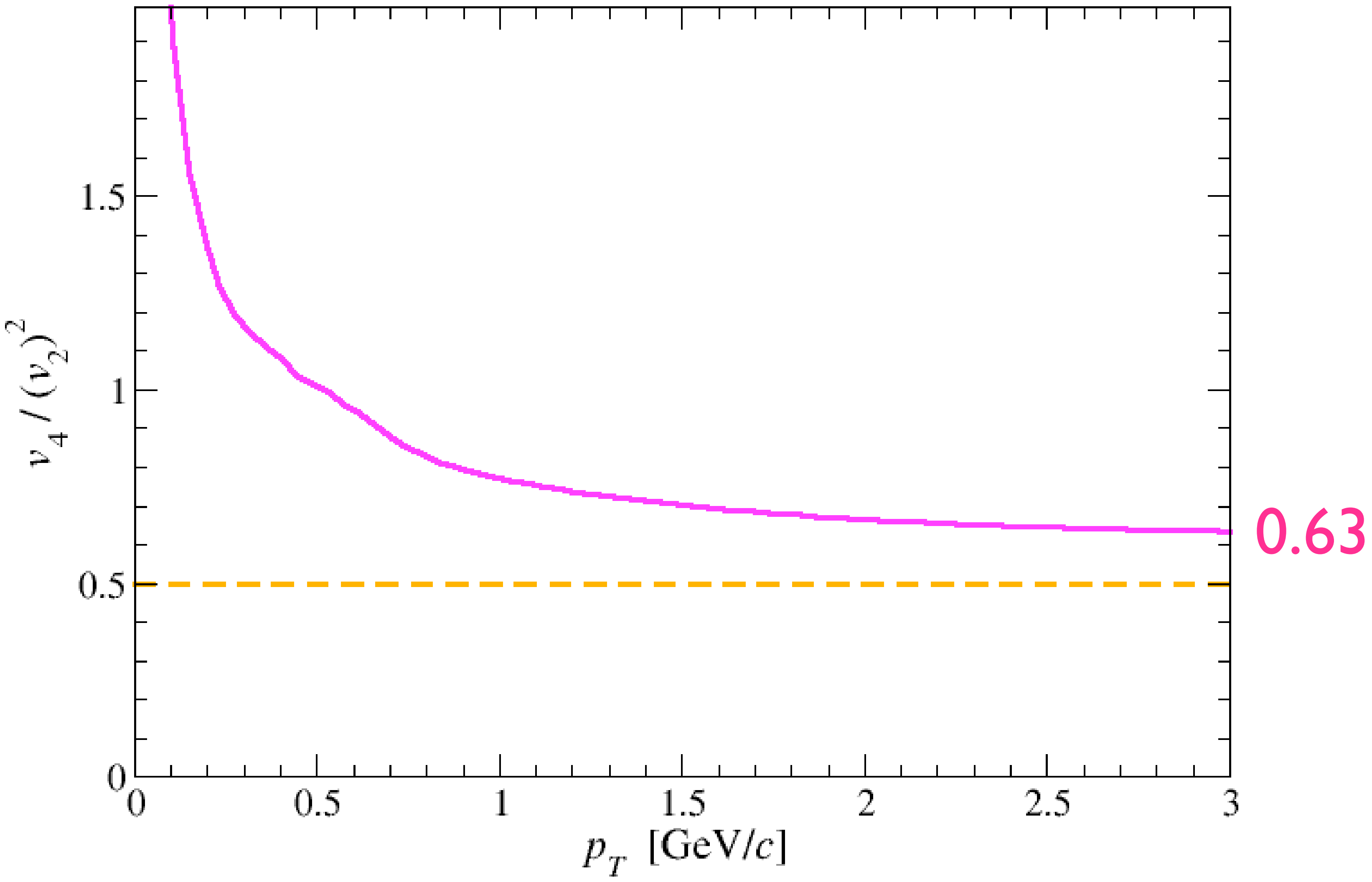
$$v_2(p_T) = \frac{V_2 u_{\max}}{T_{\rm f.o.}}(p_T - m_T v_{\max}):$$ mass-ordering of $v_2(p_T, y)$ persists;

- Assuming additionally that $V_4 \ll V_2$, one finds for p_T large enough:

$$\frac{v_4(p_T, y)}{v_2(p_T, y)^2} = \frac{1}{2}$$

Ideal fluid-dynamics: numerical results

3-dimensional (Bjorken) ideal-fluid dynamics pushed to very large times ($T_{\rm f.o.}$ is small) of a semi-peripheral Au-Au collision "at RHIC":



Due to the large eccentricity, the ratio deviates from the analytical value 1/2: difference under control (smaller for more central collisions).

Anisotropic flow: out-of-equilibrium scenario

⚠ Despite the terminology, "flow" does not imply fluid dynamics.

An exact computation of the dependence of v_2, v_4 on the number $\mathcal{N}$ of collisions undergone by particles requires a microscopic transport model, yet one can guess the general tendency.

