Hints of incomplete thermalization in RHIC data

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in collaboration with

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RHIC Au–Au results: the fashionable view

BROOKHAVEN NATIONAL LABORATORY

RHIC Scientists Serve Up "Perfect" Liquid

New state of matter more remarkable than predicted -- raising many new questions

April 18, 2005

Ideal fluid dynamics reproduce both p_t spectra and elliptic flow $v_2(p_t)$ of soft ($p_t \lesssim 2$ GeV/c) identified particles for minimum bias collisions, near central rapidity.
This agreement necessitates a soft equation of state, and very short thermalization times: $\tau_{\text{thermalization}} < 0.6$ fm/c.

$\Rightarrow$ **strongly interacting Quark-Gluon Plasma**

Ideal fluid dynamics in heavy-ion collisions

- A few reminders on fluid dynamics
- Ideal fluid dynamics in nucleus–nucleus collisions: theory
  - Overall scenario
  - General predictions of ideal fluid dynamics
    - Anisotropic flow
- Out-of-equilibrium scenario
  - Generic predictions
  - Reconciling data and theory (?)

R.S. Bhalerao, J.-P. Blaizot, N.B., J.-Y. Ollitrault, PLB **627** (2005) 49

Fluid dynamics: various types of flow

- **Thermodynamic equilibrium?** ☞ Knudsen number $Kn = \dfrac{\overbrace{\lambda}^{\text{mean free path}}}{\underbrace{L}_{\text{system size}}}$
  - $Kn \gg 1$: Free-streaming limit
  - $Kn \ll 1$: Liquid (hydro) limit
- **Viscous or Ideal?** ☞ Reynolds number $Re = \dfrac{\varepsilon L v_{\text{fluid}}}{\underbrace{\eta}_{\text{viscosity}}}$

  $\eta \sim \varepsilon \lambda c_s$
  - $Re \gg 1$: Ideal (non-viscous) flow
  - $Re \leq 1$: Viscous flow
- **Compressible or Incompressible?** ☞ Mach number $Ma = \dfrac{v_{\text{fluid}}}{\underbrace{c_s}_{\text{speed of sound}}}$
  - $Ma \ll 1$: Incompressible flow
  - $Ma > 1$: Compressible (supersonic) flow

Fluid dynamics: various types of flow

Three numbers:

$$Kn = \frac{\lambda}{L}, \qquad Re = \frac{\varepsilon L v_{\text{fluid}}}{\eta}, \qquad Ma = \frac{v_{\text{fluid}}}{c_s}$$

$\Rightarrow$ **an important relation:**

$$Kn \times Re = \frac{\varepsilon \lambda\, v_{\text{fluid}}}{\eta} \sim \frac{v_{\text{fluid}}}{c_s} = Ma$$

Compressible flow: "Liquids are Ideal"

Viscosity $\equiv$ departure from equilibrium

Ideal fluid picture of a heavy-ion collision

0. Creation of a dense "gas" of particles

1. At some time τ_0, the mean free path λ is much smaller than *all* dimensions in the system
   $\Rightarrow$ thermalization (T_0), ideal fluid dynamics applies

2. The fluid expands: density decreases, λ increases (system size also)

3. At some time, the mean free path is of the same order as the system size: ideal fluid dynamics is no longer valid
   "(kinetic) freeze-out"

Freeze-out usually parameterized in terms of a temperature $T_{\rm f.o.}$

If λ varies smoothly with temperature, consistency requires $T_{\rm f.o.} \ll T_0$

☛ analytical predictions

Ideal fluid dynamics: general predictions

Consistent ideal fluid dynamics picture requires $T_{\rm f.o.} \ll T_0$

$\Leftrightarrow$

Ideal-fluid limit = $T_{\rm f.o.} \to 0$ limit

☞ one can compute in a model-independent way

- the spectrum $$E\frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{p}} = C\int_\Sigma \exp\left(-\frac{p^\mu u_\mu(x)}{T_{\rm f.o.}}\right) p^\mu\,\mathrm{d}\sigma_\mu$$
- and its azimuthal anisotropies ("flow")

using saddle-point approximations around the minimum of

N.B. & J.-Y. Ollitrault, **nucl-th/0506045**

Ideal fluid dynamics: general predictions

Consistent ideal fluid dynamics picture requires $T_{\rm f.o.} \ll T_0$

$$\Leftrightarrow$$

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fluid velocity

- the spectrum $E\frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{p}} = C\int_\Sigma \exp\left(-\frac{\boxed{p^\mu u_\mu(x)}}{T_{\rm f.o.}}\right) p^\mu\, \mathrm{d}\sigma_\mu$

particle momentum

- and its azimuthal anisotropies ("flow")

