# MEASURING DIRECTED FLOW WITH THREE-PARTICLE CORRELATIONS

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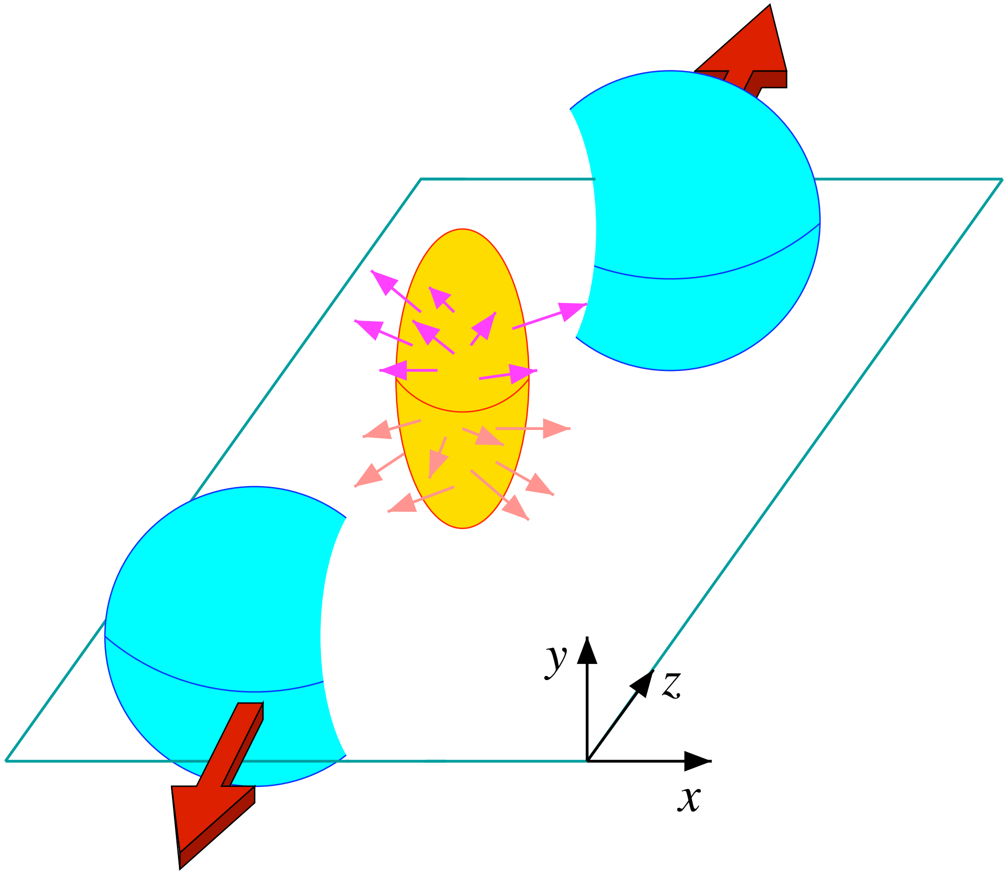
- Methods of flow analysis
  - ♠ two-particle method(s): any $v_n$
  - ♠ four-particle method: any $v_n$
  - ♡ three-particle method: $v_1$ only
- Application to NA49 data

Phys. Rev. **C66** (2002) 014905

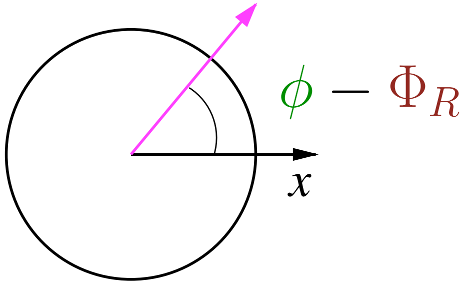
# ANISOTROPIC FLOW



Source anisotropy

⇒ anisotropic emission of particles:

(transverse) FLOW



$$\frac{\mathrm{d}N}{\mathrm{d}\phi} \propto 1 + 2\,v_1 \cos(\phi - \Phi_R) + 2\,v_2 \cos 2(\phi - \Phi_R) + \ldots$$

$v_1$ "directed", $v_2$ "elliptic"

$$v_n = \langle \cos n(\phi - \Phi_R) \rangle$$

$v_1$, $v_2$ ⇒ source equation of state

# METHODS OF FLOW ANALYSIS, part I

Flow = correlation with the reaction plane

$$v_n = \langle \cos n(\phi - \Phi_R) \rangle = \left\langle e^{in(\phi - \Phi_R)} \right\rangle$$

$\Phi_R$ reaction plane azimuth... unknown!