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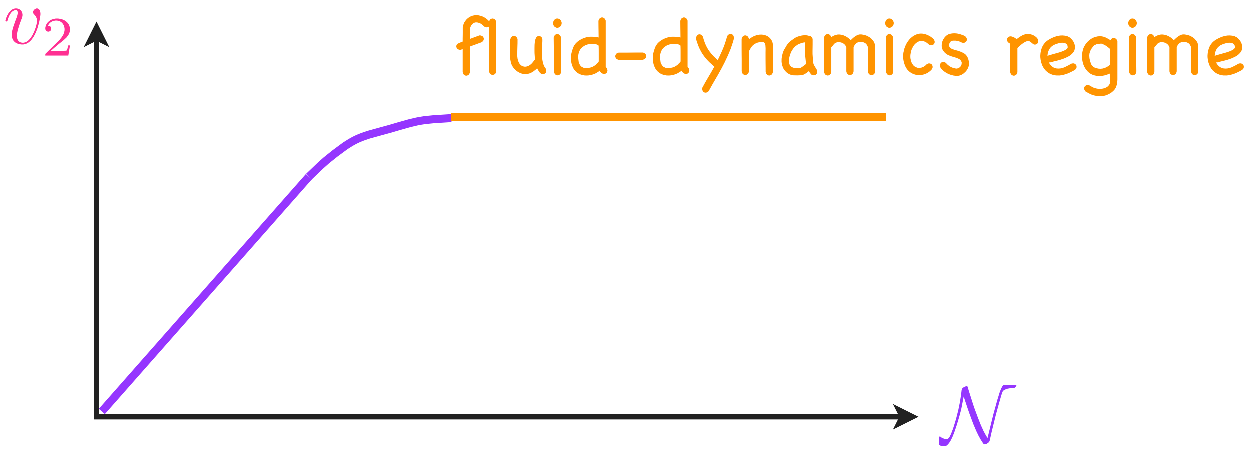
✦ For a given number of collisions, the system thermalizes: further collisions no longer increase v_2.



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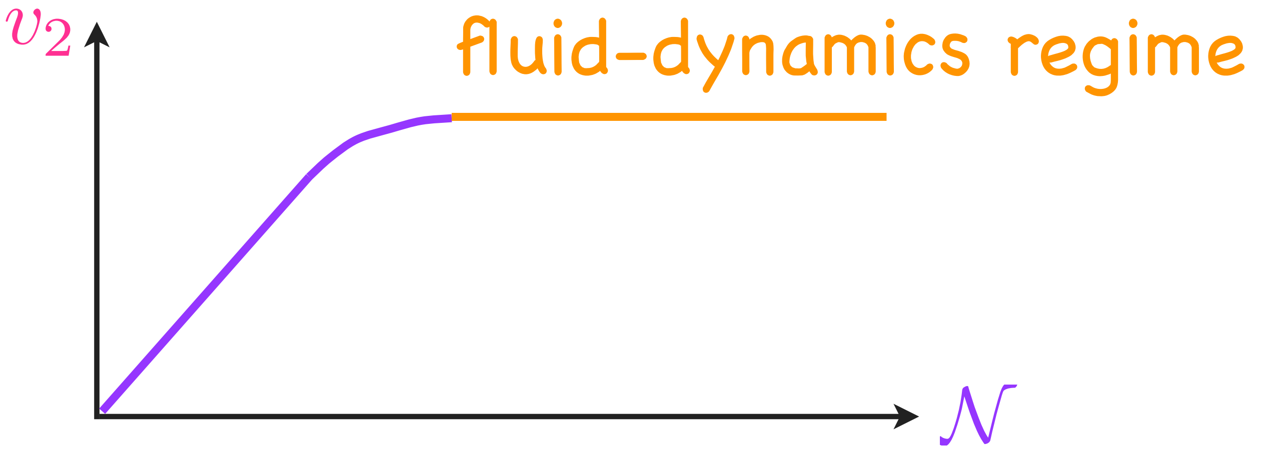