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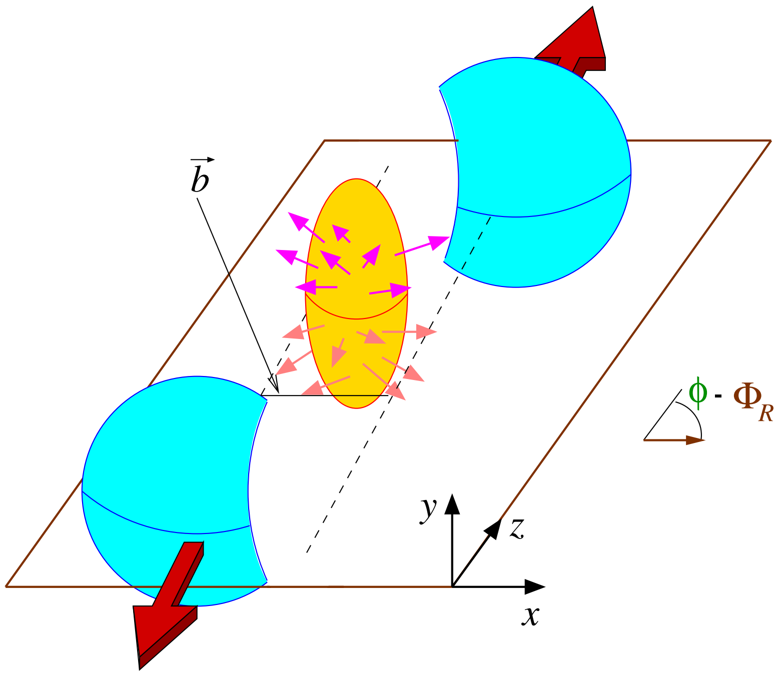
N.B. & J.-Y. Ollitrault, **nucl-th/0506045**

Heavy-ion observable: Anisotropic flow

Non-central collision:



Initial anisotropy of the source (in the transverse plane)

$\Rightarrow$ anisotropic pressure gradients, larger along the impact parameter $\vec{b}$

$\Rightarrow$ anisotropic emission of particles:

anisotropic (collective) flow

$$E\frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{p}} \propto \frac{\mathrm{d}N}{p_t\,\mathrm{d}p_t\,\mathrm{d}y}\left[1 + 2v_1\cos(\phi - \Phi_R) + 2v_2\cos 2(\phi - \Phi_R) + \ldots\right]$$

"directed" "elliptic"

"Flow": misleading terminology; does NOT imply fluid dynamics!

Non-central collisions: parameters

Initial conditions in non-central collisions, will be characterized by

- a parameter measuring the shape of the overlap region:
  - spatial eccentricity $\epsilon = \dfrac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$
- two numbers measuring the size of the overlap region:
  - "reduced" radius $\dfrac{1}{\bar{R}} = \sqrt{\dfrac{1}{\langle x^2 \rangle} + \dfrac{1}{\langle y^2 \rangle}}$

    (anisotropic flow caused by pressure *gradients*)
  - transverse area of the collision zone $S = 2\pi \sqrt{\langle x^2 \rangle \langle y^2 \rangle}$

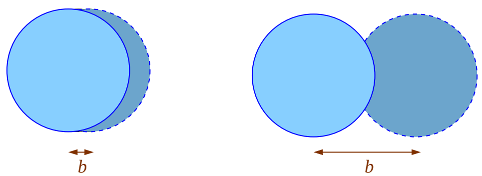
Anisotropic flow: predictions of hydro

- Characteristic build-up time of v_2 is $\bar{R}/c_s$

  typical system size   speed of sound

- v_2/ϵ constant across different centralities

  system eccentricity



- v_2 roughly independent of the system size (Au–Au vs. Cu–Cu)
- v_2 increases with increasing speed of sound c_s
- Mass-ordering of the $v_2(p_T)$ of different particles