⇒ solution (?): extract $v_n$ from 2-particle correlations

$$\begin{aligned} \left\langle e^{in(\phi_1 - \phi_2)} \right\rangle &= \left\langle e^{in(\phi_1 - \Phi_R)}\, e^{in(\Phi_R - \phi_2)} \right\rangle \\ &\approx \left\langle e^{in(\phi_1 - \Phi_R)} \right\rangle \left\langle e^{in(\Phi_R - \phi_2)} \right\rangle = \left(v_n\{2\}\right)^2 \end{aligned}$$

Assumption: the correlation between two particles is due to the correlation of each one with the reaction plane

Problem: there exist other correlation sources.

Many sources of nonflow correlations:

- ♠ quantum correlations (HBT)
- ♠ momentum conservation
- ♠ resonance decays
- ♠ strong and Coulomb interactions
- ♠ (mini)jets, etc.

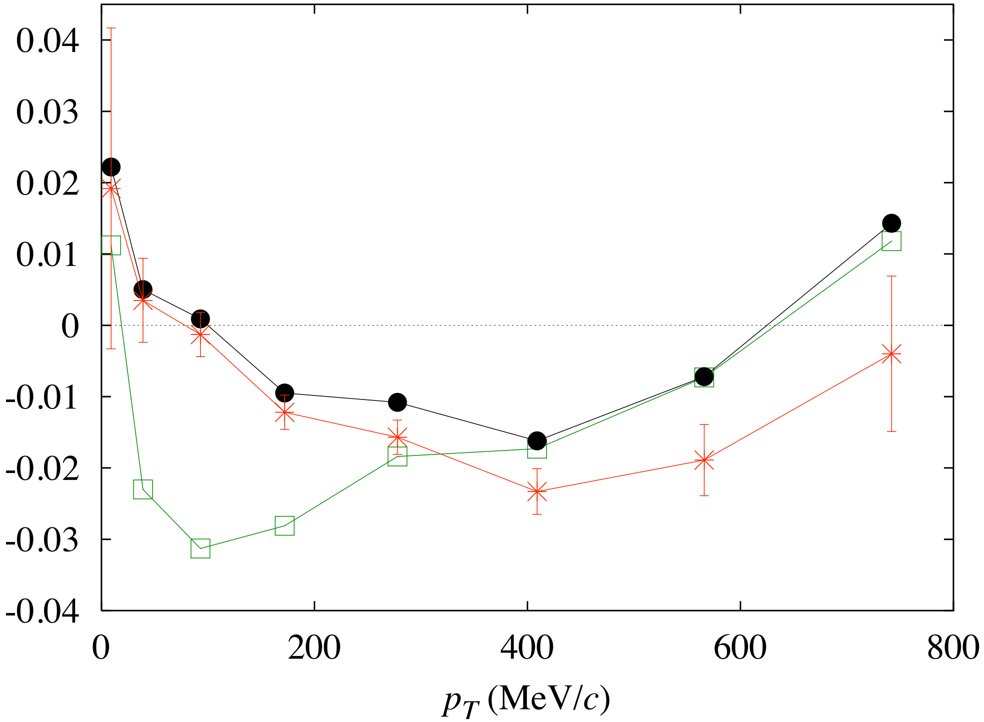
Unwanted correlations of the same magnitude as the flow correlations!

$$\mathcal{O}\left(\frac{1}{N}\right) \sim (v_n)^2$$

Solution (?):
compute & subtract the correlations

Problem: are all sources known?

charged pion $v_1$ at SPS



□ : “data” (NA49, PRL 1998)

