Jet broadening in a medium-modified parton shower approach

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Medium-induced modifications of jets

- Jet physics in collisions of elementary particles

Modified Leading Logarithmic Approximation (MLLA)

(of QCD)

- Towards a “medium-modified MLLA”: analytical approach
  - longitudinal distributions inside jets

N.B. & U.A.Wiedemann, hep-ph/0506218

  - transverse momentum distributions inside jets

N.B.... work in progress!

Analytical calculations only: no comparison with data!
☞ intended as baseline for Monte Carlo implementations.

Jets “in vacuum”

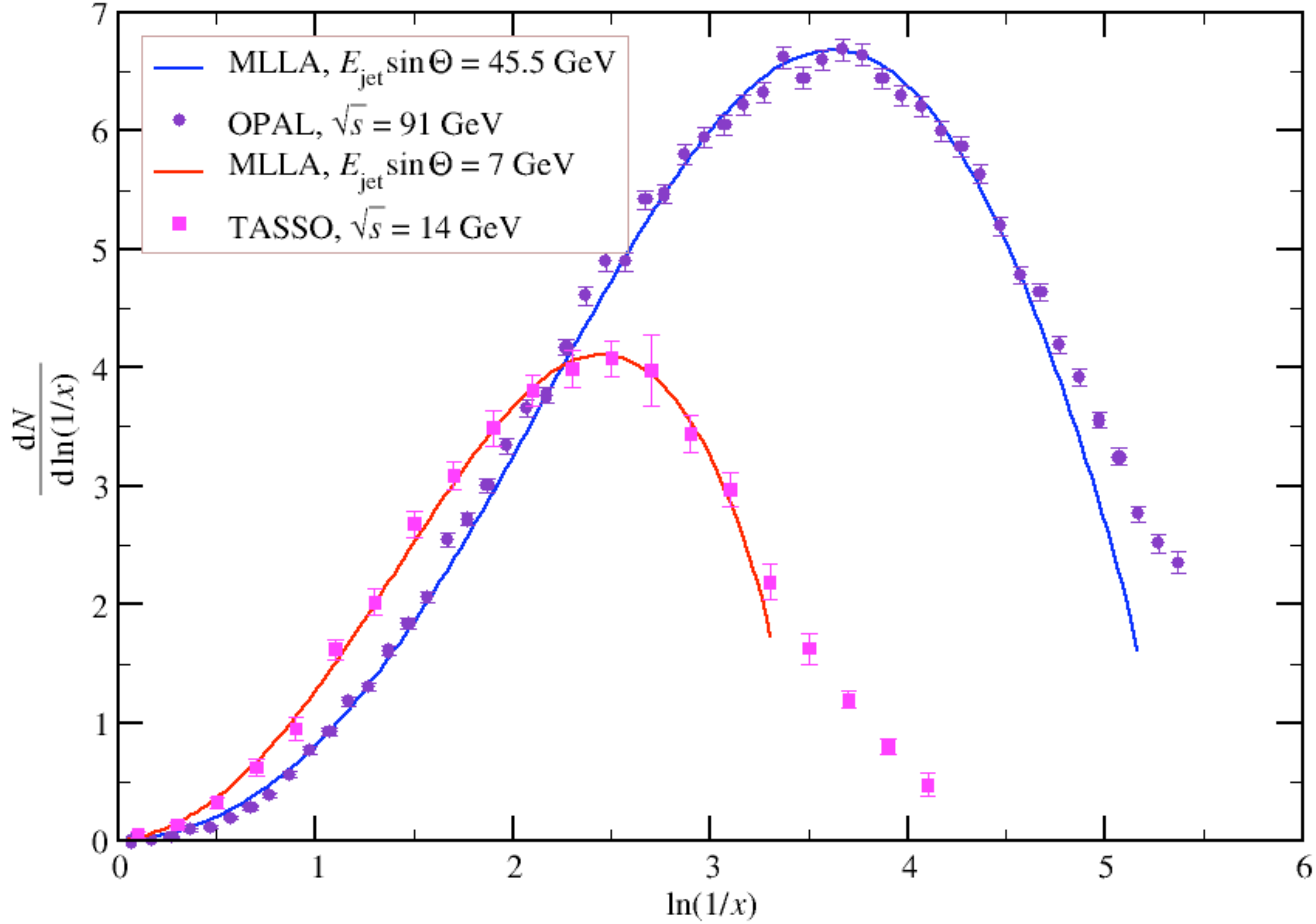
- Jet physics in collisions of elementary particles
  - longitudinal inclusive distributions inside jets

    e^+e^- collisions, $p\bar{p}$ collisions

    particle with momentum $(k_0, \vec{k})$

$$\boxed{\xi \quad \text{aka} \quad \ell \equiv \ln \frac{E_{\rm jet}}{k_0} = \ln \frac{1}{x}}$$

Jets in vacuum: successes of MLLA
e^+e^- data



TASSO Collaboration, Z. Phys. C **47** (1990) 187

OPAL Collaboration, Phys. Lett. B **247** (1990) 617 (includes comparison with MLLA)

Jets in vacuum: successes of MLLA
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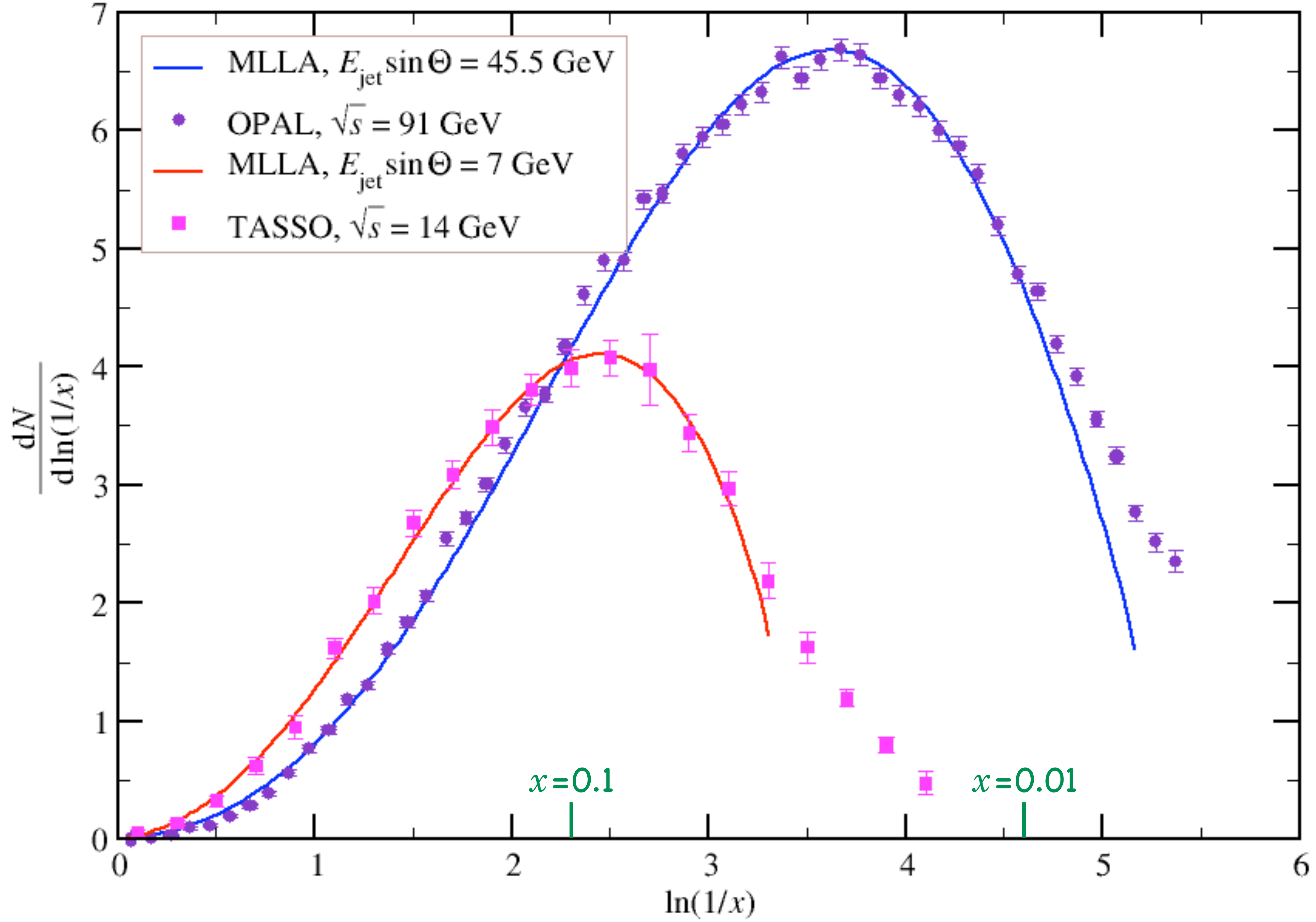