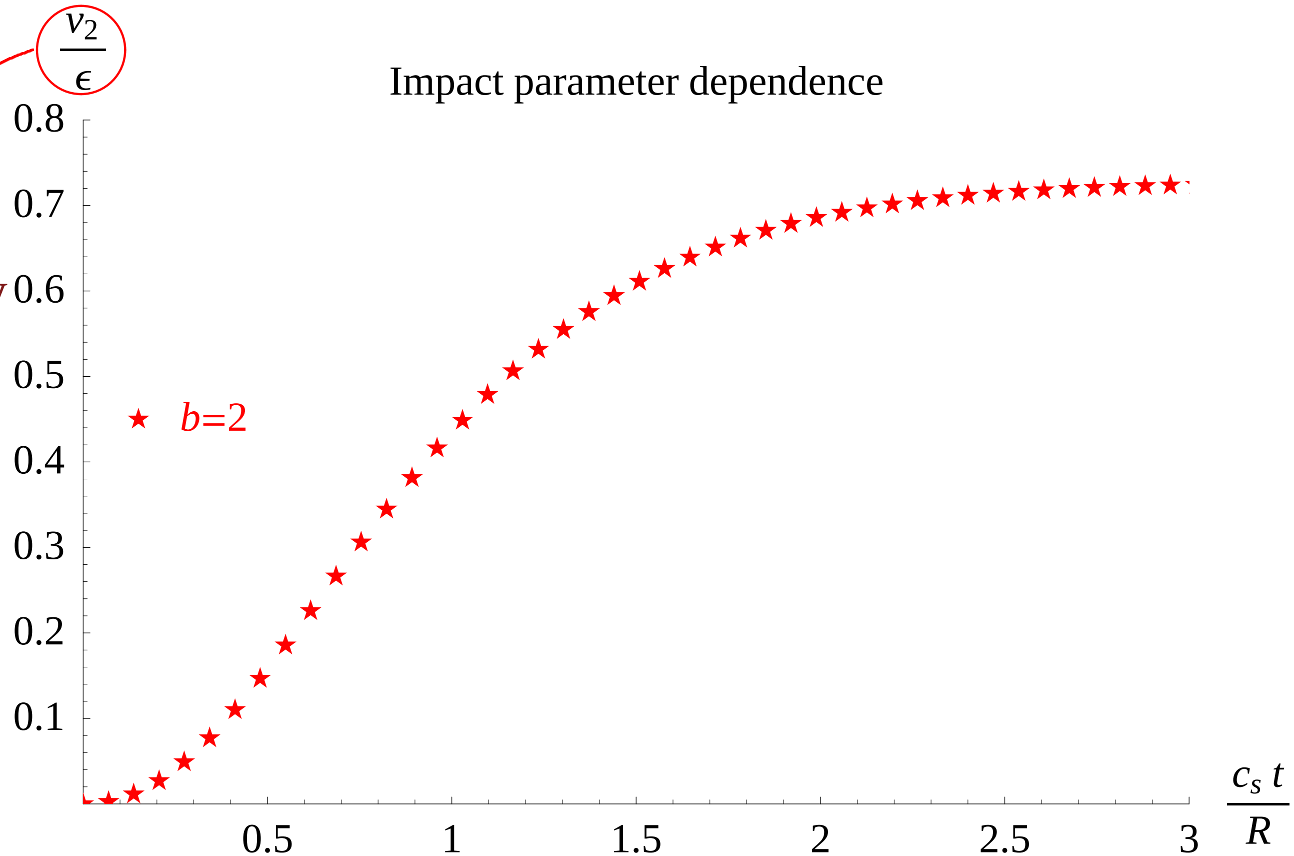
  (the heavier the particle, the smaller its v_2 at a given momentum)
- Relationship between different harmonics: $\dfrac{v_4}{(v_2)^2} = \dfrac{1}{2}$

Dependence of v_2 on centrality

The natural time scale for v_2 is $\bar{R}/c_s$:

massless particles $c_s^2 = \frac{1}{3}$

v_2 scaled by initial eccentricity $\frac{\langle y^2 - x^2 \rangle}{\langle x^2 + y^2 \rangle}$