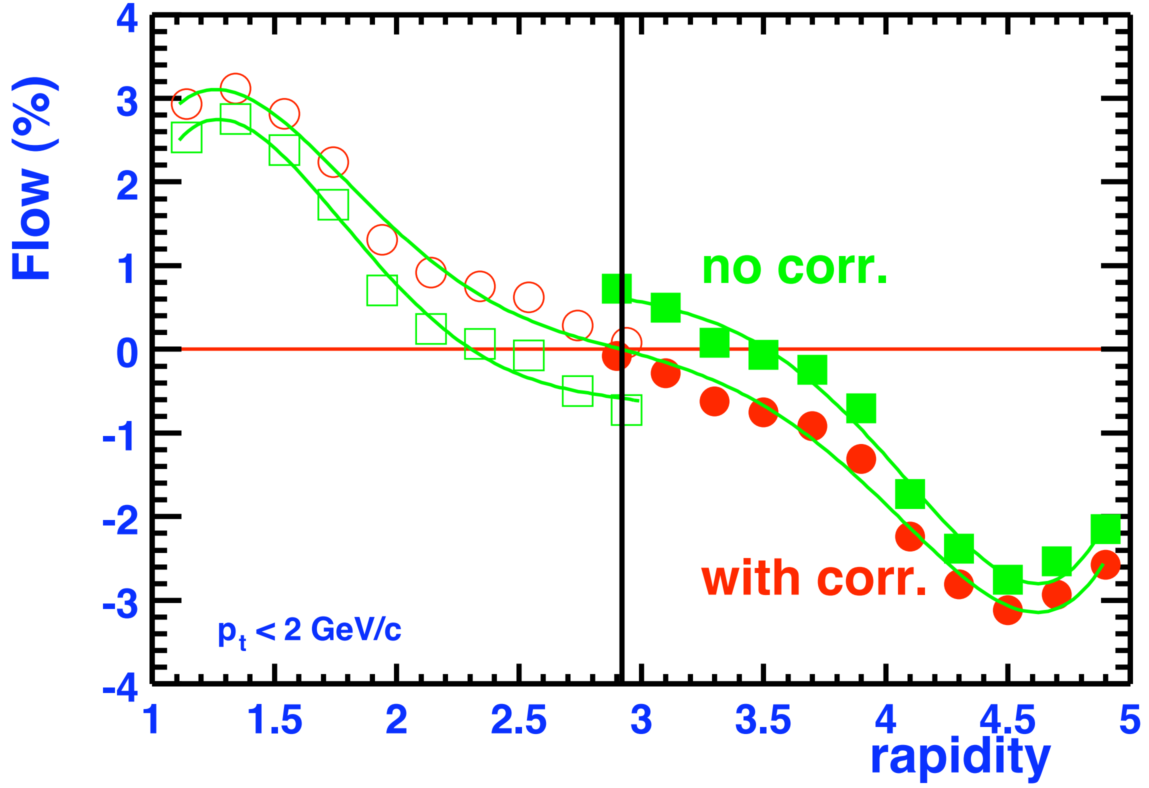
● : data − HBT

× : data − (HBT & $p_T$ conservation)

N.B., P.M. Dinh, J.-Y. Ollitrault, Phys. Lett. **B477** (2000) 51, Phys. Rev. **C62** (2000) 034902.

# NONFLOW CORRELATIONS (continued)

Influence of momentum conservation (NA49 $\pi^+, \pi^-$, 158 $A$GeV, min. bias)



N.B., P.M. Dinh, J.-Y. Ollitrault, A.M. Poskanzer, S.A. Voloshin, Phys. Rev. **C66** (2002) 014901

# METHODS OF FLOW ANALYSIS, part II

Nonflow two-particle correlations are a nuisance... let us eliminate them!

⇒ cumulant of the four-particle correlation:

$$\left\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)}\right\rangle - \left\langle e^{in(\phi_1-\phi_2)}\right\rangle\left\langle e^{in(\phi_3-\phi_4)}\right\rangle - \left\langle e^{in(\phi_1-\phi_4)}\right\rangle\left\langle e^{in(\phi_3-\phi_2)}\right\rangle \approx -(v_n\{4\})^4$$

up to nonflow FOUR-particle correlations, $\mathcal{O}(1/N^3)$, negligible.

