COLLECTIVE FLOW AND MULTIPARTICLE AZIMUTHAL CORRELATIONS

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- Standard collective flow analysis

  $\longrightarrow$ two-particle correlations

  - Limited sensitivity

  Phys. Lett. B**477** (2000) 51

  Phys. Rev. C**62** (2000) 034902

- New method

  $\longrightarrow$ multiparticle correlations

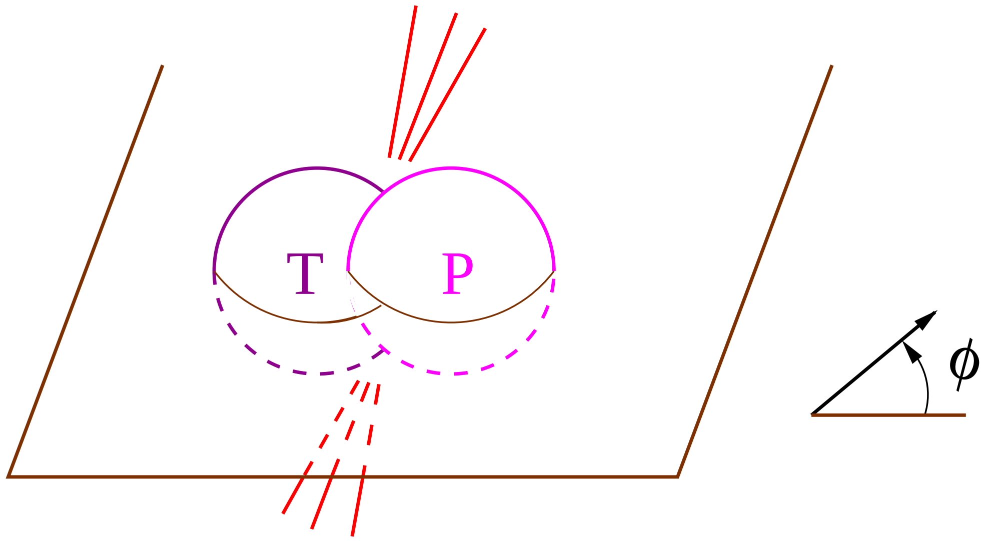
  - Integrated flow
  - Differential flow
  - Increased sensitivity
  - Acceptance corrections

  `nucl-th/0007063`, Phys. Rev. C (may 2001)

  `nucl-th/0104xxx`

FLOW

Flow $\equiv$ azimuthal correlation with the reaction plane:



Fourier expansion of the azimuthal distributions of outgoing particles with respect to the unknown reaction plane:

$$\frac{\mathrm{d}N}{\mathrm{d}\phi} = A\left(1 + v_1\,\cos\phi + v_2\,\cos 2\phi + \cdots\right)$$

where:

$$v_n = \left\langle e^{in\phi} \right\rangle.$$

$v_n(p_T, y)$ differential flow; $v_n(\mathcal{D})$ integrated flow.

v_1 "directed" flow, v_2 "elliptic" flow.

At CERN SPS, v_1 and $v_2 \simeq 3\%$ for pions and protons.
PHENIX & STAR analyses: $v_2 \simeq 5 - 6\%$.

STANDARD FLOW ANALYSIS

Coefficient v_n extracted from the measured two-particle azimuthal correlations:

$$\left\langle e^{in(\phi_1-\phi_2)}\right\rangle = \left\langle e^{in\phi_1}\right\rangle \left\langle e^{-in\phi_2}\right\rangle + \left\langle e^{in(\phi_1-\phi_2)}\right\rangle_c$$
$$\equiv v_n^2 + \left\langle e^{in(\phi_1-\phi_2)}\right\rangle_c .$$

[study of $\Delta\phi$ correlation (see Roy Lacey) or correlation between 2 subevents].