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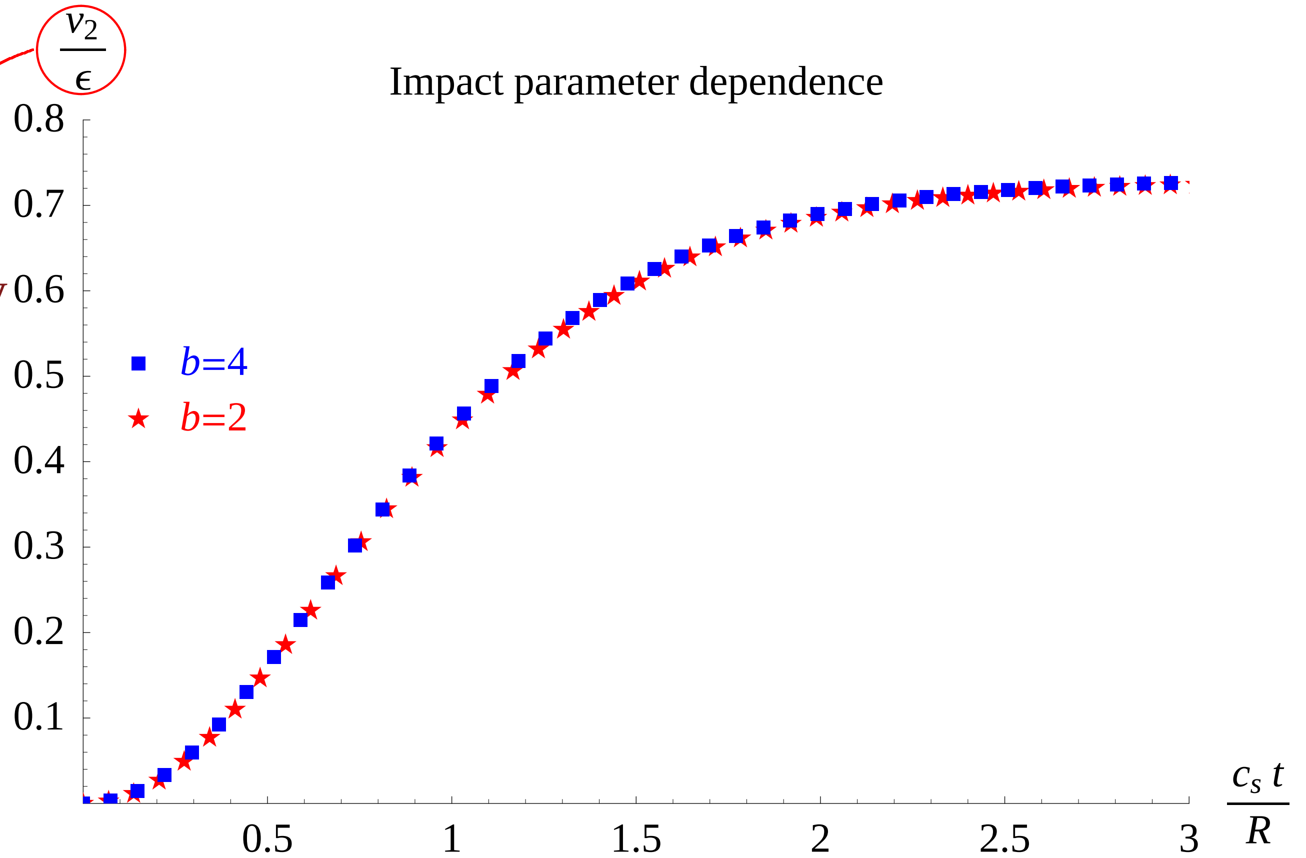

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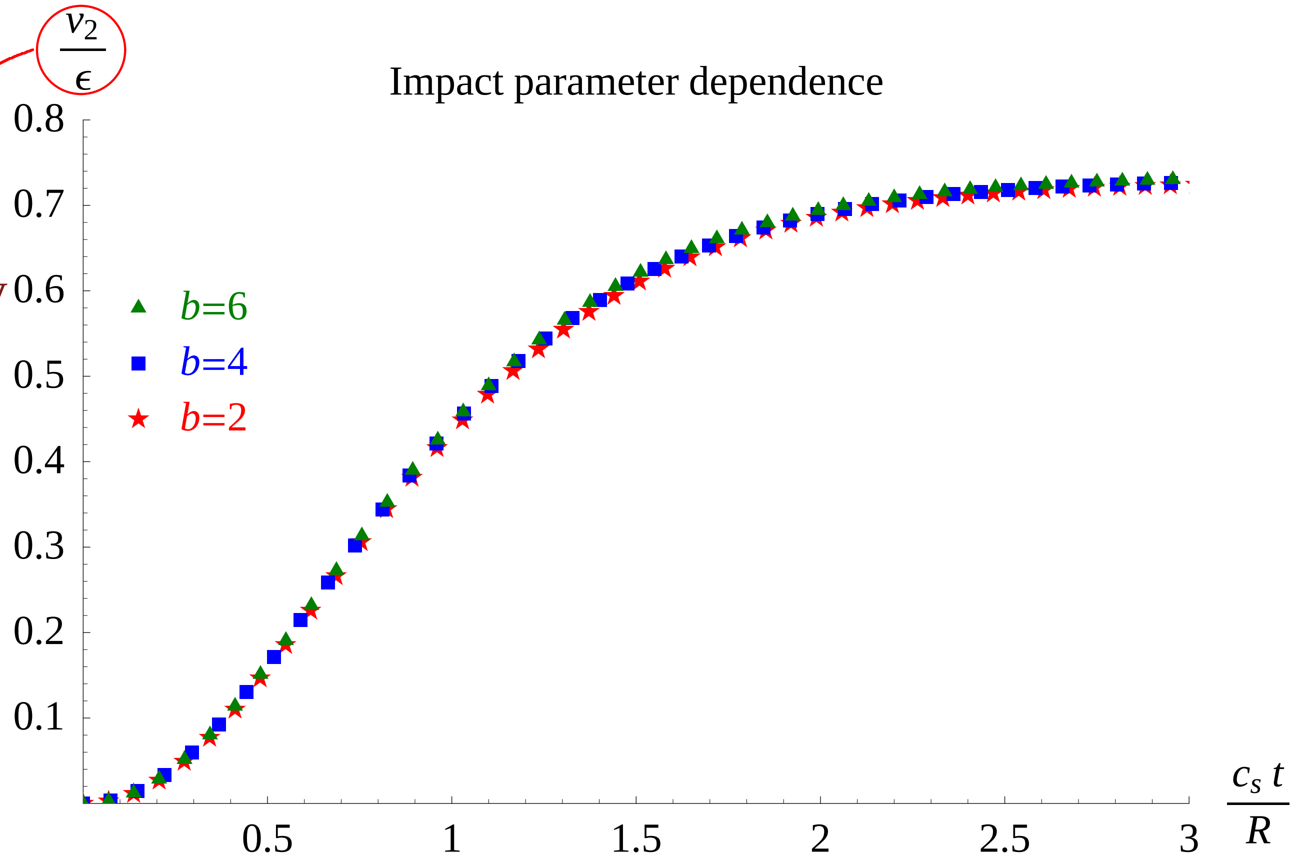