N.B., P.M. Dinh, J.-Y. Ollitrault, Phys. Rev. **C63** (2001) 054906, **C64** (2001) 054901.

Problem: the method is statistics-consuming, requires many quadruplets:

$$\frac{\delta v_n\{4\}}{v_n} \simeq \frac{1}{2\sqrt{N_{\rm evts}}}\frac{1}{(v_n\sqrt{N})^4}$$

statistical uncertainty depends on $v_n$

$(v_n\sqrt{N})^2$ with 2-particle m.

⇒ smaller $v_n$ requires more statistics!

# METHODS OF FLOW ANALYSIS, part III:

# $v_1$ from THREE-PARTICLE CORRELATIONS NEW

Idea: consider the <u>mixed</u> three-particle correlation:

$$\left\langle e^{i(\phi_1+\phi_2-2\phi_3)} \right\rangle = (v_1\{3\})^2\, v_2 + \underbrace{\mathcal{O}\left(\frac{1}{N^2}\right)}_{\text{nonflow } \underline{\mathbf{3}}\text{-particle correlation}}$$

$v_2$ measured in separate analysis $\Rightarrow$ value of $v_1$

♡ Statistical uncertainty moderate: $\dfrac{\delta v_1\{3\}}{v_1} \simeq \dfrac{1}{2\sqrt{N_{\rm evts}}} \dfrac{1}{(v_1\sqrt{N})^2\,(v_2\sqrt{N})}$

especially if $v_2$ is “large” (SPS, RHIC)

♡ Error due to nonflow correlations negligible

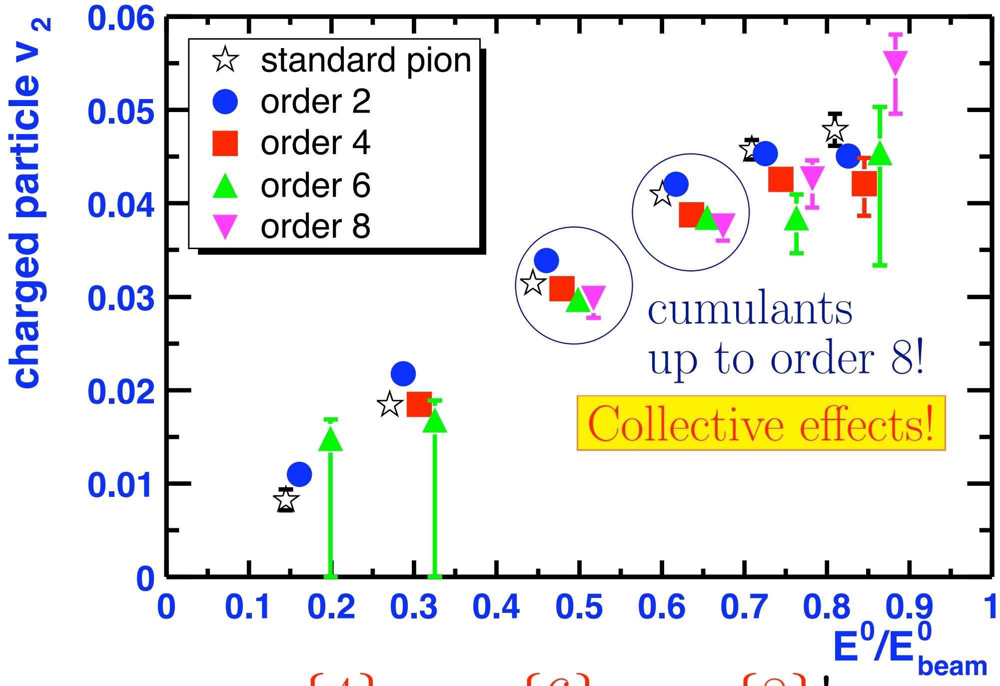
# $v_1$ from THREE-PARTICLE CORRELATIONS

See Alexander Wetzler's talk for further experimental details

Integrated $v_2$ and $v_1$ vs. centrality, 158$A$ GeV Pb-Pb

① Pick a reference (integrated) $v_2$

② and deduce the integrated $v_1$:

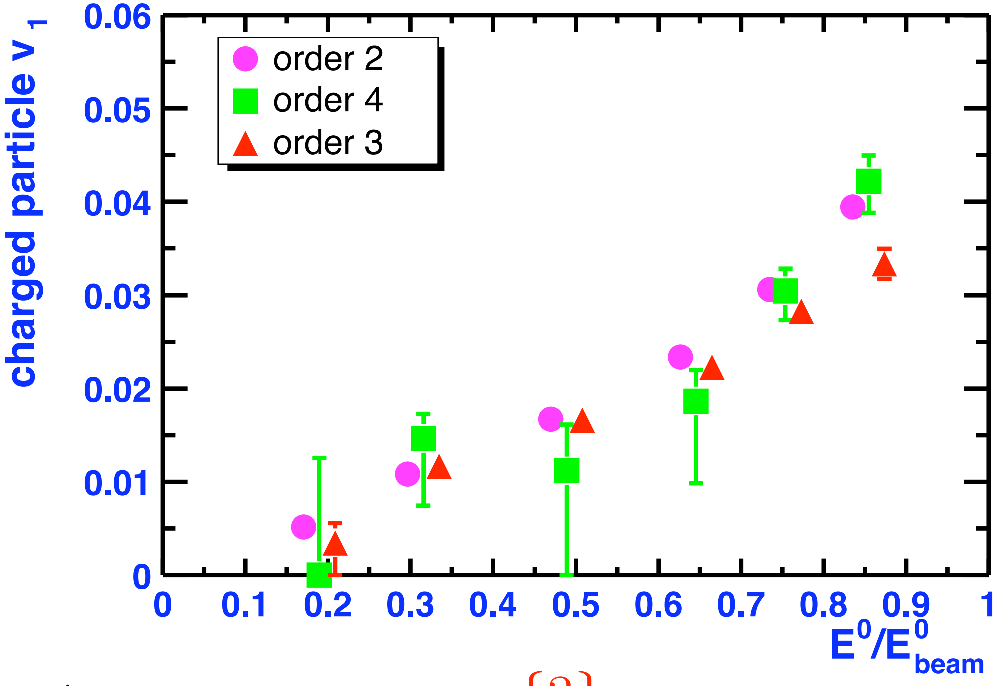


cumulants
up to order 8!

Collective effects!

$v_2\{4\} = v_2\{6\} = v_2\{8\}$!