Expansion of two-particle correlations:

measured flow nonflow

"STANDARD" ASSUMPTION: the measured two-particle azimuthal correlations are only due to flow:

$$v_n = \pm\sqrt{\left\langle e^{in(\phi_1-\phi_2)}\right\rangle}.$$

Other sources of two-particle azimuthal correlations are negligible:

$$v_n^2 \gg \left\langle e^{in(\phi_1-\phi_2)}\right\rangle_c .$$

Is that true?

TWO-PARTICLE NONFLOW ("DIRECT") CORRELATIONS

Many sources for $\left\langle e^{in(\phi_1-\phi_2)}\right\rangle_c$:

- total momentum conservation;
- quantum "HBT" correlations;
- final state (strong / Coulomb) interactions;
- resonance decays;
- other sources? (minijets...)

} order *1 / N*

$\Rightarrow$ the assumption $v_n^2 \gg \left\langle e^{in(\phi_1-\phi_2)}\right\rangle_c$ underlying the standard analysis holds only if

$$\boxed{v_n \gg \frac{1}{N^{1/2}}}.$$

Possibility: compute and subtract nonflow correlations.

OK, but nonflow correlations may not be under control. . .

Important: two-particle nonflow correlations scale as $\dfrac{1}{N}$
$\Rightarrow$ dominant for peripheral collisions.