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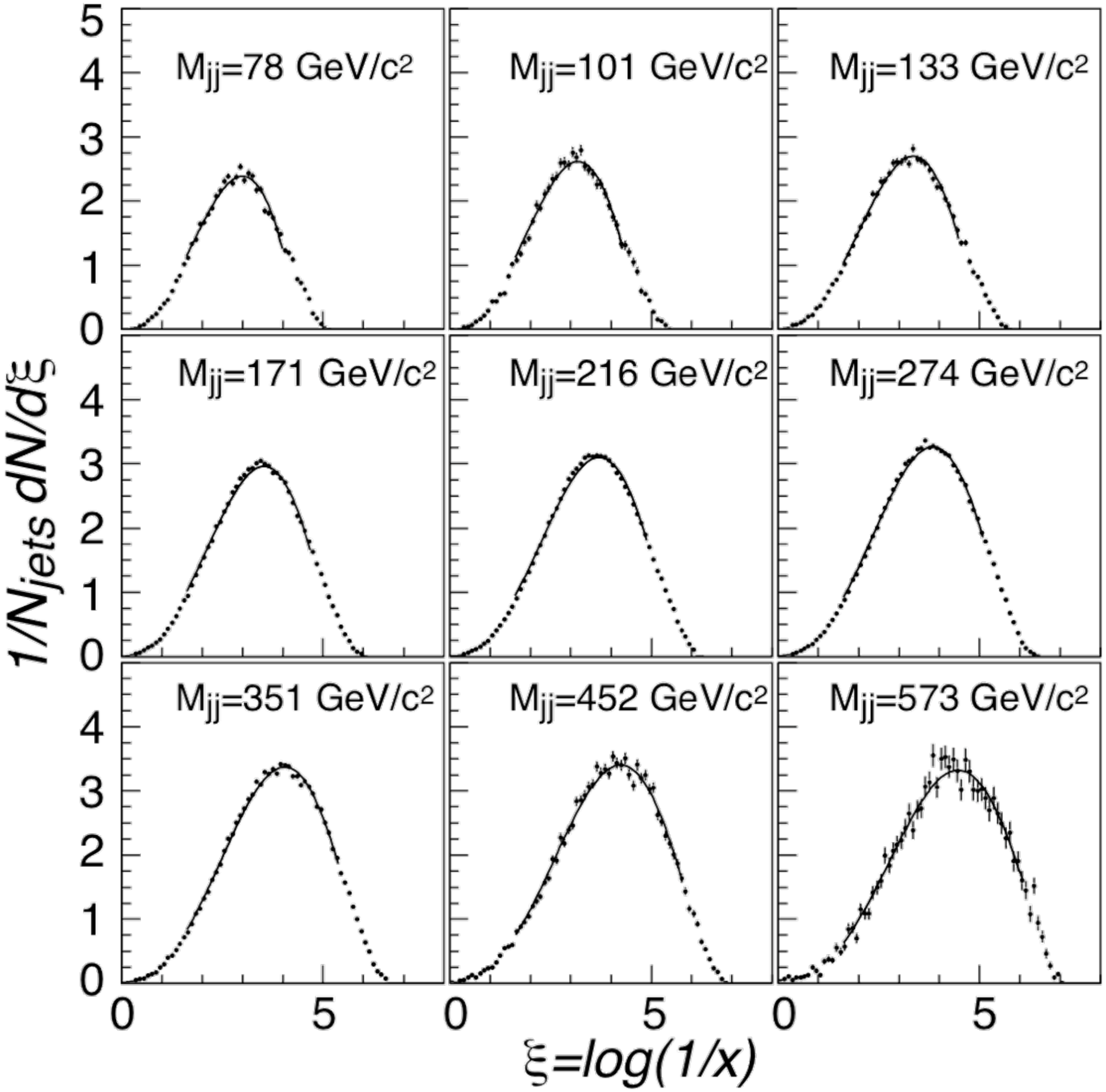
Jets in vacuum: successes of MLLA

$p\bar{p}$ data



data (here $\theta_c = 0.47$) + comparison with MLLA: CDF Collaboration, Phys. Rev. D **68** (2003) 012003

Jets in vacuum

- Jet physics in collisions of elementary particles
  - longitudinal inclusive distributions inside jets

    e^+e^- collisions, $p\bar{p}$ collisions

  - transverse momentum inclusive distributions inside jets

    $p\bar{p}$ collisions

particle with momentum ($k_0, k_\parallel \simeq k_0\,, \vec{k}_\perp$)

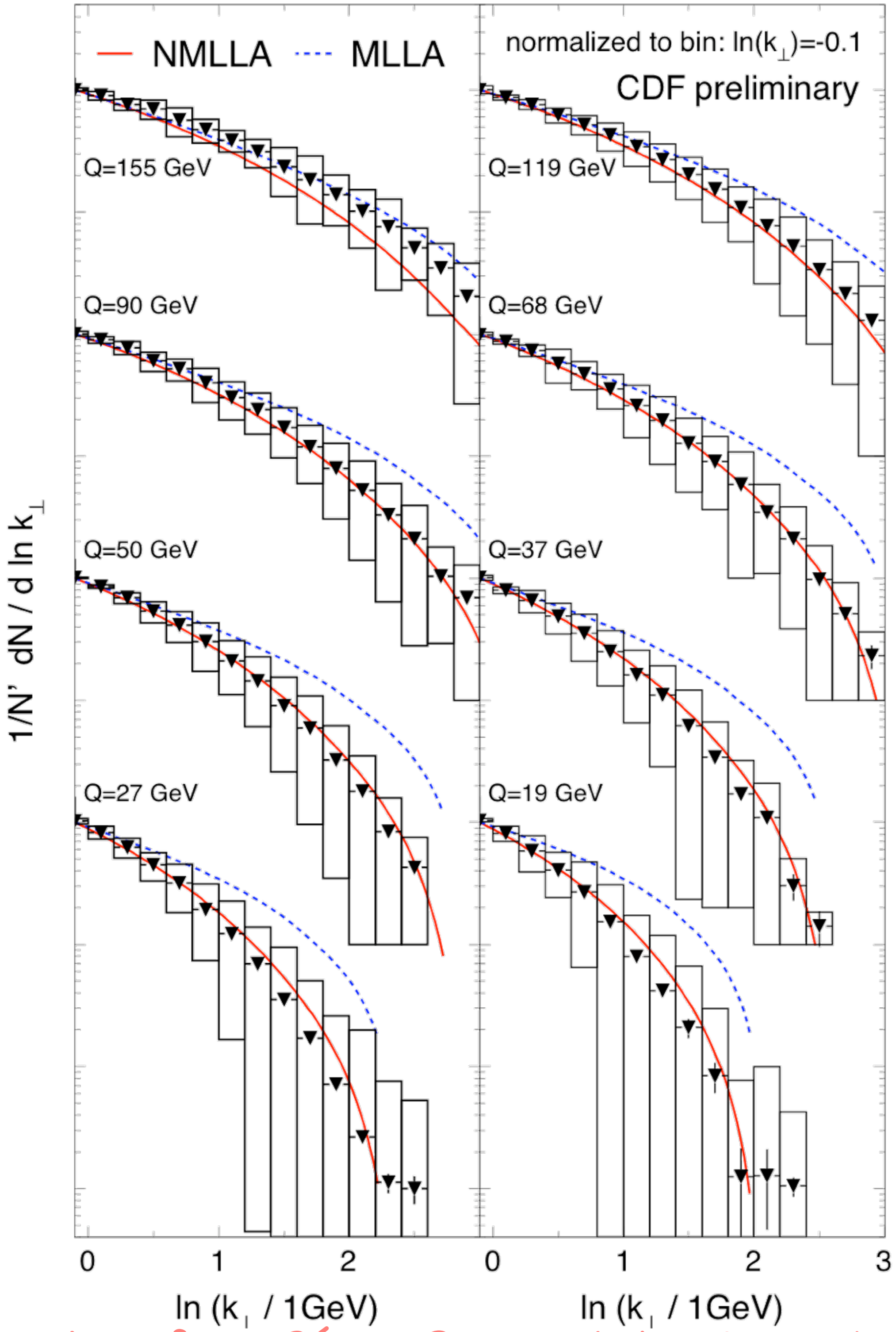
$$\boxed{y \equiv \ln\frac{k_\perp}{Q_0} \simeq \ln\frac{k_0\Theta}{Q_0}}$$

$k_\parallel$, $\vec{k}_\perp$, Θ: with respect to the jet axis (direction of energy flow)

Q_0 infrared cutoff

Jets in vacuum: successes of (N)MLLA

$p\bar{p}$ data

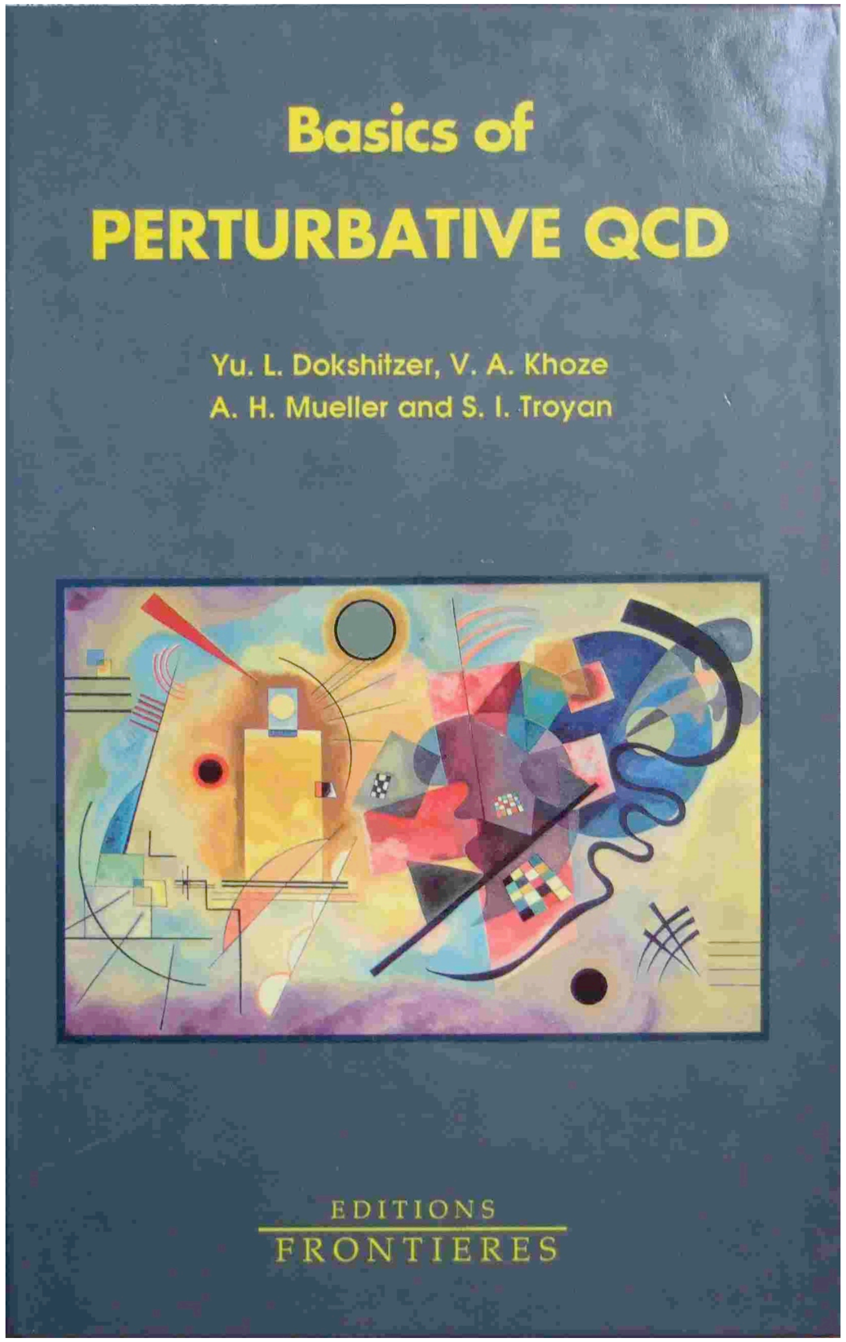


comparison from Pérez-Ramos, Arleo & Machet, Phys. Rev. D **78** (2008) 014019

Jets “in vacuum”:

Ye big Booke of pQCD

(especially for jet calculus)



Basics of

PERTURBATIVE QCD

Yu. L. Dokshitzer, V. A. Khoze
A. H. Mueller and S. I. Troyan

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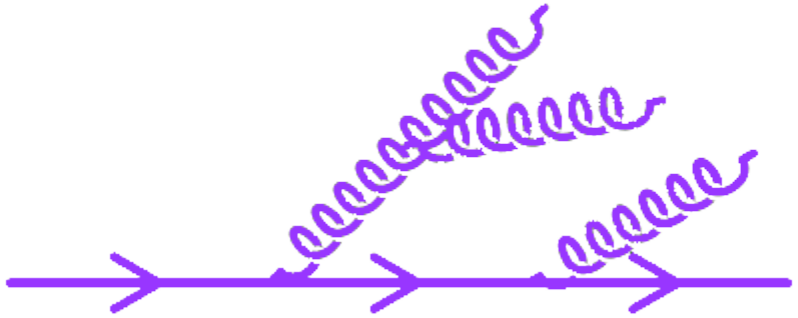
available @ http://www.lpthe.jussieu.fr/~yuri/BPQCD/cover.html

MLLA in four/five slides

Main ingredients:

- Resummation of double- and single-logarithms in $\ln\frac{1}{x}$ and $\ln\frac{E_{\rm jet}}{Q_0}$;
- Takes into account the running of α_s along the parton shower evolution;
- Probabilistic interpretation (results from intra-jet colour coherence):
  - independent successive branchings $g\rightarrow gg\,,\,g\rightarrow q\bar{q}\,,\,q\rightarrow qg$;
  - with angular ordering of the sequential parton decays:

    at each step in the evolution,
    the angle between father and
    offspring partons decreases.



- Includes in a systematic way $\underline{\text{next-to-leading-order corrections}}$.

$$\mathcal{O}(\sqrt{\alpha_s})\ !$$

MLLA in four/five slides

Central object: generating functional $Z_i[Q, \Theta; u(k)]$

☞ generates the various cross-sections ($\rightarrow ggg$, $\rightarrow ggq\bar{q}$ ...) for a jet initiated by a parton i ($= g, q, \bar{q}$) with energy Q in a cone of angle Θ.

$$\begin{aligned} Z_i[Q, \Theta; u(k)] \;=\; & \mathrm{e}^{-w_i(Q,\Theta)}\, u(Q) \\ & + \sum_j \int^{\Theta} \frac{\mathrm{d}\Theta'}{\Theta'} \int_0^1 \mathrm{d}z\; \mathrm{e}^{w_i(Q,\Theta') - w_i(Q,\Theta)}\, \frac{\alpha_s(k_\perp)}{2\pi} \\ & \quad \times P_{ji}(z)\, Z_j[zQ, \Theta'; u]\, Z_k[(1-z)Q, \Theta'; u] \end{aligned}$$

MLLA in four/five slides

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angular ordering

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probability to have
no branching with angle < Θ

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between Θ and Θ'

angular ordering

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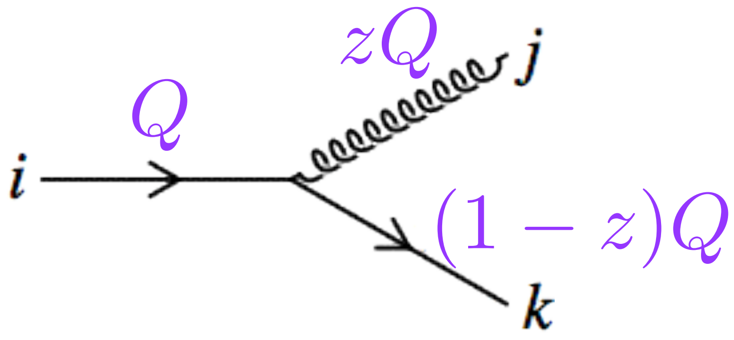
probability to have no branching with angle < Θ

between Θ and Θ'

angular ordering

LO splitting function $i \to jk$



MLLA in four/five slides

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probability to have
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angular
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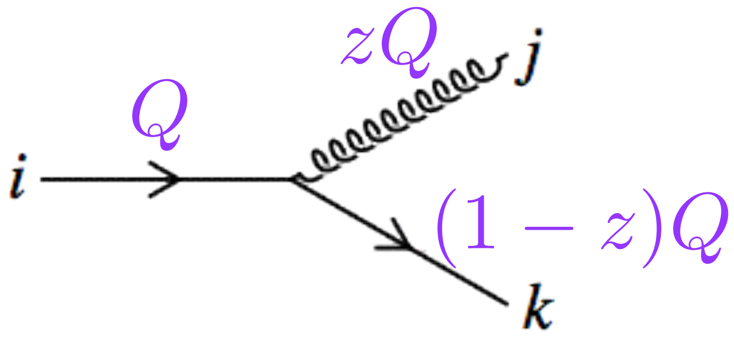
LO splitting function $i \to jk$

$k_\perp \approx z(1-z)Q$



MLLA in four/five slides

The 1st derivative of the generating function gives (after some detours in Mellin space) the inclusive longitudinal distribution of partons inside a (gluon) jet:

"limiting spectrum"

$$\bar{D}_G^{\lim}(x, Y_\Theta, Q_0) = \frac{4N_c Y_\Theta}{bB(B+1)} \int_{\epsilon-\mathrm{i}\infty}^{\epsilon+\mathrm{i}\infty} \frac{\mathrm{d}\nu}{2\pi\mathrm{i}}\, x^{-\nu}\, \Phi(-A+B+1, B+2; -\nu Y_\Theta)$$

with

$$A \equiv \frac{4N_c}{b\nu}, \qquad B \equiv \frac{a}{b}, \qquad a \equiv \frac{11}{3}N_c + \frac{2N_f}{3N_c^2}, \qquad b \equiv \frac{11}{3}N_c - \frac{2}{3}N_f$$

(these coefficients follow from the prefactors of the leading-order splitting functions)

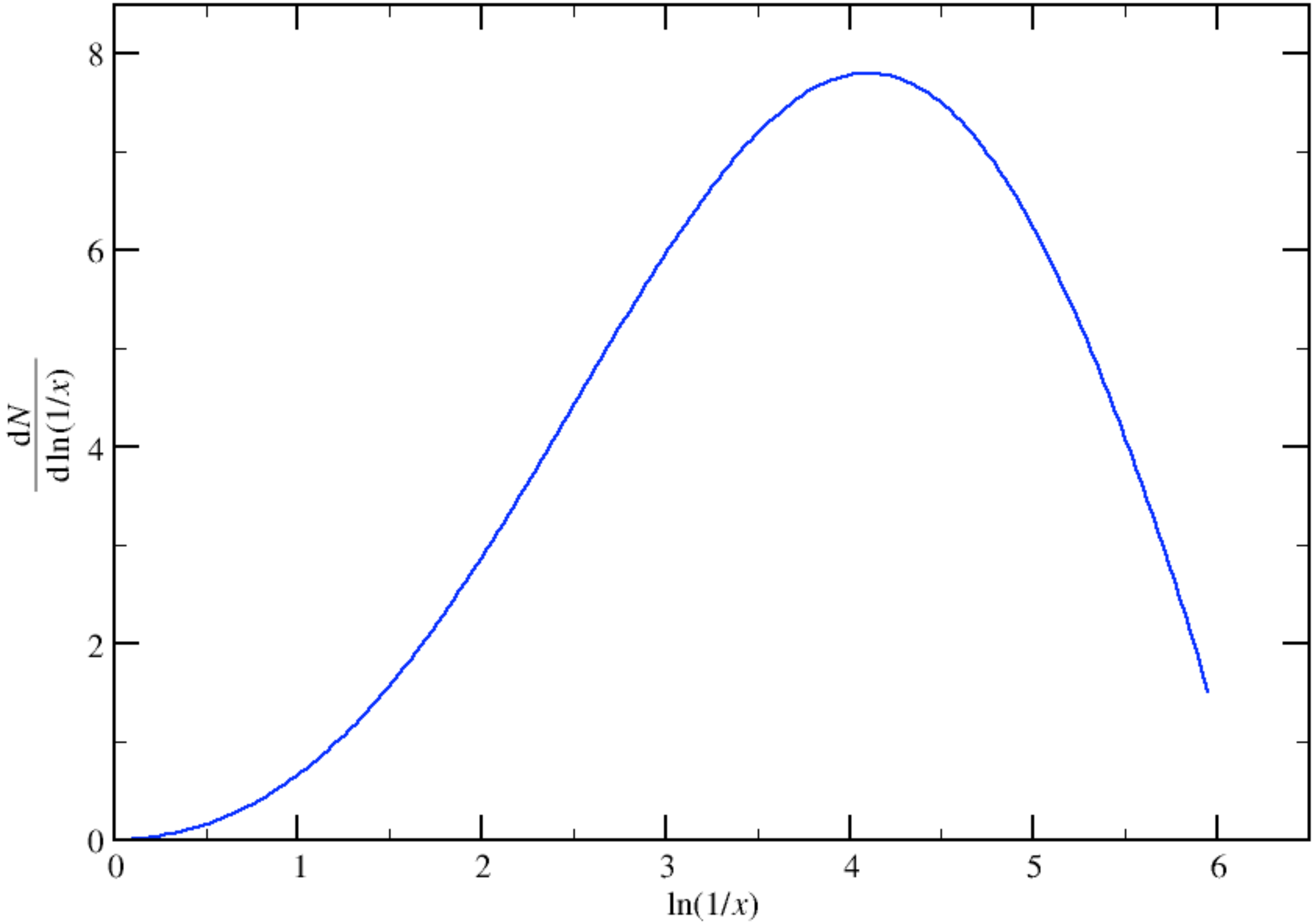
and

$$Y_\Theta \equiv \ln \frac{E_{\mathrm{jet}} \sin \Theta}{Q_0}$$

Impressive expression... which can be dealt with!

