FLOW ANALYSIS FROM MULTIPARTICLE CORRELATIONS

N. Borghini (Brussels),
P.M. Dinh (C.E.A., Saclay),
J.-Y. Ollitrault (C.E.A., Saclay)

- Standard analysis methods

  → two-particle correlations

  – Limited sensitivity

  Phys. Lett. B**477** (2000) 51
  Phys. Rev. C**62** (2000) 034902

- New method(s)

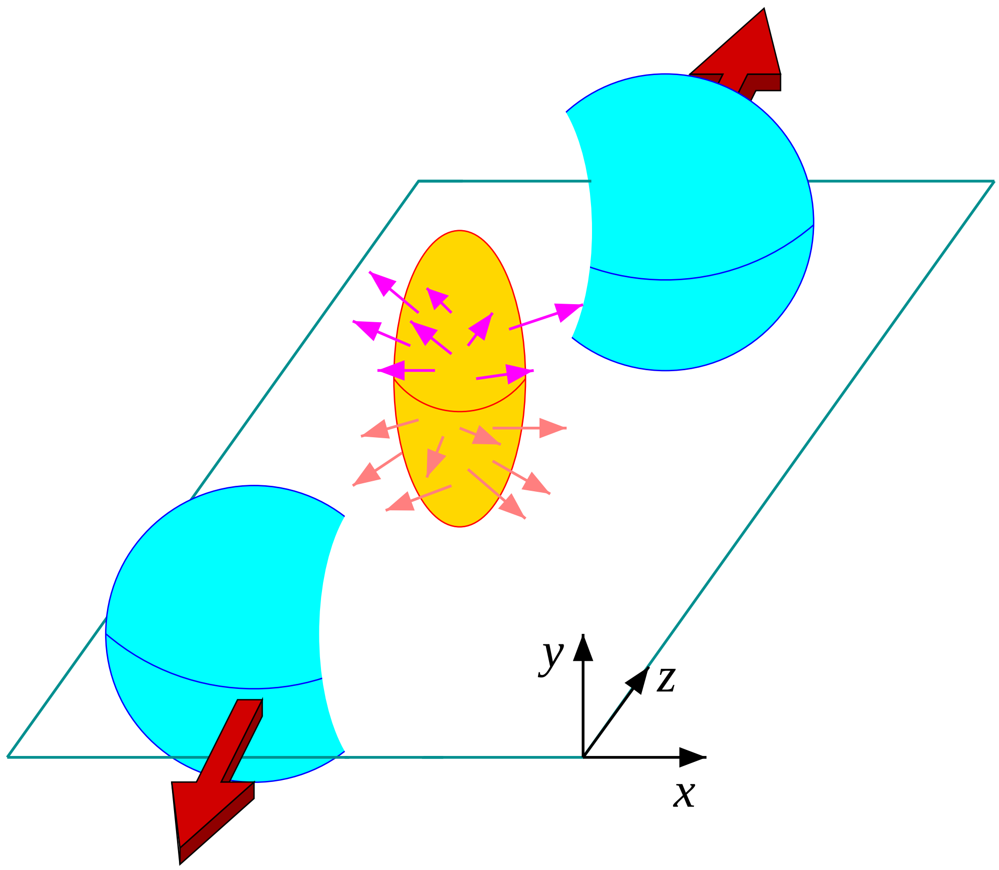
  → multiparticle correlations

  – Integrated flow
  – (Differential flow)
  – Increased sensitivity
  – Acceptance corrections

  Phys. Rev. C**63** (2001) 054906
  Phys. Rev. C**64** (2001) 054901
  `nucl-th/0204017`

(ANISOTROPIC) FLOW

Flow $\equiv$ azimuthal correlation with the reaction plane:



Fourier expansion of the azimuthal distributions of outgoing particles with respect to the unknown reaction plane:

$$\frac{\mathrm{d}N}{\mathrm{d}(\phi - \Phi_R)} \propto \left(1 + 2v_1 e^{i(\phi-\Phi_R)} + 2v_2 e^{2i(\phi-\Phi_R)} + \cdots\right)$$

where:

$$v_n = \left\langle e^{in(\phi-\Phi_R)} \right\rangle .$$

v_1 "directed" flow, v_2 "elliptic" flow.

$\Longrightarrow$ Constraints on the equation of state, signal of thermalization...