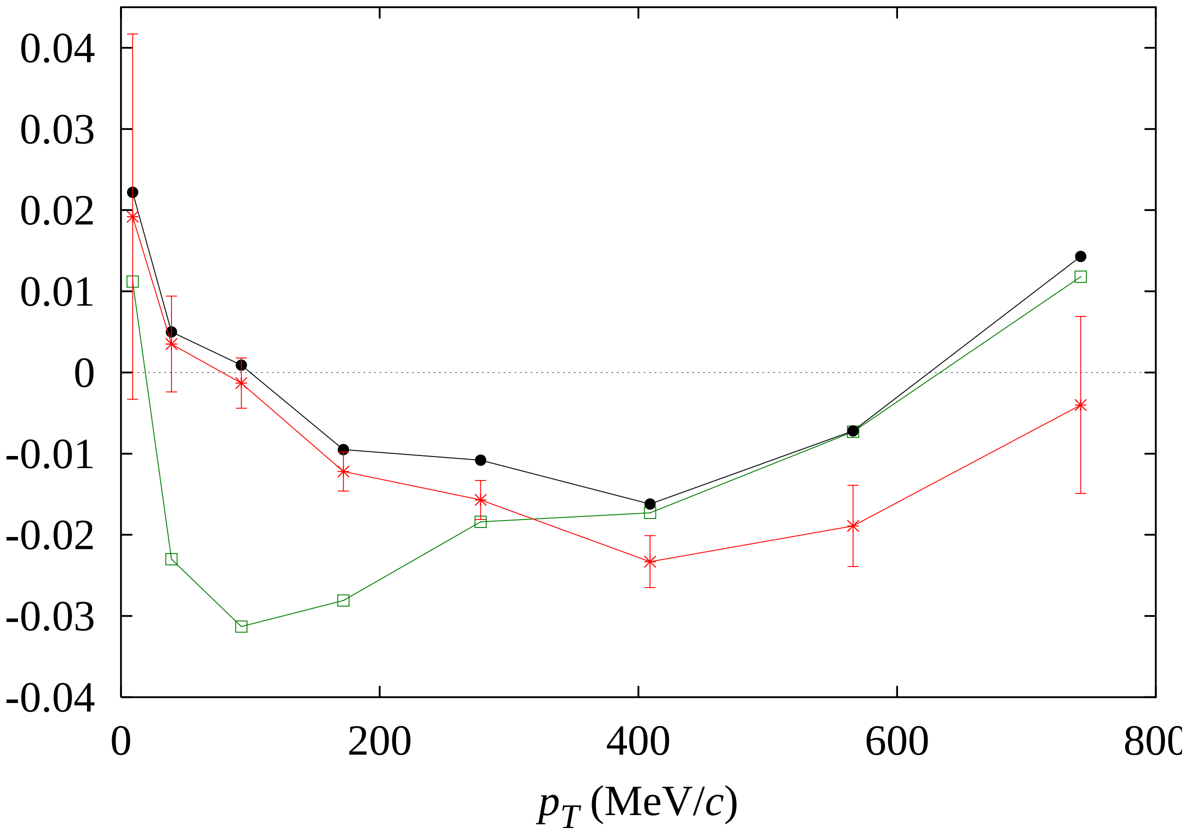
STANDARD FLOW ANALYSIS AT SPS

"Standard" assumption: $v_n^2 \gg \left\langle e^{in(\phi_1-\phi_2)}\right\rangle_c \sim \frac{1}{N}$.

- v_1 and $v_2 \simeq 3\%$ for pions and protons;
- total multiplicity in the collision $N \simeq 2500$.

$\Rightarrow$ the assumption is not valid.

Pion directed flow at SPS



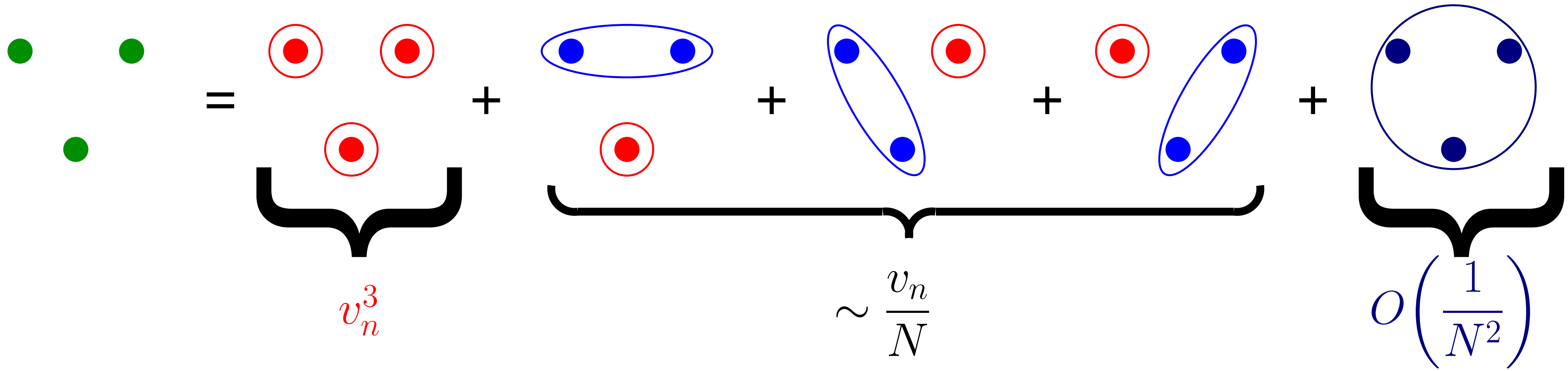
□ : "data"

•: data − HBT

×: data − (HBT & p_T conservation)

NEW METHOD

Idea: extract flow from multiparticle azimuthal correlations.