MLLA in four/five slides

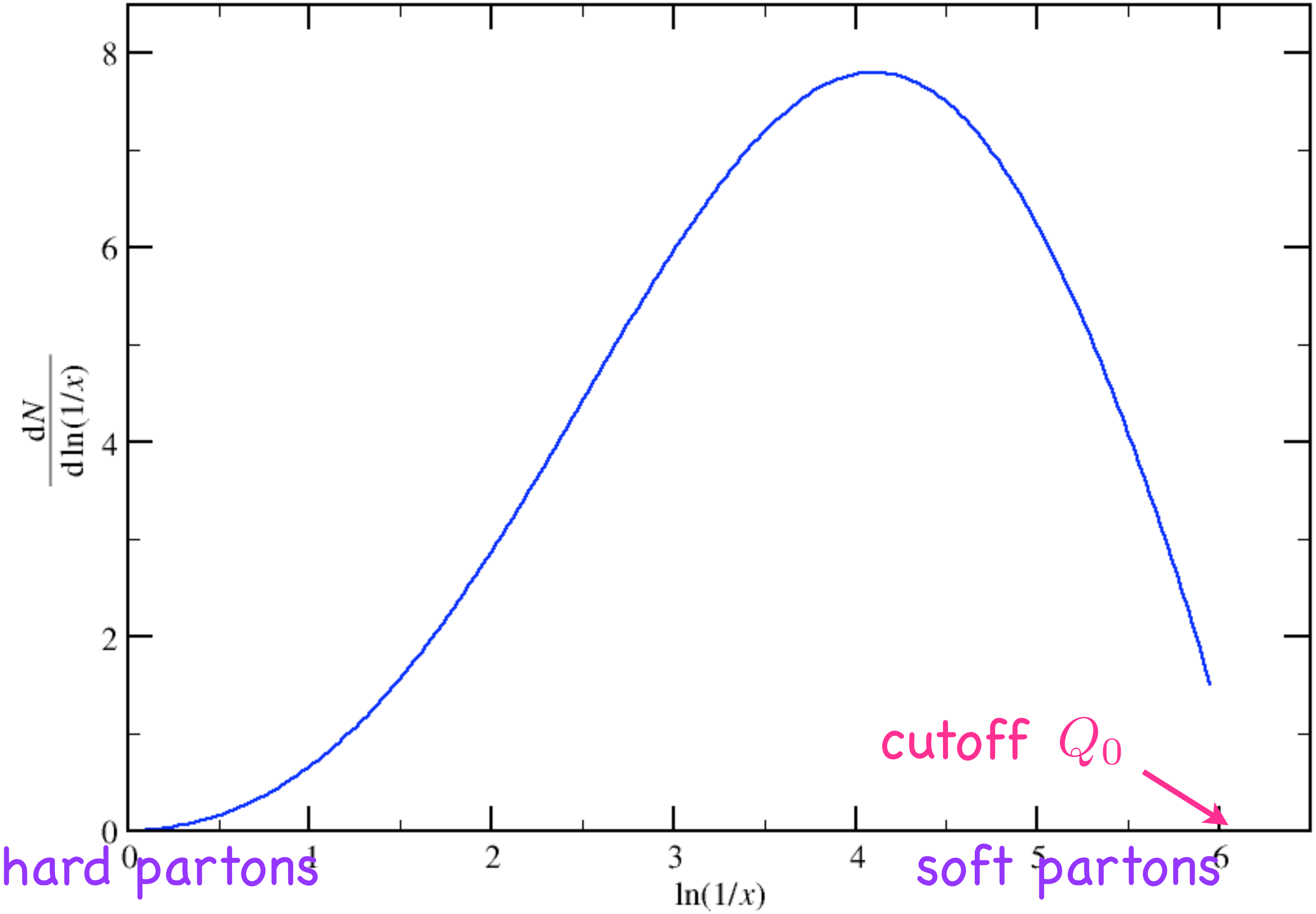
For a 100 GeV parton:



☞ "hump-backed plateau"

MLLA in four/five slides

For a 100 GeV parton:



cutoff Q_0

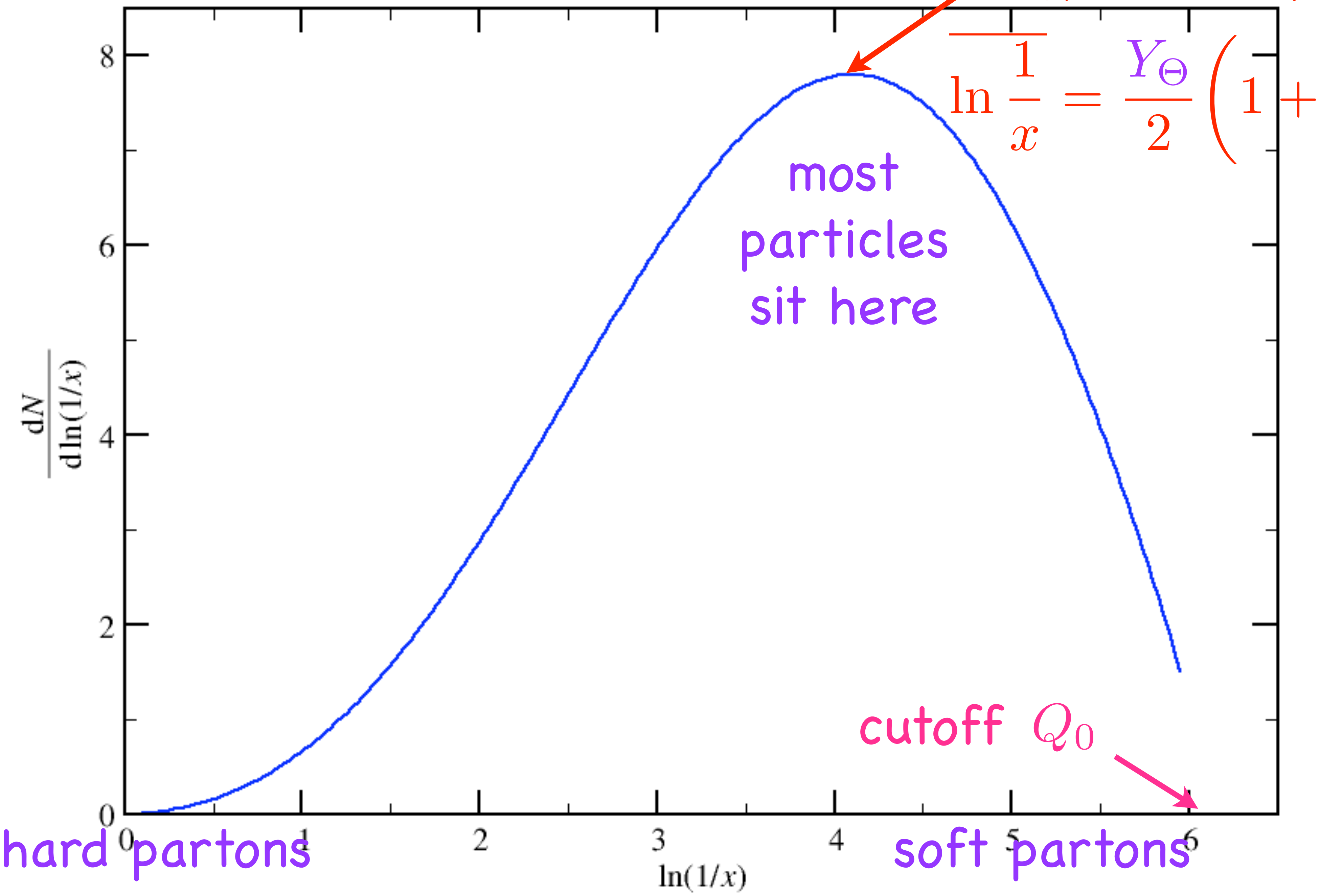
hard partons

soft partons

☞ “hump-backed plateau”

MLLA in four/five slides

For a 100 GeV parton:



maximum multiplicity for

$$\overline{\ln\frac{1}{x}} = \frac{Y_\Theta}{2}\left(1 + a\sqrt{\frac{\alpha_s}{8\pi N_c}}\right)$$

most particles sit here

cutoff Q_0

hard partons

soft partons

☞ “hump-backed plateau”

Note: the hump is dominated by the singular parts of the $P_{ji}(z)$.

MLLA in four/five slides

Modified Leading Logarithmic Approximation:

- Successive independent parton splittings, with a constraint on the emission angles

⇒ limiting spectrum $\bar{\mathcal{D}}^{\rm lim}(x, Y_\Theta, Q_0)$

- The spectrum is exact in the asymptotic $Y_\Theta \to \infty$ limit and includes in a systematic way $\underbrace{\text{corrections to subleading order}}_{\mathcal{O}(\sqrt{\alpha_s})}$

MLLA in four/five slides

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- What about hadronization? ($\bar{\mathcal{D}}^{\rm lim}(x, Y_\Theta, Q_0)$ is a parton spectrum)

☞ Local parton-hadron duality (LPHD)

$$\bar{\mathcal{D}}^{\rm h}(x, Y_\Theta, Q_0) = K^{\rm h}\bar{\mathcal{D}}^{\rm lim}(x, Y_\Theta, Q_0)$$

⇒ two parameters Q_0 and $K^{\rm h}$.