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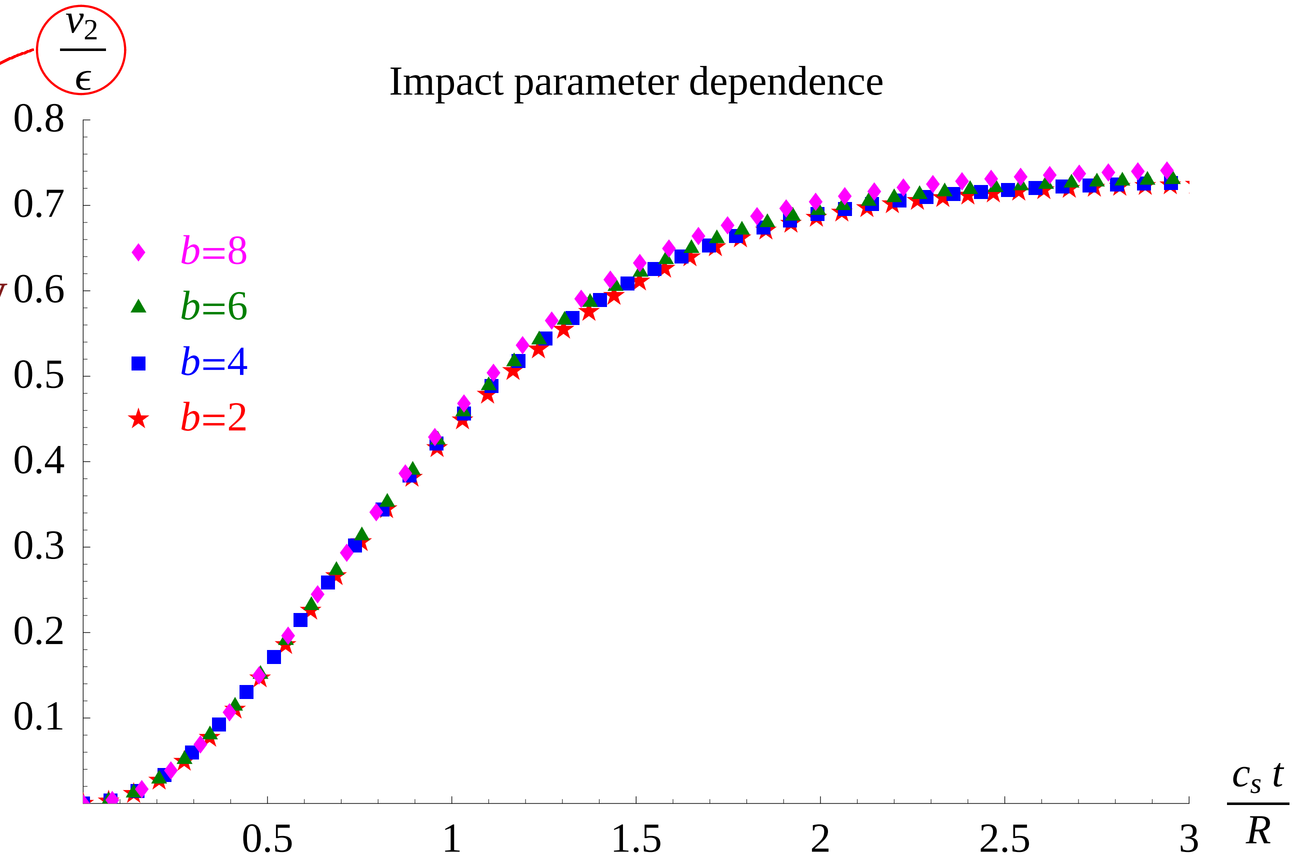