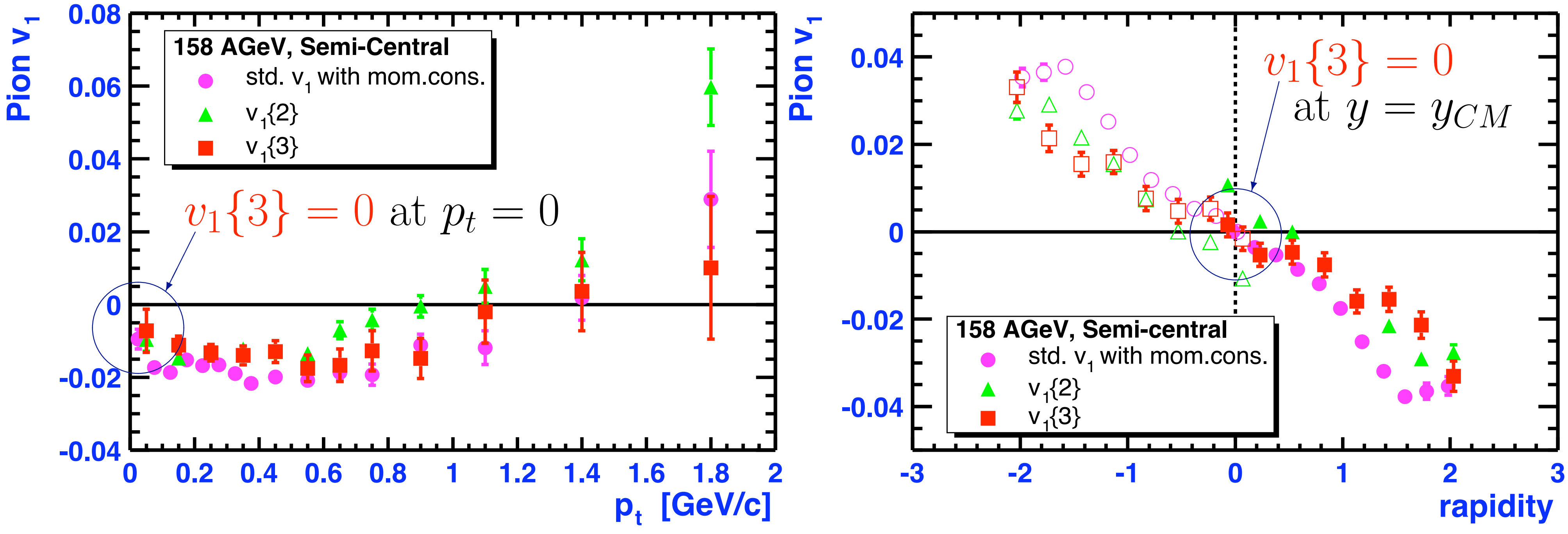
$\Longrightarrow$



$v_1\{3\}$

# $v_1$ from THREE-PARTICLE CORRELATIONS

③ Reference $v_2$ + integrated $v_1\{3\}$ yield differential $v_1\{3\}$:
$v_1(p_t)$ and $v_1(y)$ for charged pions, semicentral 158$A$ GeV Pb-Pb