Modeling the medium influence: a suggestion

- The hump of the limiting spectrum is mostly due to the singular parts of the splitting functions.
- In medium, the emission of a soft gluons by a fast parton increases.

☞ One can model medium-induced effects by modifying the parton splitting functions $P_{ji}(z)$ and especially their singular $\frac{1}{z}$ parts:

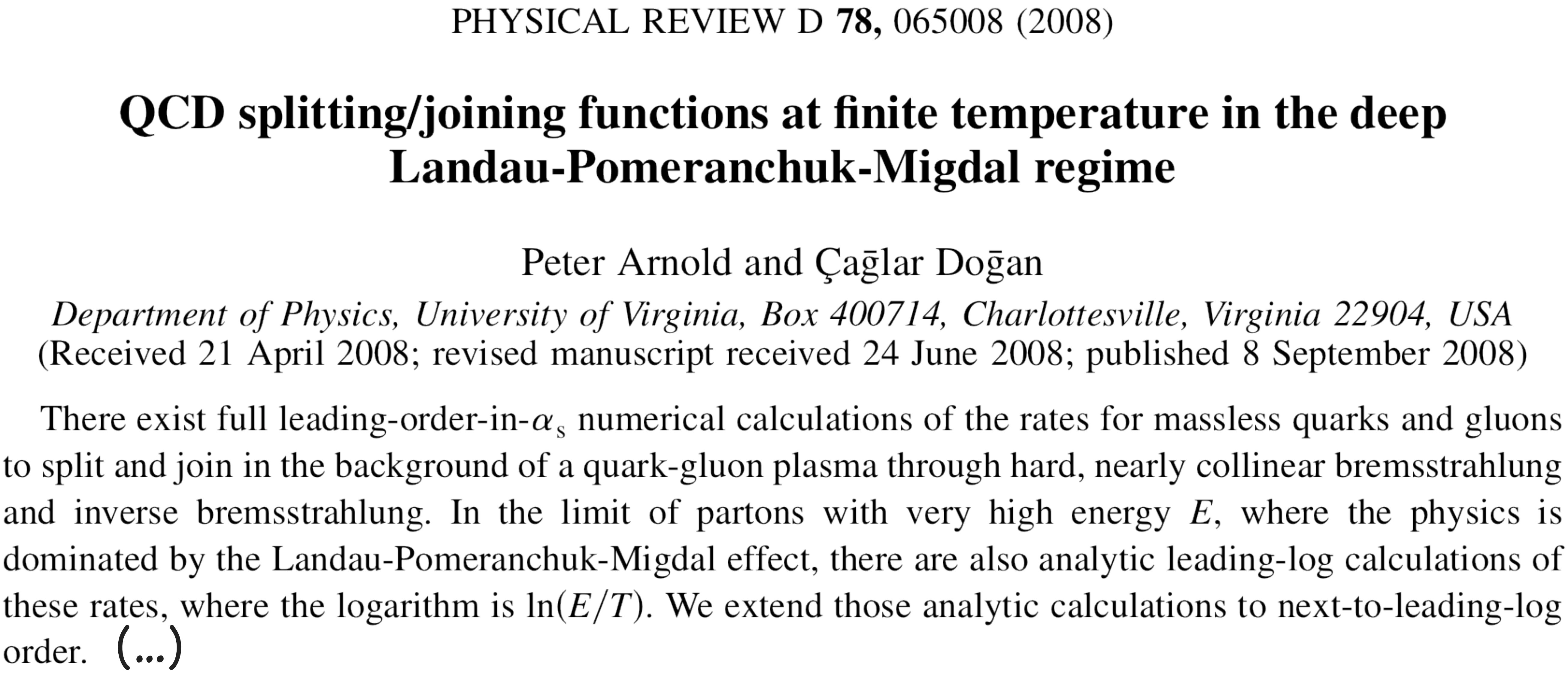
$$P_{Gq}(z) = \frac{4}{3}\left[\frac{2(1+f_{\rm med})}{z} - 2 + z\right]$$ and so on.

$f_{\rm med} > 0 \quad \Rightarrow$ Bremsstrahlung increases

⚠ I am messing up with the splitting functions...

☞ usual sum rules no longer hold.

Medium-modified splitting functions vs. the "usual" approach to jet quenching



PHYSICAL REVIEW D **78,** 065008 (2008)

QCD splitting/joining functions at finite temperature in the deep Landau-Pomeranchuk-Migdal regime

Peter Arnold and Çağlar Doğan

Department of Physics, University of Virginia, Box 400714, Charlottesville, Virginia 22904, USA

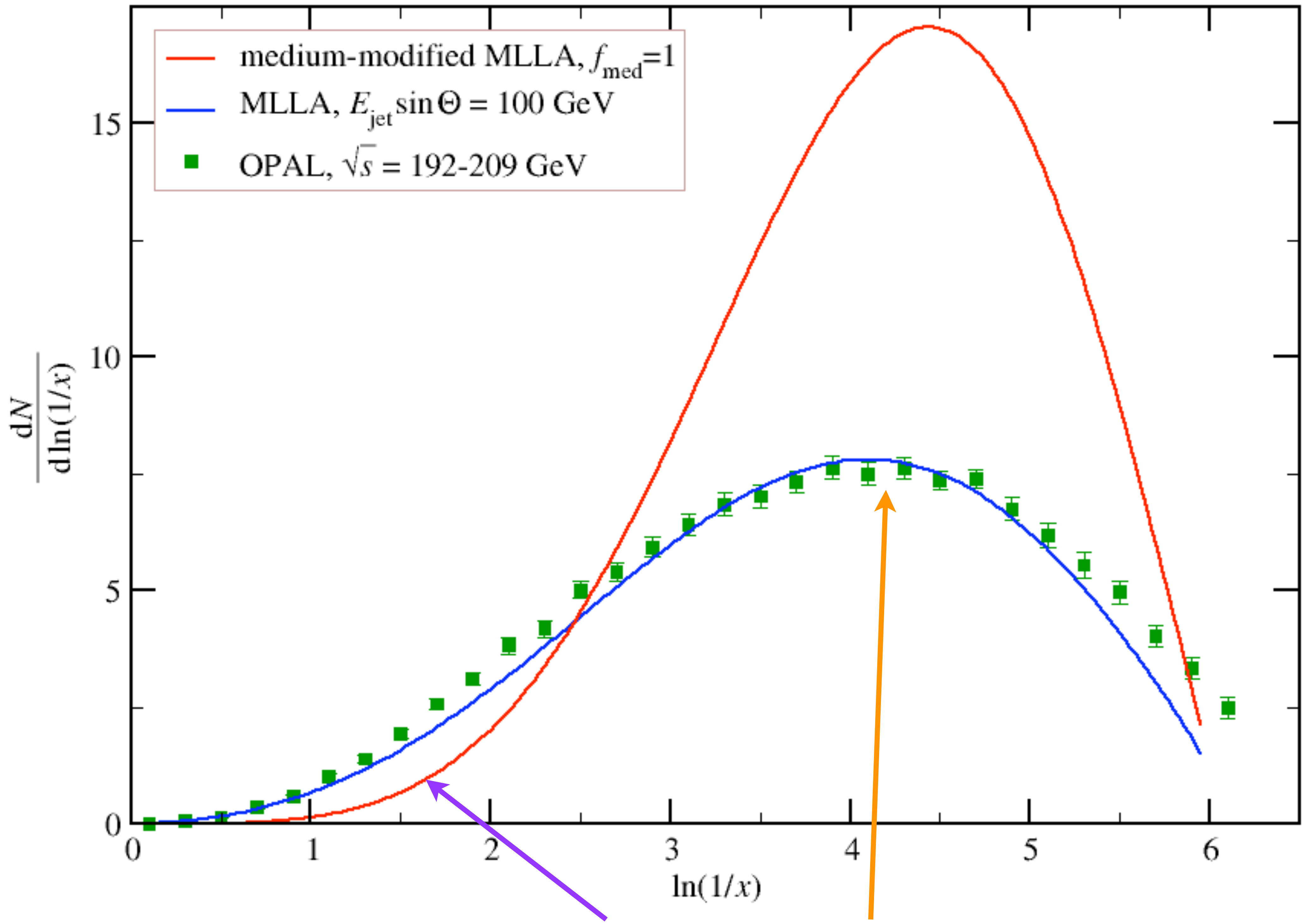
(Received 21 April 2008; revised manuscript received 24 June 2008; published 8 September 2008)

There exist full leading-order-in-α_s numerical calculations of the rates for massless quarks and gluons to split and join in the background of a quark-gluon plasma through hard, nearly collinear bremsstrahlung and inverse bremsstrahlung. In the limit of partons with very high energy E, where the physics is dominated by the Landau-Pomeranchuk-Migdal effect, there are also analytic leading-log calculations of these rates, where the logarithm is $\ln(E/T)$. We extend those analytic calculations to next-to-leading-log order. (...)

"My" constant $f_{\rm med}$: irrealistic, but allows analytical computations, to check (future) Monte-Carlo cascades.