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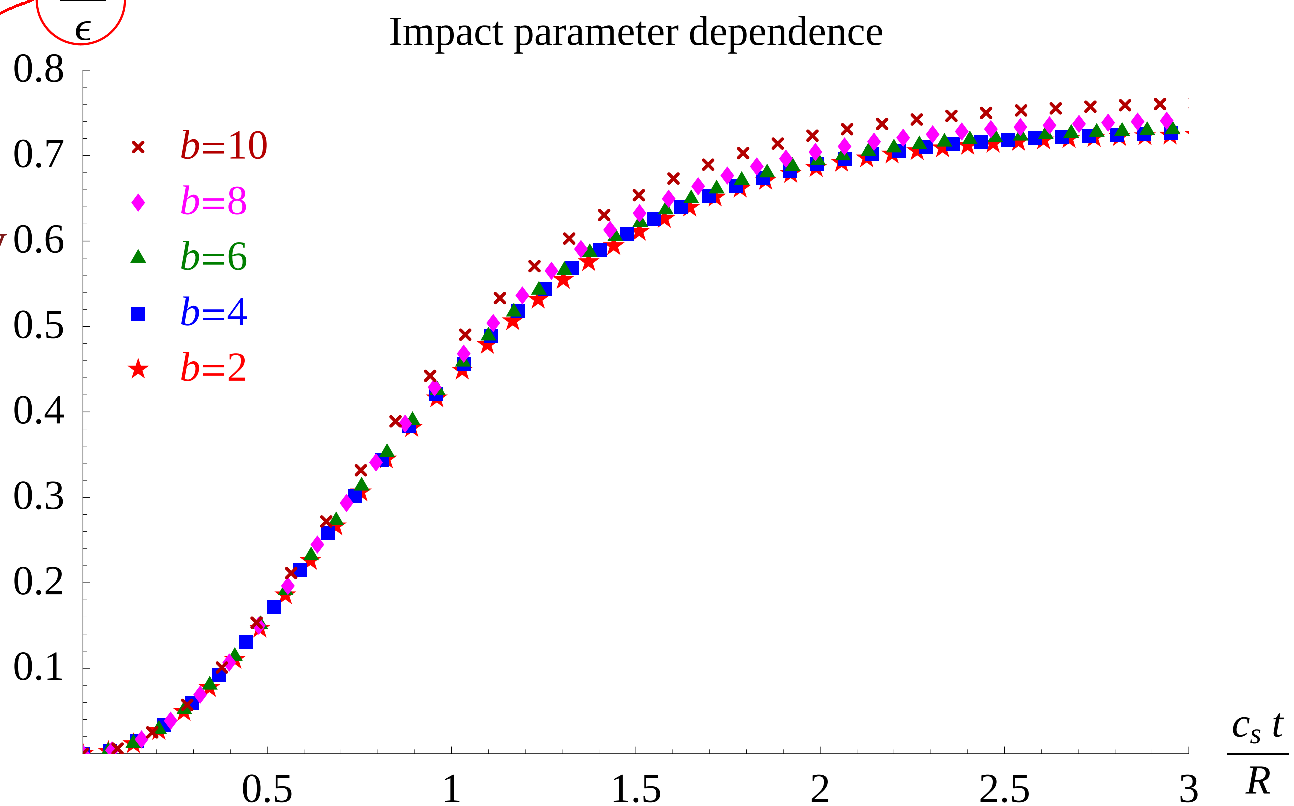
$\frac{v_2}{\epsilon}$