FLOW ANALYSIS METHODS (simplified)

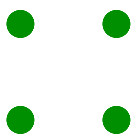
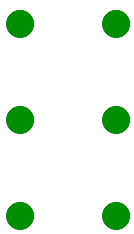
♣ Two-particle methods ( $= \left\langle e^{in(\phi_1-\phi_2)} \right\rangle$):

♣ "subevent" method (P. Danielewicz and G. Odyniec):

→ correlation between **2** subevents;

♣ **two**-particle correlation function analysis (R. Lacey): $C(\Delta\phi)$;

♡ Multiparticle methods NEW!

 $= \left\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)} \right\rangle$,  $= \left\langle e^{in(\phi_1+\phi_2+\phi_3-\phi_4-\phi_5-\phi_6)} \right\rangle$ $\cdots$

(♡) cumulants of the event flow vector;

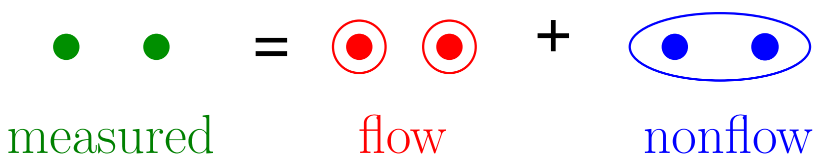
♡♡ cumulants of multiparticle azimuthal correlations.

STANDARD FLOW ANALYSIS

Coefficient v_n extracted from the measured two-particle azimuthal correlations:

$$\left\langle e^{in(\phi_1-\phi_2)}\right\rangle = \left\langle e^{in(\phi_1-\Phi_R)}\right\rangle\left\langle e^{-in(\phi_2-\Phi_R)}\right\rangle + \left\langle e^{in(\phi_1-\phi_2)}\right\rangle_c$$
$$\equiv v_n^2 + \left\langle e^{in(\phi_1-\phi_2)}\right\rangle_c .$$

Expansion of two-particle correlations:



"STANDARD" ASSUMPTION: nonflow sources of two-particle azimuthal correlations are negligible:

$$v_n^2 \gg \left\langle e^{in(\phi_1-\phi_2)}\right\rangle_c .$$

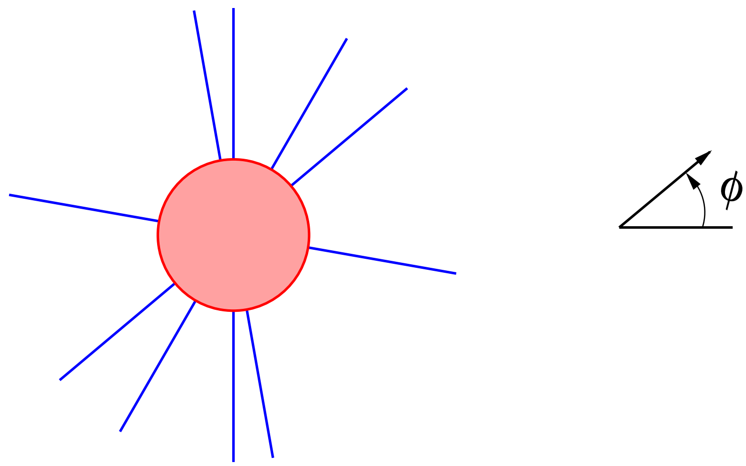
The measured two-particle azimuthal correlations are only due to flow:

$$v_n = \pm\sqrt{\left\langle e^{in(\phi_1-\phi_2)}\right\rangle} .$$

TWO-PARTICLE NONFLOW CORRELATIONS a simple example

Central collision → NO flow, $v_n = 0$.

Strong direct back-to-back correlations:

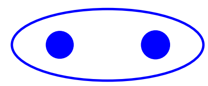


$\Rightarrow \langle \cos 2(\phi_1 - \phi_2) \rangle > 0.$

The standard analysis assumes $v_2 = \sqrt{\langle \cos 2(\phi_1 - \phi_2) \rangle}$...

$$\Rightarrow v_2 \neq 0$$

TWO-PARTICLE
NONFLOW ("DIRECT")
CORRELATIONS

Many sources for $\left\langle e^{in(\phi_1-\phi_2)}\right\rangle_c =$  :

- ◇ total momentum conservation;
- ◇ quantum "HBT" correlations;
- ◇ final state (strong/Coulomb) interactions;
- ◇ resonance decays;
- ◇ other sources? (minijets...)

} order $\dfrac{1}{N}$

$\Rightarrow$ the assumption $v_n^2 \gg \left\langle e^{in(\phi_1-\phi_2)}\right\rangle_c$ underlying the standard analysis holds only if

$$\boxed{v_n \gg \frac{1}{N^{1/2}}}.$$

Possibility: compute and subtract nonflow correlations.

OK, but nonflow correlations may not be under control. . .