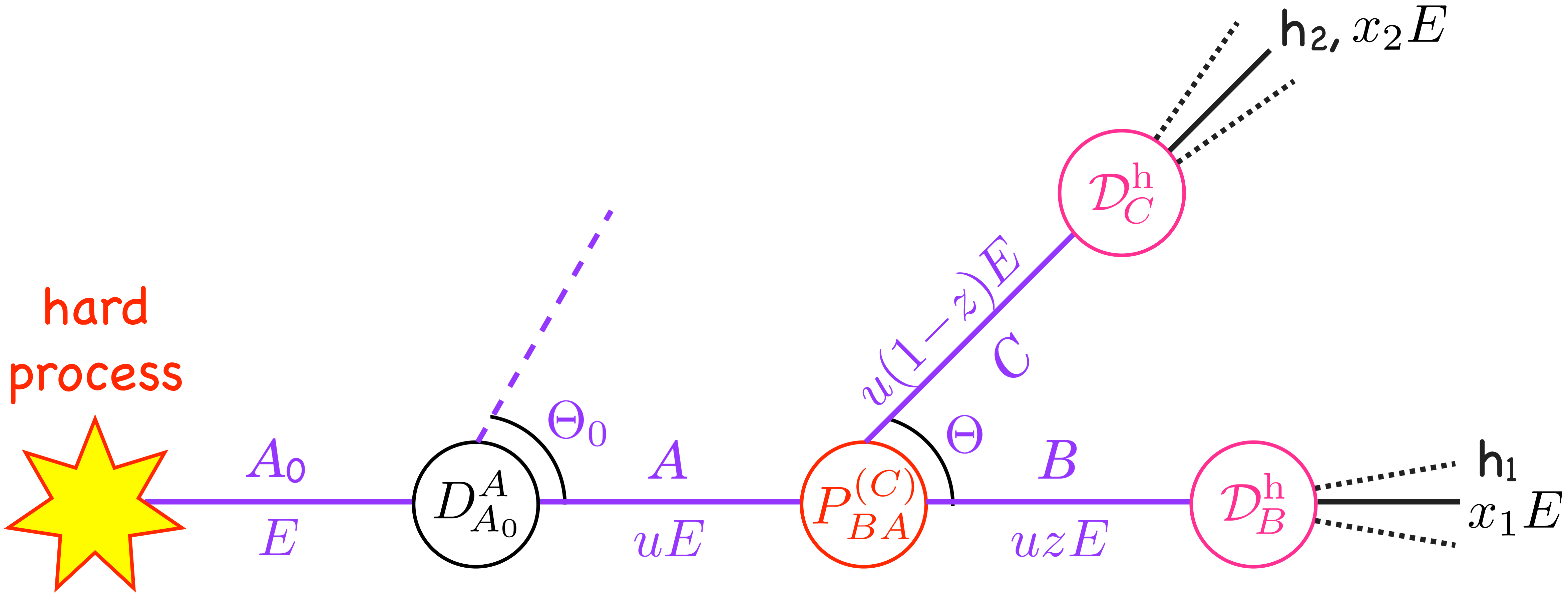
Medium-modified hump-backed plateau

inclusive longitudinal distribution



Partons are redistributed from large x to small x.

Two further slides on MLLA…



- Probability $D_{A_0}^{A}(u, E\Theta_0, uE\Theta)$ that parton A_0 gives rise to A.
- A splits into B (energy fraction z) and C: splitting function $P_{BA}(z)$.
- Hadronization from B into h₁ (energy x_1E) & C into h₂ (energy x_2E).

☞ double differential (x_1, x_2,Θ) 2-particle cross-section

Two further slides on MLLA...

Integrating the double differential two-particle cross-section over one of the hadrons (weighted with its energy) yields the double differential single-particle inclusive distribution:

$$\frac{\mathrm{d}^2 N}{\mathrm{d}x\,\mathrm{d}\ln\Theta} = \frac{\mathrm{d}}{\mathrm{d}\ln\Theta}\left[\sum_A \int \mathrm{d}u\, D_{A_0}^{A}(u, E\Theta_0, uE\Theta)\, \mathcal{D}_A^{\mathrm{h}}\!\left(\frac{x}{u}, uE\Theta, Q_0\right)\right]$$

Θ angle with respect to the direction of energy flow (jet axis).

detailed computations in Pérez-Ramos & Machet, JHEP **04** (2006) 043

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Θ angle with respect to the direction of energy flow (jet axis).

After some algebraic manipulations...

$$\left(\frac{\mathrm{d}^2N}{\mathrm{d}x\,\mathrm{d}\ln\Theta}\right)_{G,q} = \frac{\mathrm{d}}{\mathrm{d}\ln\Theta}\left[\frac{\langle C\rangle_{G,q}}{N_c}\,\bar{\mathcal{D}}_G^{\mathrm{lim}}\left(\ln\frac{1}{x},\ln\frac{E_{\mathrm{jet}}\Theta}{Q_0},Q_0\right)\right]$$

where $\langle C\rangle_{A_0}$ is the average color current of partons.

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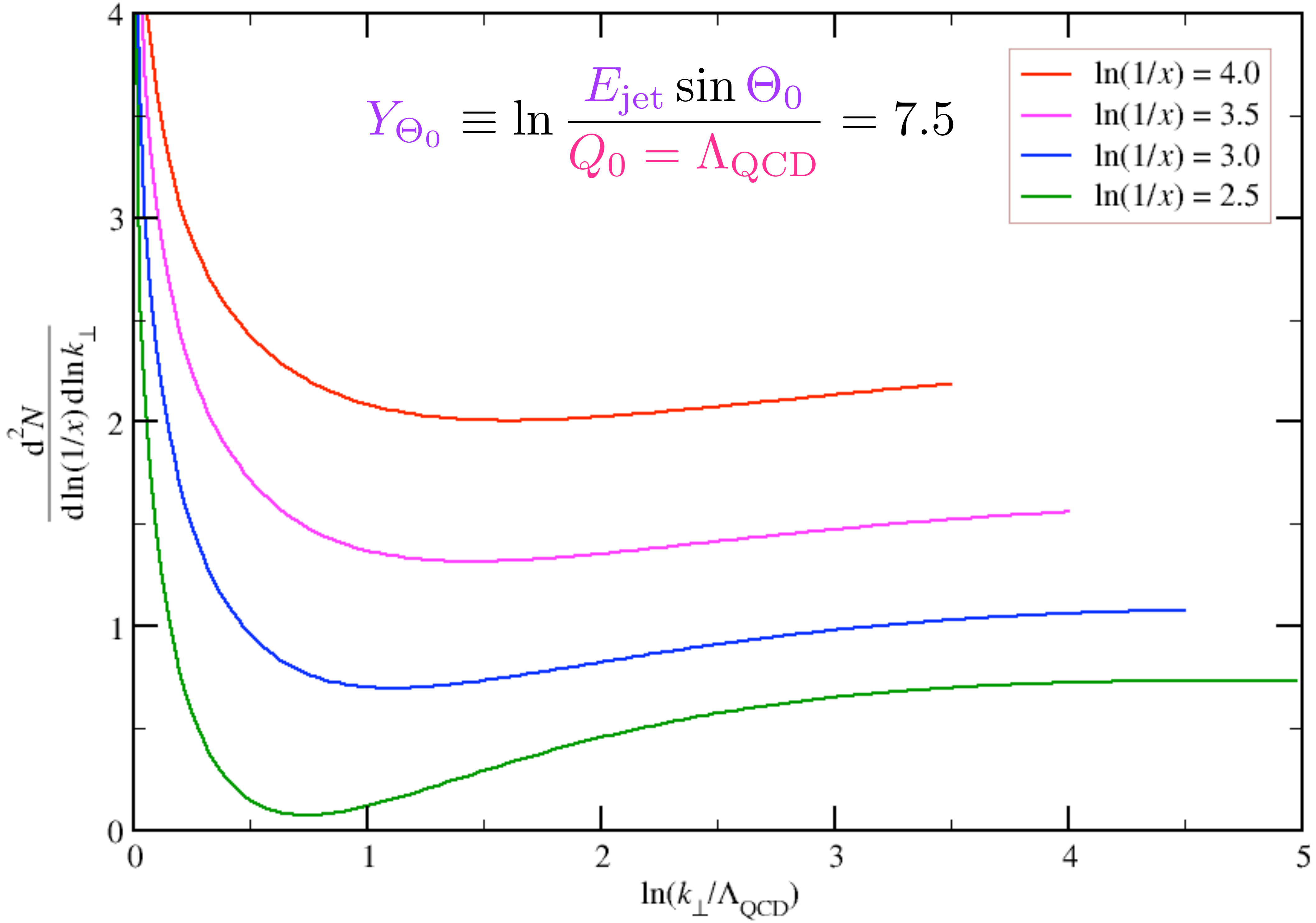
where $\langle C\rangle_{A_0}$ is the average color current of partons.

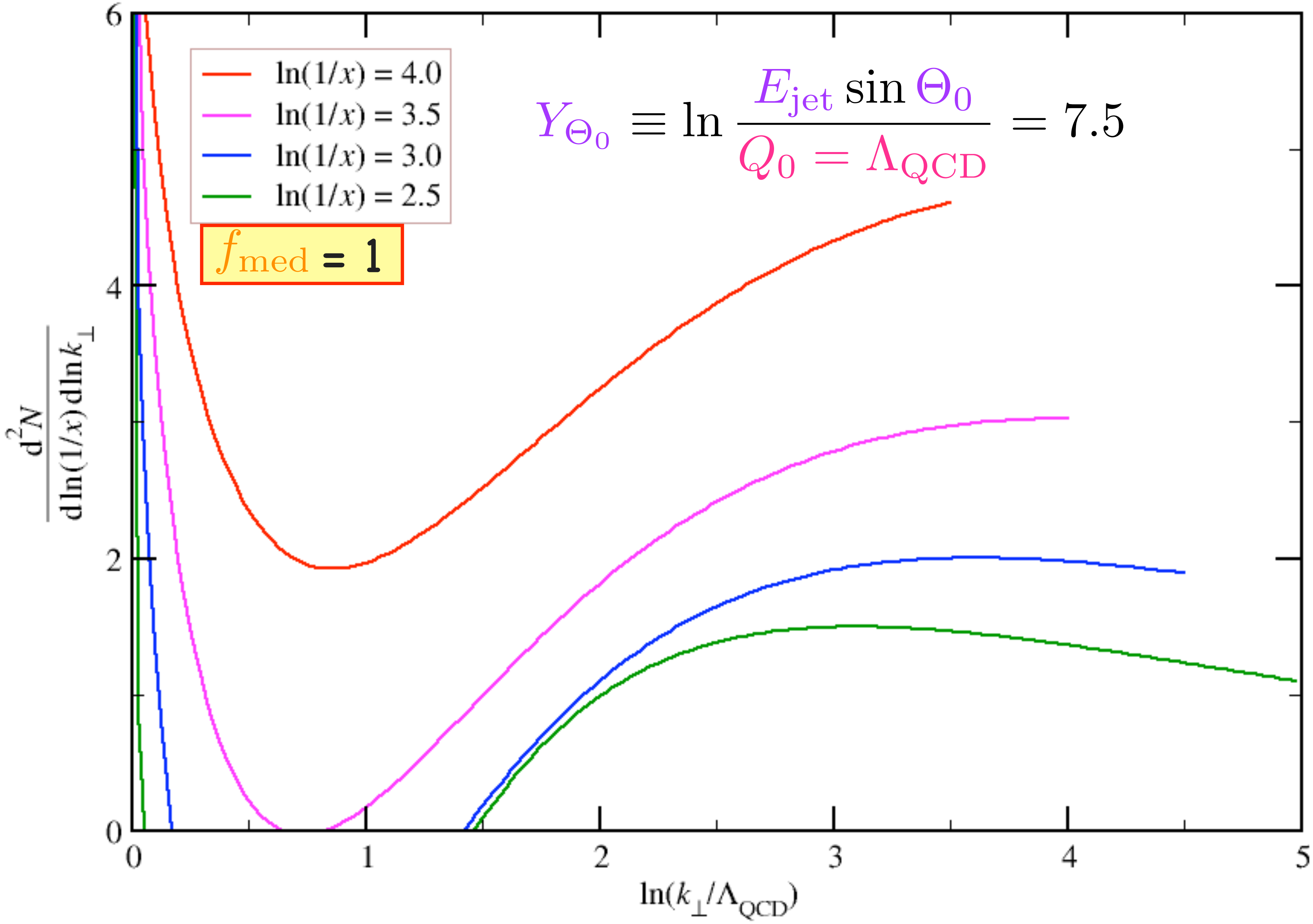
Eventually $\frac{\mathrm{d}N}{\mathrm{d}\ln\Theta} = \int \mathrm{d}x\, \frac{\mathrm{d}^2 N}{\mathrm{d}x\,\mathrm{d}\ln\Theta}$ ... which I shall not do here!