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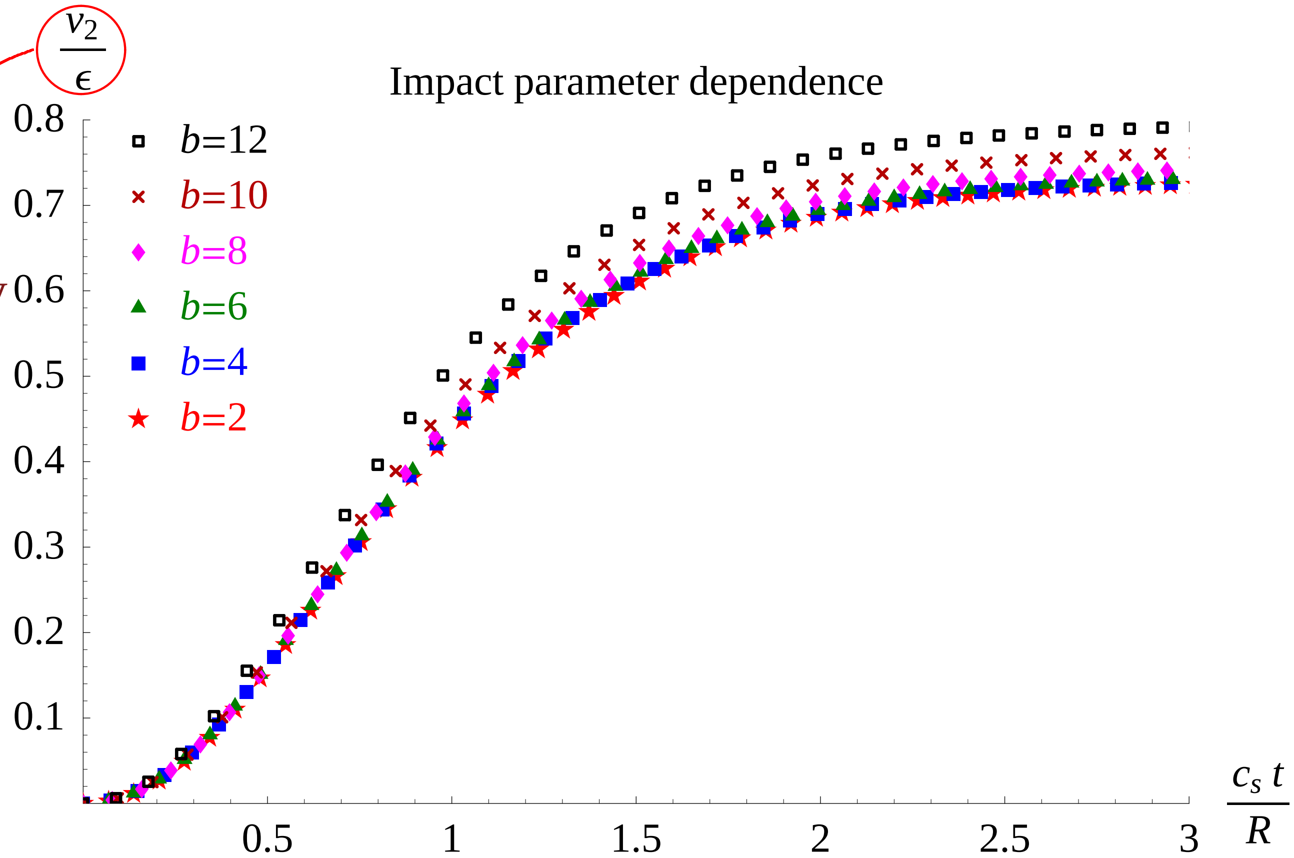
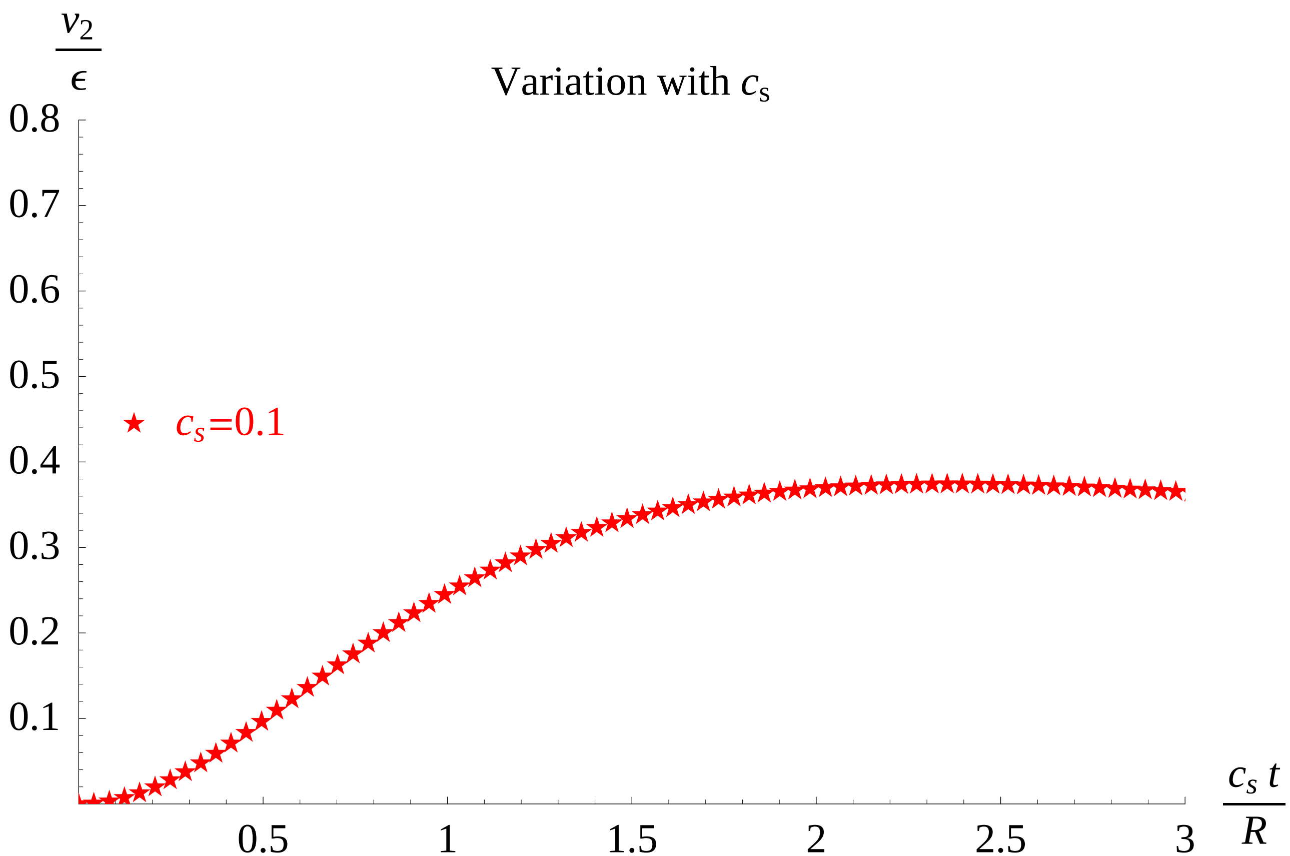