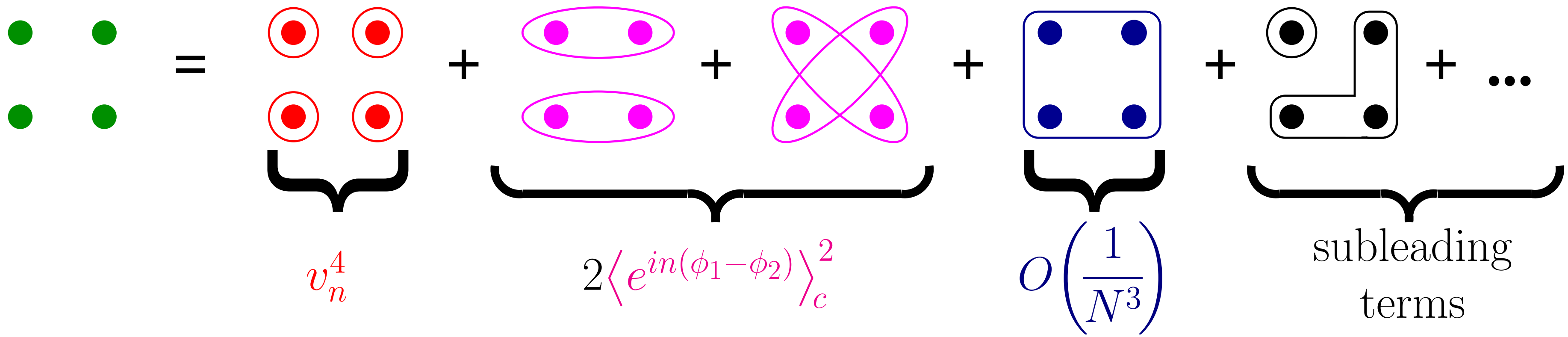


Method: compare flow with direct 3-particle correlations

⇒ eliminate (non-negligible) extra terms:

cumulant of the multiparticle correlations.

NEW METHOD: INTEGRATED FLOW $v(\mathcal{D})$



Cumulant of the four-particle azimuthal correlation:

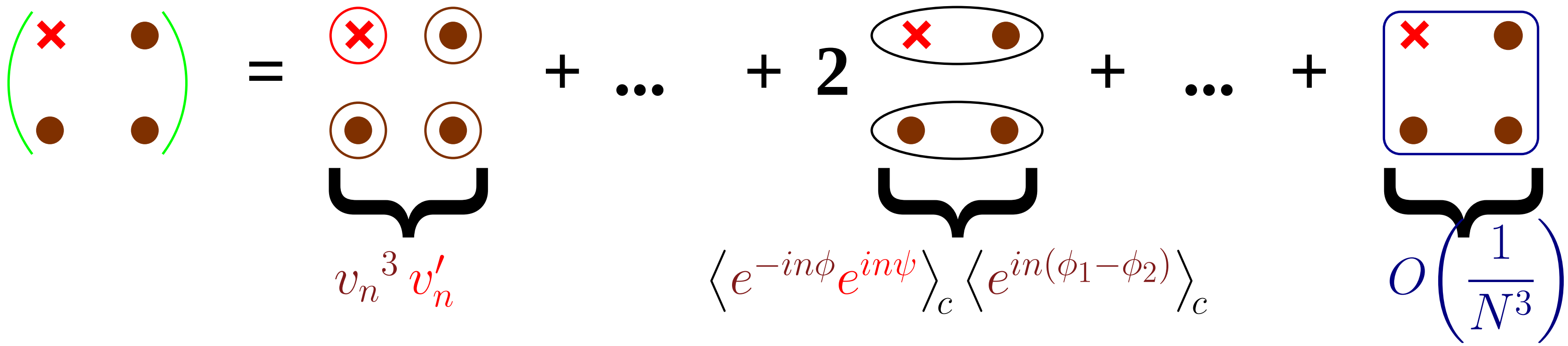
$$\langle\langle e^{in(\phi_1-\phi_2+\phi_3-\phi_4)}\rangle\rangle \equiv \langle e^{in(\phi_1-\phi_2+\phi_3-\phi_4)}\rangle - 2\langle e^{in(\phi_1-\phi_2)}\rangle^2$$
$$= -v_n^4 + O\left(\frac{1}{N^3}\right)$$

Increased sensitivity: analysis valid if $\boxed{v_n \gg \frac{1}{N^{3/4}}}$.