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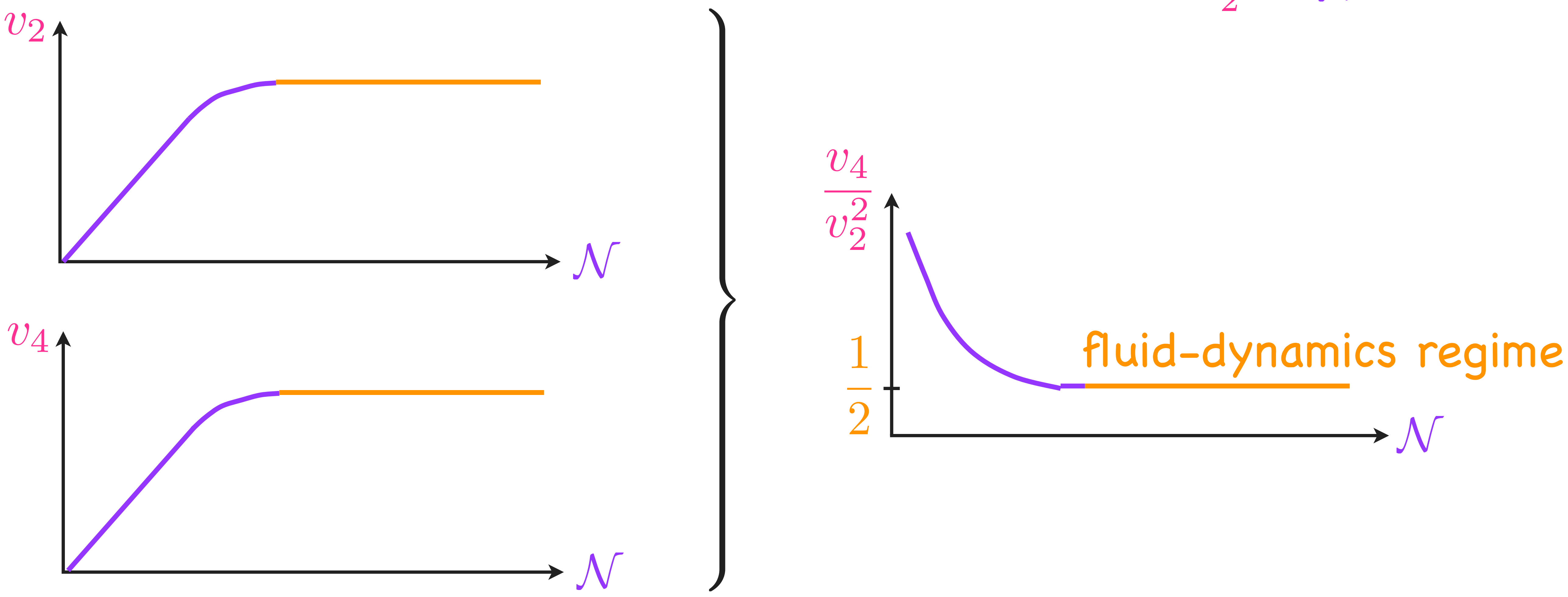
v_2, v_4 proportional to the number of collisions $\mathcal{N}$ $\Rightarrow \dfrac{v_4}{v_2^2} \propto \dfrac{1}{\mathcal{N}}$.



☞ in the non-equilibrium scenario $\dfrac{v_4(p_T, y)}{v_2(p_T, y)^2} > \dfrac{1}{2}$.

Anisotropic flow:
out-of-equilibrium scenario

One expects the number of collisions $\mathcal{N}$ that create anisotropic flow to become smaller and smaller

- with increasing transverse momentum;
- with increasing rapidity;
- with increasing impact parameter;
- when decreasing the system size by going to smaller systems;
- when decreasing the beam energy;

leading to larger and larger $\dfrac{v_4(p_T, y)}{v_2(p_T, y)^2}$ ratios.

more detail in N.B., Eur. Phys. J. A **29** (2006) 27

& in R.S.Bhalerao, J.-P.Blaizot, N.B., J.-Y.Ollitrault, PLB **627** (2005) 49

Testing fluid dynamics with the relationship between v_2 and v_4

The experience from RHIC has taught us that one more anisotropic flow harmonic is measurable and non-zero... Good news!

However, v_4 has not attracted much attention from phenomenologists.

- An analytical ideal-fluid dynamics calculation predicts scaling laws:
  - $v_n\left(\frac{p_T}{m}, y\right)$ identical for all "slow" particles: $\frac{v_4}{v_2^2}\left(\frac{p_T}{m}, y\right)$ universal;
  - for fast particles $\frac{v_4(p_T, y)}{v_2(p_T, y)^2} = \frac{1}{2}$.

- Qualitative arguments suggest that an out-of-equilibrium evolution leads to $\frac{v_4(p_T, y)}{v_2(p_T, y)^2} > \frac{1}{2}$.

(checked in a transport model by C.Gombeaud & J.-Y.Ollitrault NEW)

Constraining models with v_2 and v_4: some caveats

Comparisons of experimental data on $\frac{v_4}{v_2^2}$ with the ideal-fluid dynamics prediction require some care...

Ideal hydro tells "to leading order in the fluid-velocity anisotropies, $v_4(p_T, y) = \frac{1}{2} v_2(p_T, y)^2$ for each type of fast particle".

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- Corrections to the leading-order approximation can lead to a factor significantly different from $\frac{1}{2}$ (cf. the numerical simulation above).
- Elliptic flow v_2 is known to vary strongly with the particle type, transverse momentum and rapidity, so that any averaging spoils the simple relationship:

$$\frac{\langle v_4(p_T, y)\rangle}{\langle v_2(p_T, y)\rangle^2} > \frac{\langle v_4(p_T, y)\rangle}{\langle v_2(p_T, y)^2\rangle} = \frac{1}{2}$$

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- Don't expect $\frac{v_4}{v_2^2} = \frac{1}{2}$ for low-p_T particles!