Important: two-particle nonflow correlations scale as $\dfrac{1}{N}$
$\Rightarrow$ dominant for peripheral collisions

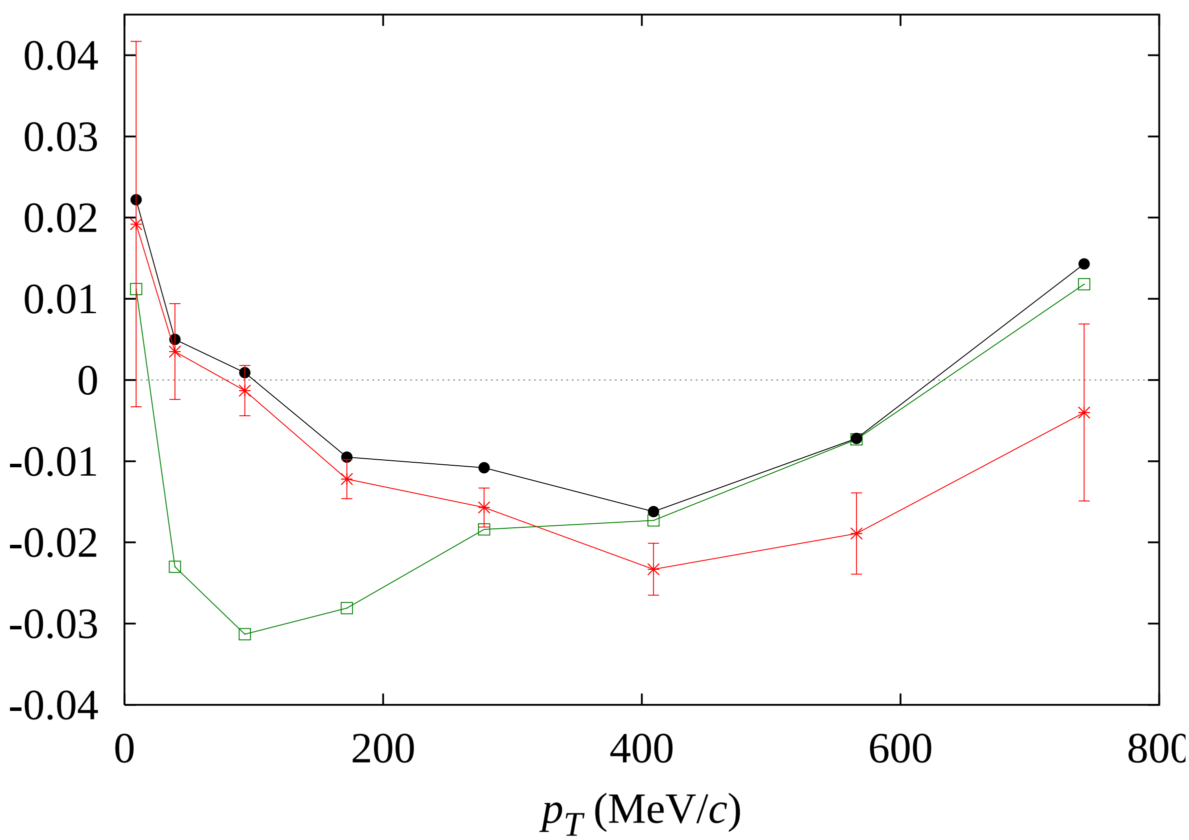
STANDARD FLOW ANALYSIS AT SPS

"Standard" assumption: $v_n^2 \gg \left\langle e^{in(\phi_1-\phi_2)} \right\rangle_c \sim \frac{1}{N}$.

- v_1 and $v_2 \simeq 3\%$ for pions and protons;
- total multiplicity in the collision $N \simeq 2500$.

$\Rightarrow$ the assumption is not valid.

Pion directed flow at SPS (1996 data)



□: "data" [NA49, Phys. Rev. Lett. **80** (1998) 4136]