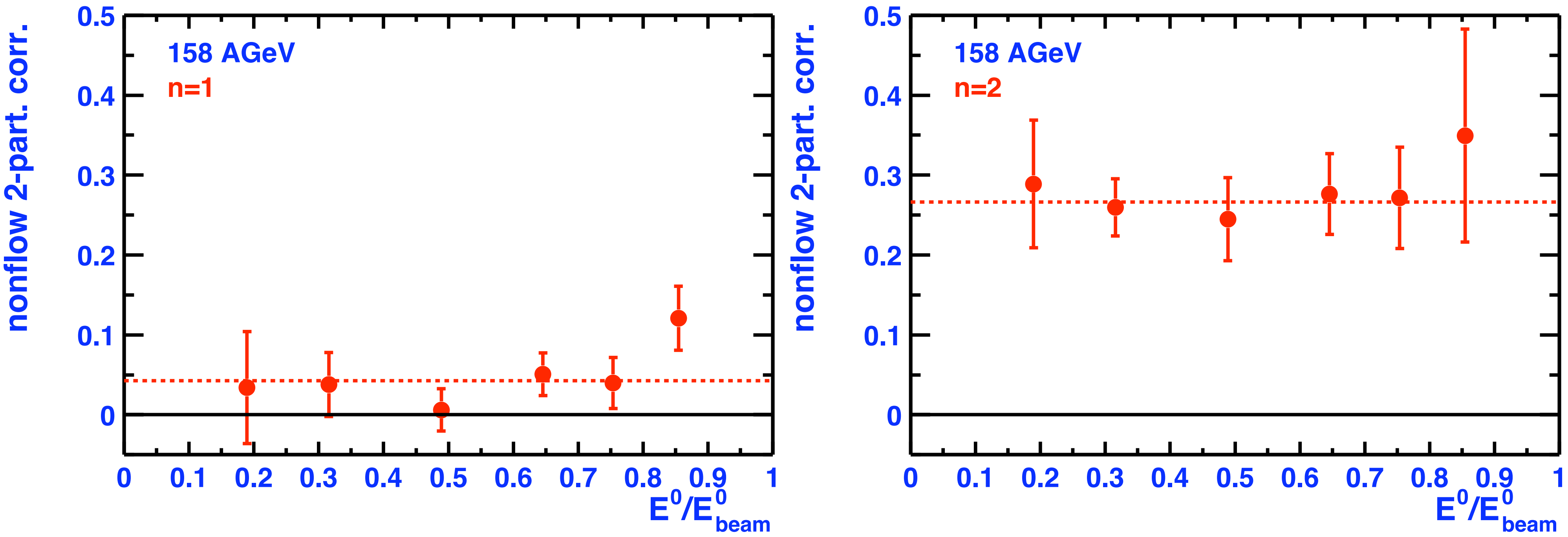
$v_1\{3\} = 0$ at $p_t = 0$

$v_1\{3\} = 0$ at $y = y_{CM}$

# TWO-PARTICLE NONFLOW EFFECTS

$$\left.\begin{aligned}(v_n\{2\})^2 - {v_n}^2 &= \mathcal{O}(1/N)\\ {v_n}^2 &\simeq (v_n\{k>2\})^2\end{aligned}\right\} \begin{array}{l} N\left[(v_n\{2\})^2 - (v_n\{k>2\})^2\right] = \mathcal{O}(1), \\ \text{independent of centrality}\end{array}$$

$$N\left[(v_1\{2\})^2 - (v_1\{3\})^2\right] \qquad N\left[(v_2\{2\})^2 - (v_2\{4\})^2\right]$$



# MEASURING DIRECTED FLOW WITH THREE-PARTICLE CORRELATIONS

At ultrarelativistic energies, $v_1$ is very small, and thus difficult to measure:

☹ Two-particle methods biased by systematic errors (2-particle nonflow effects)

☹ Four-particle cumulant method plagued by statistical uncertainty

☺ Three-particle method NEW

- ♡ free from nonflow effects
- ♡ moderate statistical uncertainty
- ♡ successful application to NA49 data

# PRACTICAL $v_1$ ANALYSIS: GENERATING FUNCTION

① For each event, compute the generating function

$$G(z_1, z_2) = \prod_{k=1}^{M} \left( 1 + \frac{z_1^*\, e^{i\phi_k} + z_1\, e^{-i\phi_k} + z_2^*\, e^{2i\phi_k} + z_2\, e^{-2i\phi_k}}{M} \right),$$

then average $G(z_1, z_2)$ over events:

$$\langle G(z_1, z_2) \rangle = \cdots + \frac{z_1^{*2} z_2}{M^3} \left\langle \sum_{j,k,l} e^{i(\phi_j + \phi_k - 2\phi_l)} \right\rangle + \cdots$$

② Deduce the cumulants, taking

$$M \left( \langle G(z_1, z_2) \rangle^{1/M} - 1 \right) = \cdots + \frac{z_1^{*2} z_2}{2} \left\langle\!\!\left\langle e^{i(\phi_1 + \phi_2 - 2\phi_3)} \right\rangle\!\!\right\rangle + \cdots$$

③ With a reference value of $v_2$, extract (the integrated) $v_1$, using

$$\left\langle\!\!\left\langle e^{i(\phi_1 + \phi_2 - 2\phi_3)} \right\rangle\!\!\right\rangle = \begin{cases} (v_1)^2\, v_2 \quad \text{(good acceptance)} \\ \qquad\qquad\qquad \text{depends on the detector} \\ \alpha\, (v_1)^2\, v_2 \quad \text{(}\xcancel{\text{PHENIX}}\text{ uneven acceptance)} \end{cases}$$

# PRACTICAL $v_1$ ANALYSIS: GENERATING FUNCTION

④ To obtain the differential $v'_1$, compute the generating function

differential particle

average over all differential particles

$$\mathcal{D}(z_1, z_2) = \frac{\left\langle e^{i\psi}\, G(z_1, z_2)\right\rangle}{\langle G(z_1, z_2)\rangle} = \cdots + z_1^* z_2 \left\langle\!\!\left\langle e^{i(\psi+\phi_1-2\phi_2)} \right\rangle\!\!\right\rangle + \cdots$$

average over all events

⑤ With the reference $v_2$ and integrated $v_1$ (step ③), deduce the differential $v'_1$ [either $v_1(p_t)$ or $v_1(y)$]:

$$\left\langle\!\!\left\langle e^{i(\psi+\phi_1-2\phi_2)} \right\rangle\!\!\right\rangle = \begin{cases} v'_1\, v_1\, v_2 \quad \text{(good acceptance)} \\ \\ \alpha'\, v'_1\, v_1\, v_2 + \beta'\, (v_1)^2\, v'_2 \ \text{(bad acceptance)} \end{cases}$$

⑥ Post your paper on `nucl-ex` and collect citations.