DIFFERENTIAL FLOW $v'(p_T, y)$

① Measure the integrated flow $\langle e^{in\phi} \rangle = v_n$ using many particles ("pions"): reaction plane determination.

② Study the correlation between the azimuth ψ of a given particle ("proton") and the reaction plane: $\langle e^{-in\phi} e^{in\psi} \rangle$.



Idea: compare the flow term with the direct multiparticle azimuthal correlation.

$\Rightarrow$ Cumulant of the (1+3)-particle azimuthal correlation:

$$\left\langle\!\left\langle e^{in(\phi_1-\phi_2-\phi_3)} e^{in\psi} \right\rangle\!\right\rangle \equiv \left\langle e^{in(\phi_1-\phi_2-\phi_3)} e^{in\psi} \right\rangle - 2\left\langle e^{-in\phi} e^{in\psi} \right\rangle \left\langle e^{in(\phi_1-\phi_2)} \right\rangle$$

$$= -v_n{}^3 \left[v'_n + O\left(\frac{1}{(N v_n)^3} \right) \right].$$

COLLECTIVE FLOW
AND
MULTIPARTICLE AZIMUTHAL CORRELATIONS

- At SPS energies, two-particle azimuthal correlations due either to collective flow or nonflow effects are of the same magnitude. $\Rightarrow$ the standard analysis is close to its validity limit $v_n \gg 1/N^{1/2}$.

- New method, using four-particle azimuthal correlations, allows measurements of smaller integrated flow values $v_n \gg 1/N^{3/4}$.

  Sensitivity can still be improved, with multiparticle (involving $2k$ particles, $k > 4$) correlations.

- Detector acceptance corrections.

- Differential flow.

Two different methods to extract flow are available...

$\Rightarrow$ **HANDS WANTED!**

Both methods may yield different results...

"NEW" (unthought of) two-particle correlations!

EVENT FLOW VECTOR

For a given event:

$$Q_n = \frac{1}{\sqrt{M}} \sum_{k=1}^{M} e^{in\phi_k}$$

M as large as possible.

- Flow $\Leftrightarrow$ $\langle Q_n \rangle = \sqrt{M} v_n \neq 0$: random walk with a preferred direction;
- Powers of $|Q_n|^2$ involve multiparticle azimuthal correlations:

$$|Q_n|^2 = \frac{1}{M} \sum_{j,k=1}^{M} e^{in(\phi_j - \phi_k)}\,;$$

- Cumulants of the $|Q_n|$ distribution yield the flow:

$$\langle\langle |Q_n|^4 \rangle\rangle \equiv \langle |Q_n|^4 \rangle - 2 \langle |Q_n|^2 \rangle^2 = - \langle Q_n \rangle^4 + O\left(\frac{1}{M}\right)$$

Method valid if $\boxed{\langle Q_n \rangle^4 \gg \frac{1}{M} \quad \Leftrightarrow \quad v_n \gg \frac{1}{M^{3/4}}}$

- Increasing sensitivity using higher order cumulants $\langle\langle |Q_n|^{2p} \rangle\rangle$.

CUMULANTS $\langle\langle |Q_n|^{2p} \rangle\rangle$: PRACTICAL FLOW ANALYSIS

"old version": Phys. Rev. C**63** (may 2001)

1. Compute $\bar{Q}_n = \frac{1}{\sqrt{M}} \sum_{k=1}^{M} e^{in\bar{\phi}_k}$ for a given event ($\bar{\phi}_k$ measured angle).