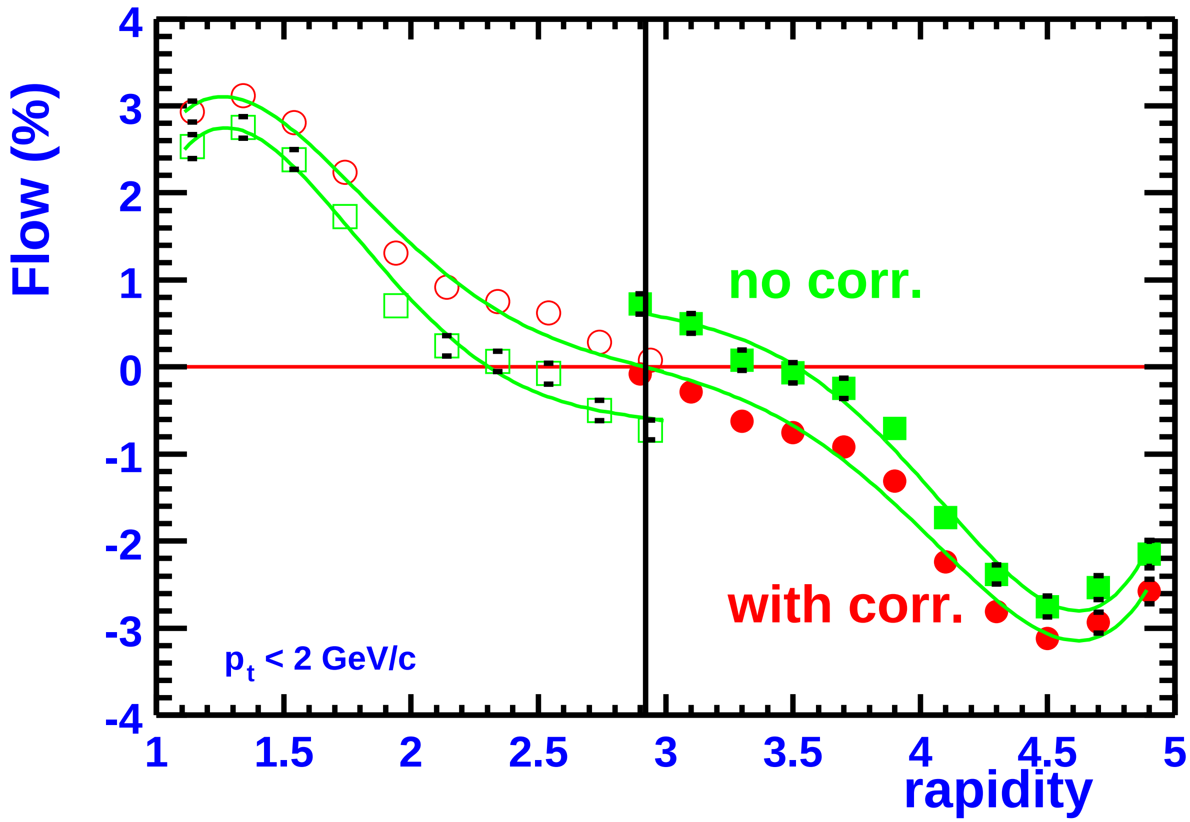
•: data − HBT [Phys. Lett. B**477** (2000) 51]

×: data − (HBT & p_T conservation) [PRC**62**, 034902]

STANDARD ANALYSIS AT THE SPS (continued)

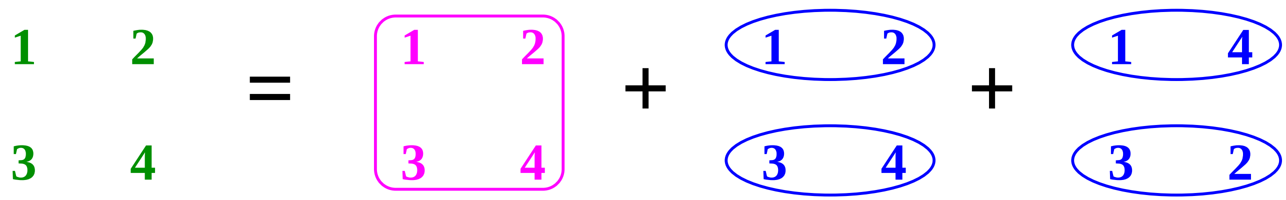
Correction for momentum conservation (NA49 full data set, 158 GeV)

N.B., P.M. Dinh, J.-Y. Ollitrault, A.M. Poskanzer, S.A. Voloshin,
`nucl-th/0202013` (Phys. Rev. C in press).



NEW METHOD

Idea: extract flow from multiparticle azimuthal correlations.



Method: compare flow with direct **4**-particle correlations

$\Rightarrow$ eliminate (non-negligible) extra terms:

cumulant of the multiparticle correlations.