detailed computations in Pérez-Ramos & Machet, JHEP **04** (2006) 043

MLLA double-differential single-particle distribution



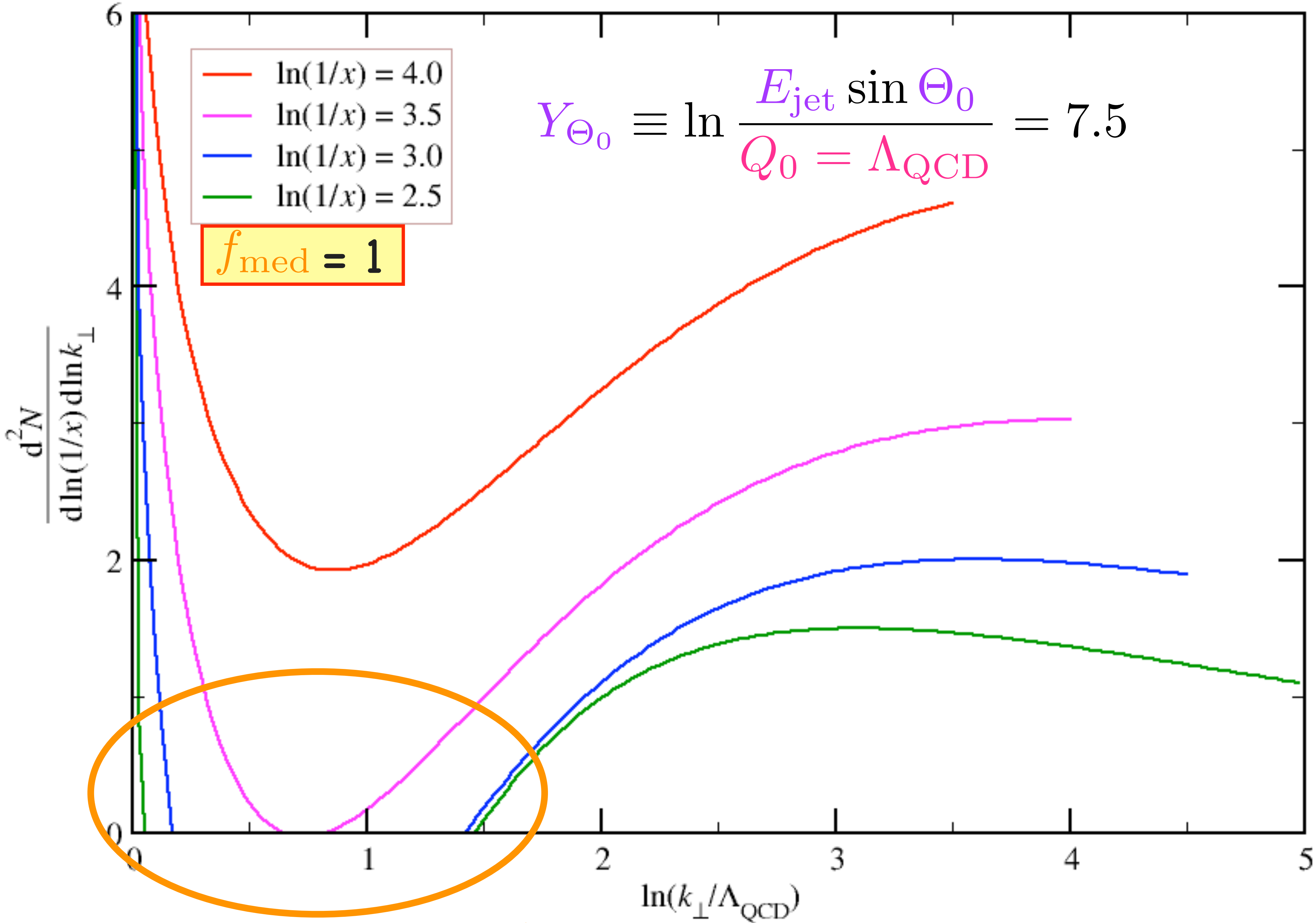
Medium-modified double-differential single-particle distribution



$$Y_{\Theta_0} \equiv \ln \frac{E_{\rm jet} \sin \Theta_0}{Q_0 = \Lambda_{\rm QCD}} = 7.5$$

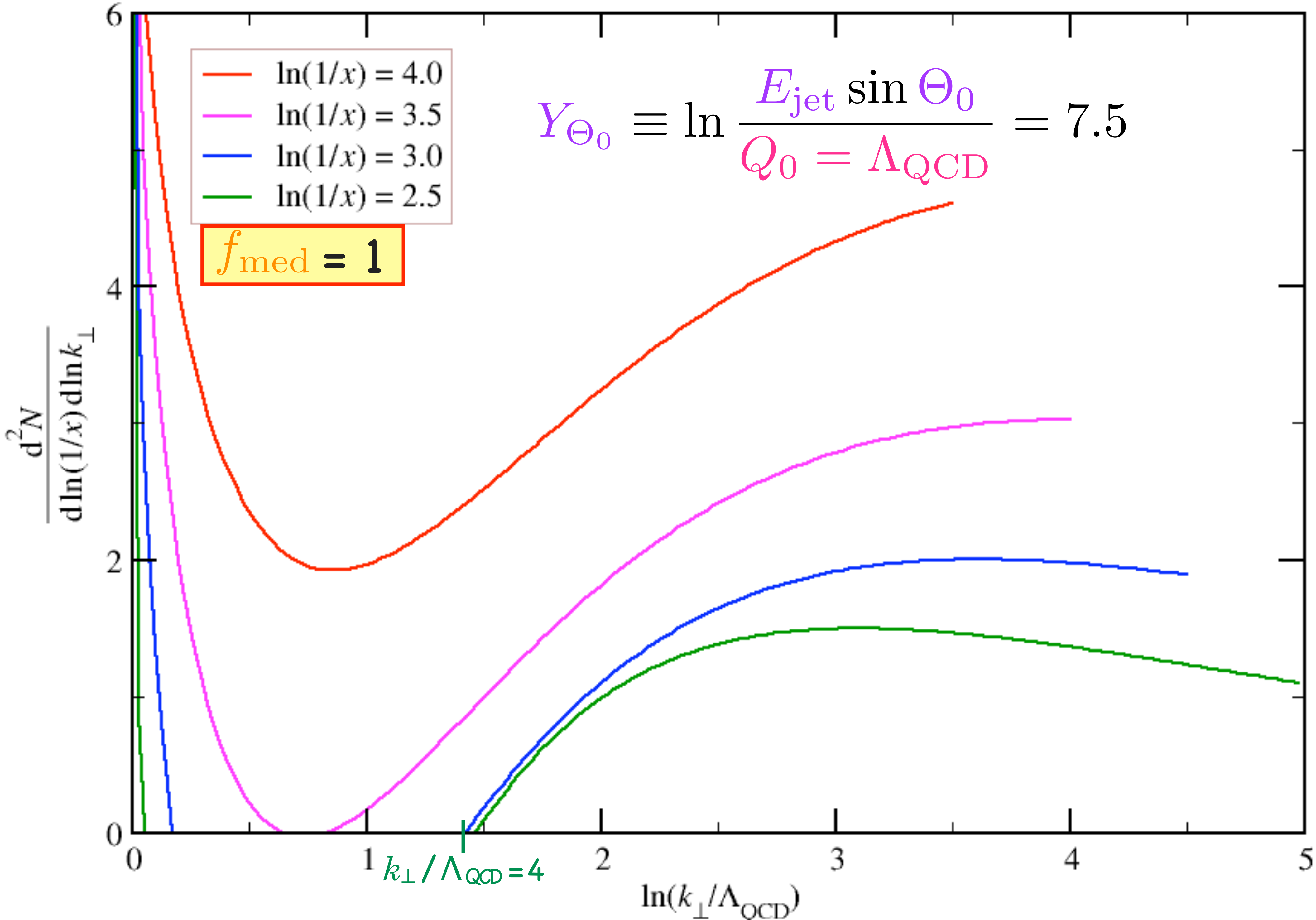
$f_{\rm med}$ = 1

Medium-modified double-differential single-particle distribution



What's happening here?!!