2. Calculate the generating function $\mathcal{G}(z) = e^{z^*\bar{Q}_n + z\bar{Q}_n^*}$, then average over events.

Why? because $\langle \mathcal{G}(z) \rangle = 1 + \cdots + |z|^2 \left\langle |\bar{Q}_n|^2 \right\rangle + \cdots + \frac{|z|^4}{4} \left\langle |\bar{Q}_n|^4 \right\rangle + \cdots$

3. Deduce the cumulants, taking $\ln \langle \mathcal{G}(z) \rangle$:

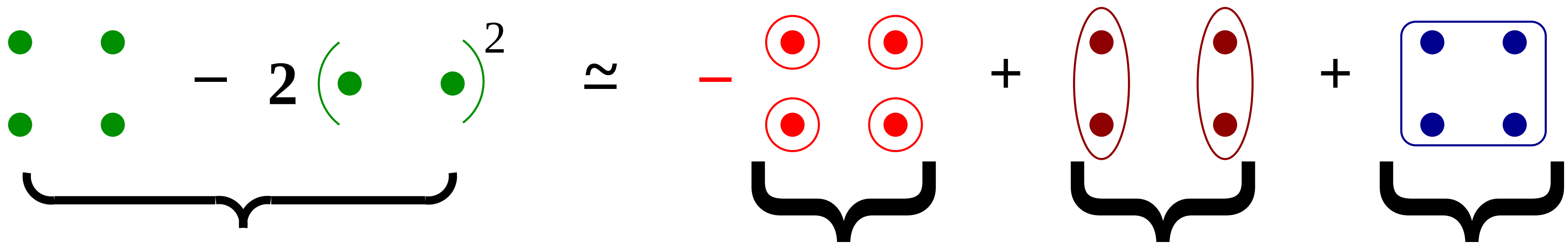
$$\ln \langle \mathcal{G}(z) \rangle = 1 + \cdots + |z|^2 \left\langle\!\left\langle |\bar{Q}_n|^2 \right\rangle\!\right\rangle + \cdots + \frac{|z|^4}{4} \left\langle\!\left\langle |\bar{Q}_n|^4 \right\rangle\!\right\rangle + \cdots$$

4. Extract the flow, using $\ln \langle \mathcal{G}(z) \rangle = \ln I_0(2|z|\langle Q_n \rangle)$.

$\longrightarrow$ for instance, $\left\langle\!\left\langle |\bar{Q}_n|^6 \right\rangle\!\right\rangle \equiv \left\langle |\bar{Q}_n|^6 \right\rangle - 9\left\langle |\bar{Q}_n|^4 \right\rangle \left\langle |\bar{Q}_n|^2 \right\rangle + 12\left\langle |\bar{Q}_n|^2 \right\rangle^3 = 4\langle Q_n \rangle^6$.

5. Put your paper on nucl-ex.

INTERFERENCE BETWEEN v_1 AND v_2



$$\langle\!\langle e^{in(\phi_1-\phi_2+\phi_3-\phi_4)}\rangle\!\rangle \qquad v_n^4 \qquad O\left(\frac{v_{2n}^2}{N^2}\right) \qquad O\left(\frac{1}{N^3}\right)$$

$\Rightarrow$ Measurements of v_n require $|v_{2n}| \ll N\, v_n^2$.

Problem for directed flow at RHIC, not for elliptic flow.

BETTER CUMULANTS: ANY HARMONIC

"new version": keep an eye on nucl-th

1. Calculate the generating function $\mathcal{G}(z) = \prod_{k=1}^{M} \left(1 + z^* e^{in\phi_k} + z e^{-in\phi_k}\right)$,

then average over events (ϕ_k measured angle).

$$\langle \mathcal{G}(z) \rangle = 1 + \cdots + |z|^2 \left\langle \sum_{j \neq k} e^{in(\phi_j - \phi_k)} \right\rangle + \cdots + \frac{|z|^4}{4} \left\langle \sum_{j,k,l,m} e^{in(\phi_j + \phi_k - \phi_l - \phi_m)} \right\rangle + \cdots$$