NEW METHOD: INTEGRATED FLOW $v(\mathcal{D})$

Cumulant of the four-particle azimuthal correlation:

$$\left\langle\!\!\left\langle e^{in(\phi_1-\phi_2+\phi_3-\phi_4)} \right\rangle\!\!\right\rangle \equiv \left\langle e^{in(\phi_1-\phi_2+\phi_3-\phi_4)} \right\rangle - 2\left\langle e^{in(\phi_1-\phi_2)} \right\rangle^2$$
$$= -v_n^4 + O\left(\frac{1}{N^3}\right)$$

Increased sensitivity: analysis valid if $\boxed{v_n \gg \frac{1}{N^{3/4}}}$, better than $v_n \gg \frac{1}{N^{1/2}}$.

$$\text{systematic error } \delta(v_n^4) \simeq \frac{1}{N^3}$$

Statistics: N_{evts} events, M particles per event $\to N_{\text{evts}} M^4$ quadruplets

$$\text{statistical error } \delta(v_n^4) \simeq \frac{1}{M^2\sqrt{N_{\text{evts}}}}.$$

CUMULANTS $\langle\langle |Q_n|^{2p} \rangle\rangle$: PRACTICAL FLOW ANALYSIS

"old version": Phys. Rev. C**63** (2001) 054906

1. Compute $Q_n = \frac{1}{\sqrt{M}} \sum_{k=1}^{M} e^{in\phi_k}$ for a given event.

2. Calculate the generating function $\mathcal{G}(z) = e^{z^* Q_n + z Q_n^*}$, then average over events.

Why? because $\langle \mathcal{G}(z) \rangle = 1 + \cdots + |z|^2 \langle |Q_n|^2 \rangle + \cdots + \frac{|z|^4}{4} \langle |Q_n|^4 \rangle + \cdots$, and

the $|Q_n|^{2p}$ give the multiparticle azimuthal correlations: $|Q_n|^2 = \frac{1}{M} \sum_{j,k=1}^{M} e^{in(\phi_j - \phi_k)}$

3. Deduce the cumulants, taking $\ln \langle \mathcal{G}(z) \rangle$:

$$\ln \langle \mathcal{G}(z) \rangle = 1 + \cdots + |z|^2 \langle\langle |Q_n|^2 \rangle\rangle + \cdots + \frac{|z|^4}{4} \langle\langle |Q_n|^4 \rangle\rangle + \cdots$$

4. Extract the flow, using $\ln \langle \mathcal{G}(z) \rangle = \ln I_0(2|z| \langle \bar{Q}_n \rangle)$.

$\longrightarrow$ for instance, $\langle\langle |Q_n|^4 \rangle\rangle \equiv \langle |Q_n|^4 \rangle - 2 \langle |Q_n|^2 \rangle^2 = -\langle \bar{Q}_n \rangle^4 = -M^2 v_n^4$.

BETTER CUMULANTS: ANY HARMONIC

"new version": Phys. Rev. C**64** (2001) 054901

(1.) Calculate the generating function $G_n(z) = \prod_{k=1}^{M}\left(1 + \frac{z^* e^{in\phi_k} + z e^{-in\phi_k}}{M}\right)$,
then average over events.

$$\langle G_n(z)\rangle = 1 + \cdots + \frac{|z|^2}{M}\left\langle \sum_{j\neq k} e^{in(\phi_j-\phi_k)} \right\rangle + \cdots + \frac{|z|^4}{4M^4}\left\langle \sum_{j,k,l,m} e^{in(\phi_j+\phi_k-\phi_l-\phi_m)} \right\rangle + \cdots$$

(2.) Deduce the cumulants, taking $M\left(\langle G_n(z)\rangle^{1/M} - 1\right) = |z|^2 \left\langle\!\left\langle e^{in(\phi_j-\phi_k)} \right\rangle\!\right\rangle + \cdots$

(3.) Extract the flow, using (→ STAR) $M\left(\langle G_n(z)\rangle^{1/M} - 1\right) = \ln I_0(2v_n|z|)$,
and/or performing the appropriate acceptance corrections (→ PHENIX).

(4.) Post your paper on `nucl-ex`.

COLLECTIVE FLOW
AND
MULTIPARTICLE AZIMUTHAL CORRELATIONS

- At SPS energies, two-particle azimuthal correlations due either to collective flow or nonflow effects are of the same magnitude. ⇒ the standard analysis is close to its validity limit $v_n \gg 1/N^{1/2}$.

- New method, using four-particle azimuthal correlations, allows measurements of smaller integrated flow values $v_n \gg 1/N^{3/4}$.

  Sensitivity (and accuracy) can still be improved, with $2p$-particle ($p > 2$) correlations ($\longrightarrow$ higher statistics).

- Detector acceptance corrections.

- Differential flow.

Method currently tested/used by E895, NA49, PHENIX, STAR. First results available!

Two-particle and multiparticle methods may yield different values $v_n\{2\} \neq v_n\{4\}$...

"NEW" (unthought of) two-particle correlations!