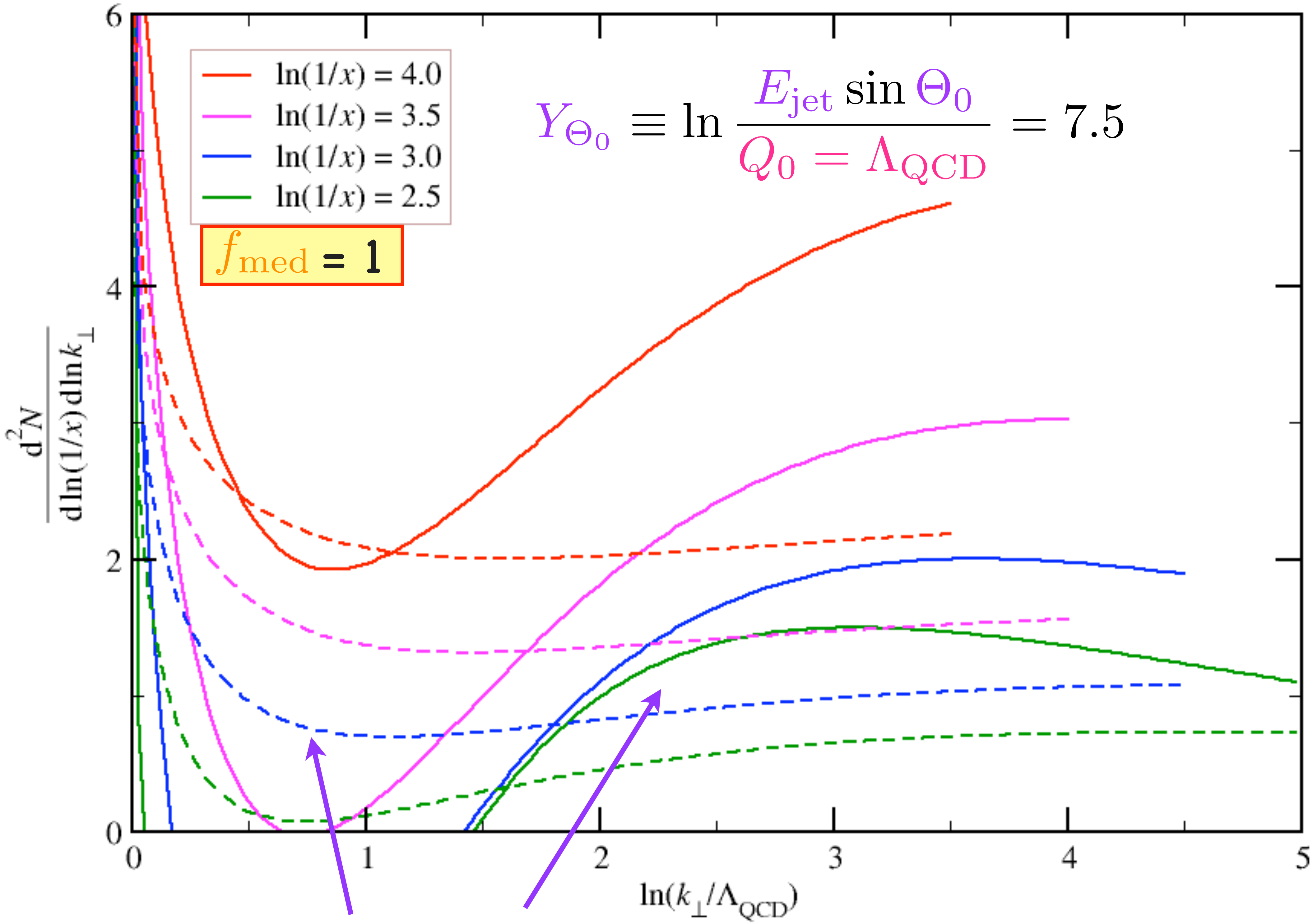
Medium-modified double-differential single-particle distribution



$$Y_{\Theta_0} \equiv \ln \frac{E_{\rm jet} \sin \Theta_0}{Q_0 = \Lambda_{\rm QCD}} = 7.5$$

Easy handwaving answer: should we trust results for $k_\perp$ < 1 GeV/c?

Medium-modified double-differential single-particle distribution



$$Y_{\Theta_0} \equiv \ln \frac{E_{\rm jet} \sin \Theta_0}{Q_0 = \Lambda_{\rm QCD}} = 7.5$$

Redistribution from low to higher $k_\perp$... or total nonsense.

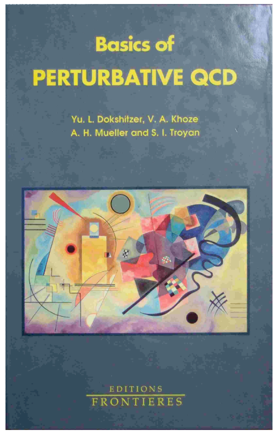
Medium-modified double-differential single-particle distribution

Jet broadening? Guess why I was not eager to integrate $\frac{\mathrm{d}^2 N}{\mathrm{d}x\,\mathrm{d}\ln\Theta}$ ...

Should I worry about negative probability distributions? (well, I do!)

- They also come up — albeit elsewhere — in the computations by Pérez-Ramos, Arleo & Machet that match the CDF data.
- They might become less severe... if I sit down, work harder and go to Next-to-MLLA.
  - Better accounting of energy-momentum conservation at high z;
  - In the absence of a medium, this does not affect the longitudinal spectrum much, only the $k_\perp$ distribution.
- But the problem might be much deeper...

Does it all make sense?


Basics of
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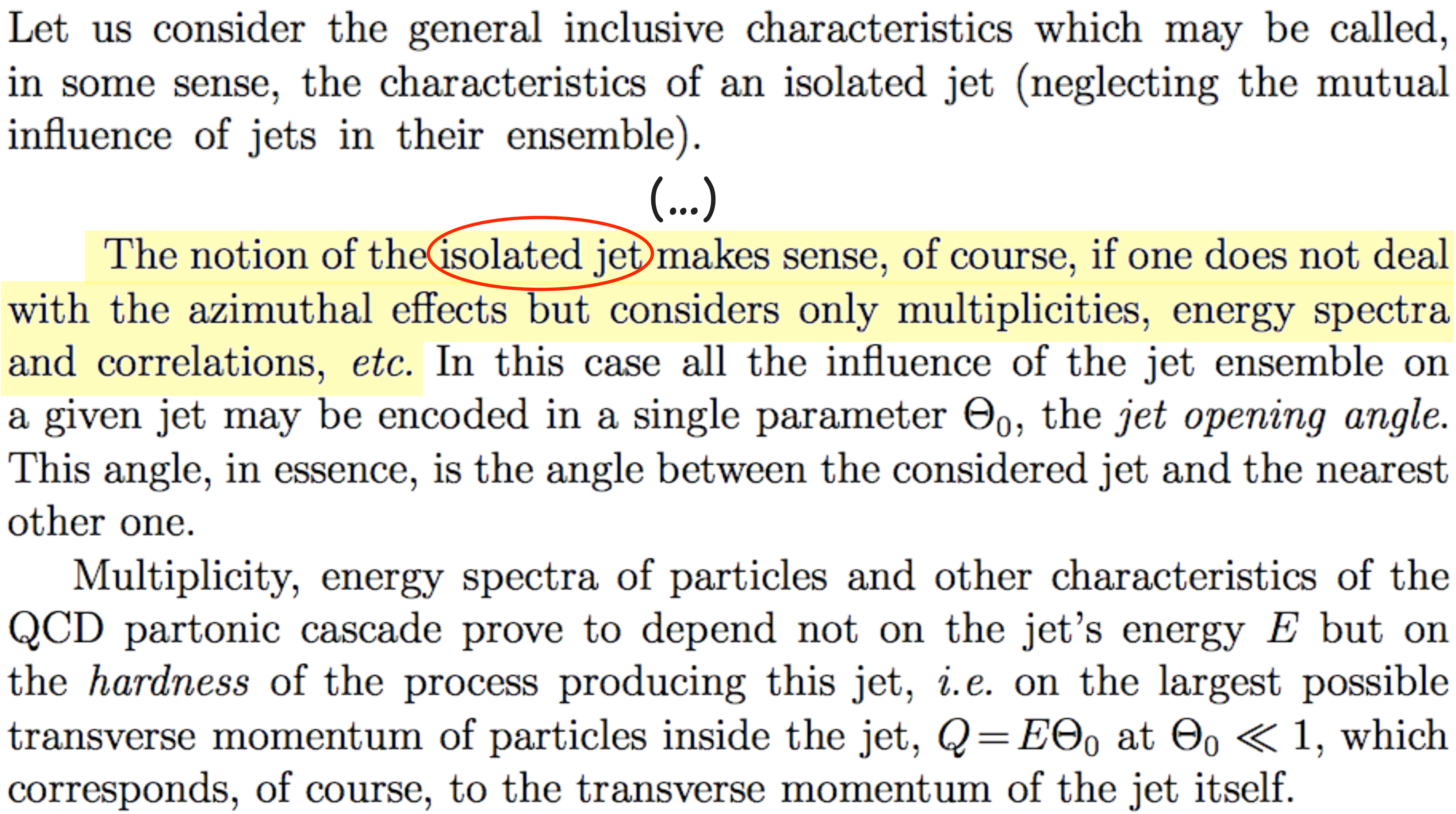
9.2 QCD Portrait of an "Individual Jet"


Let us consider the general inclusive characteristics which may be called, in some sense, the characteristics of an isolated jet (neglecting the mutual influence of jets in their ensemble).

(...)

The notion of the isolated jet makes sense, of course, if one does not deal with the azimuthal effects but considers only multiplicities, energy spectra and correlations, *etc.* In this case all the influence of the jet ensemble on a given jet may be encoded in a single parameter Θ_0, the *jet opening angle*. This angle, in essence, is the angle between the considered jet and the nearest other one.

Multiplicity, energy spectra of particles and other characteristics of the QCD partonic cascade prove to depend not on the jet's energy E but on the *hardness* of the process producing this jet, *i.e.* on the largest possible transverse momentum of particles inside the jet, $Q = E\Theta_0$ at $\Theta_0 \ll 1$, which corresponds, of course, to the transverse momentum of the jet itself.