2. Deduce the cumulants, taking $\langle \mathcal{G}(z) \rangle^{1/M} - 1$:

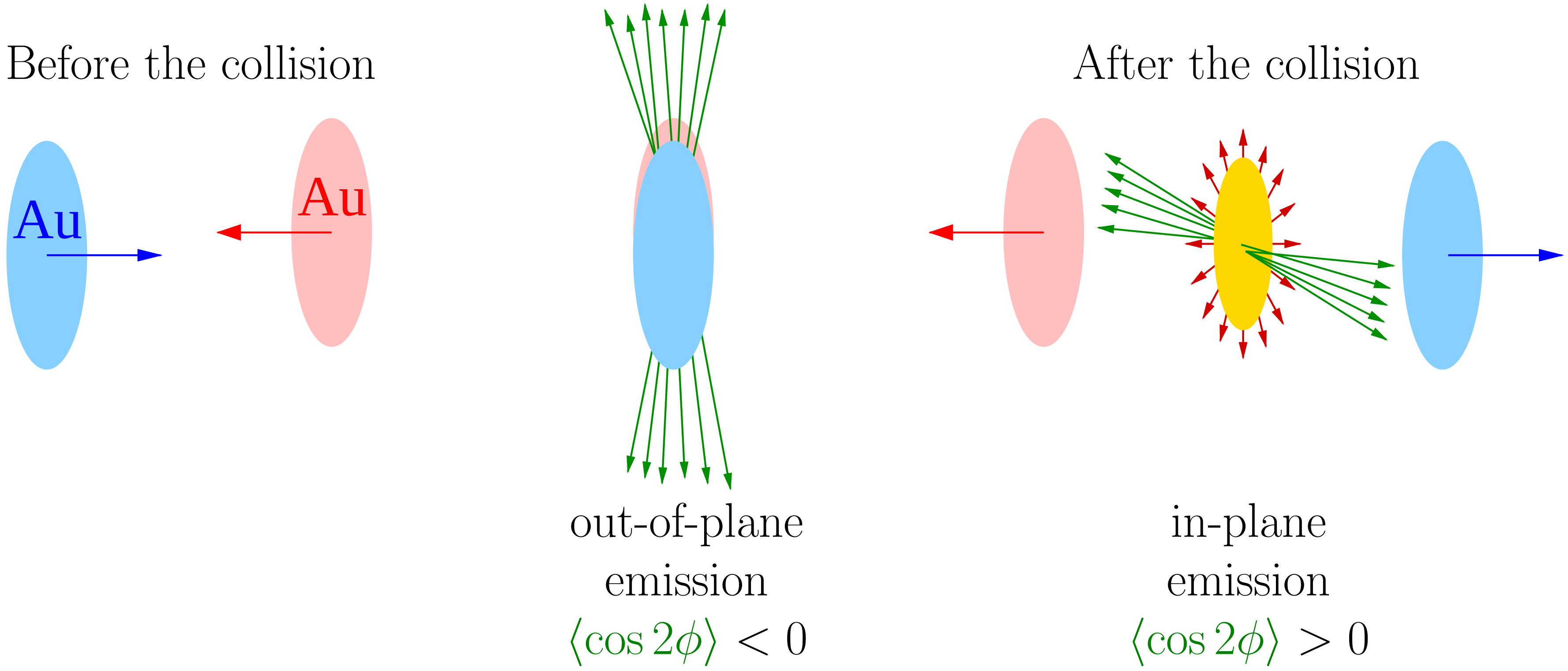
$$\langle \mathcal{G}(z) \rangle^{1/M} = 1 + \cdots + |z|^2 (M-1) \left\langle\!\left\langle e^{in(\phi_j - \phi_k)} \right\rangle\!\right\rangle + \cdots$$

3. Extract the flow, using $\langle \mathcal{G}(z) \rangle^{1/M} - 1 = I_0(2 M v_n |z|)^{1/M} - 1$.

Work still in progress (improving acceptance corrections).

WHY FLOW?

- Flow determination ⇒ equation of state:



- Influence of flow on two-particle correlations (HBT, Coulomb...).
- Observation of possible parity violation requires accurate flow determination.