Dependence of v_2 on the speed of sound



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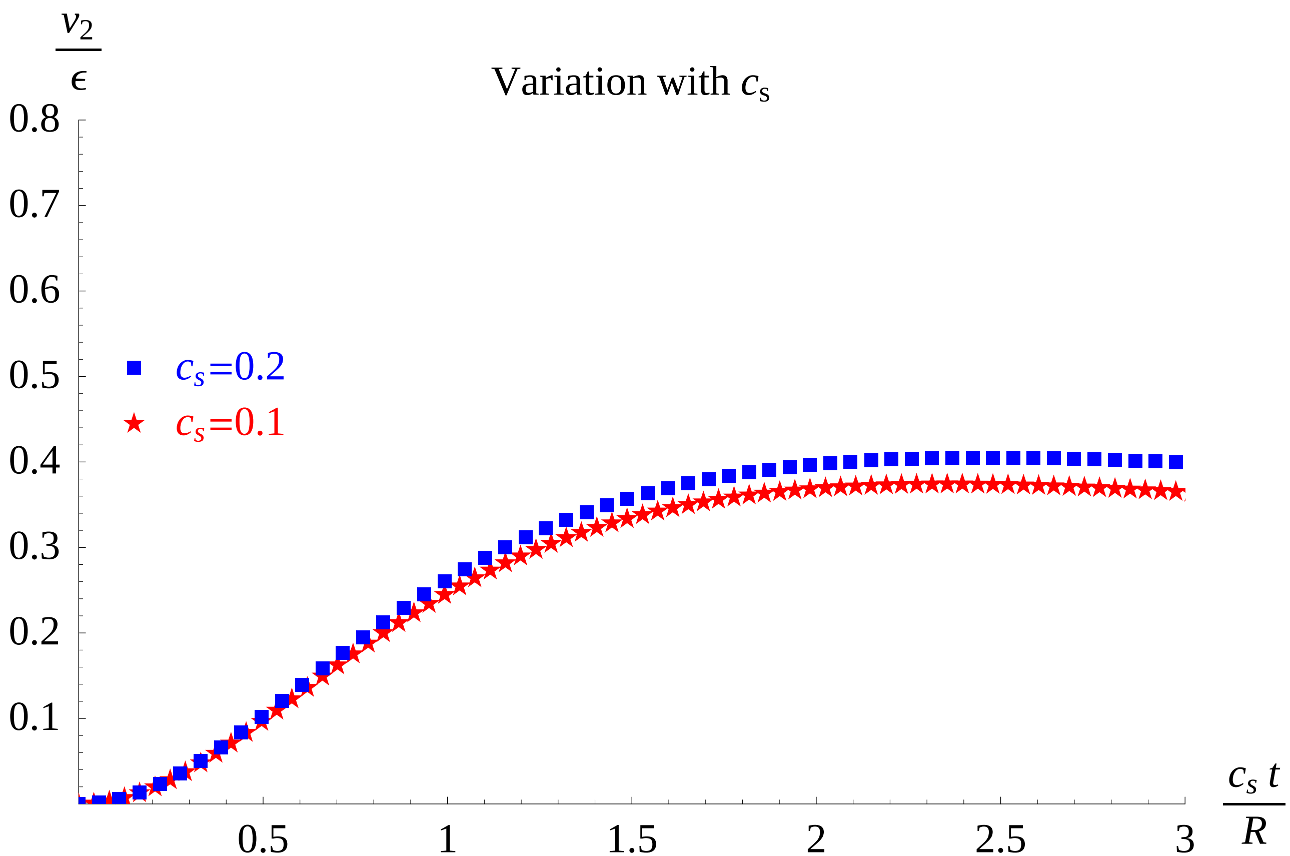

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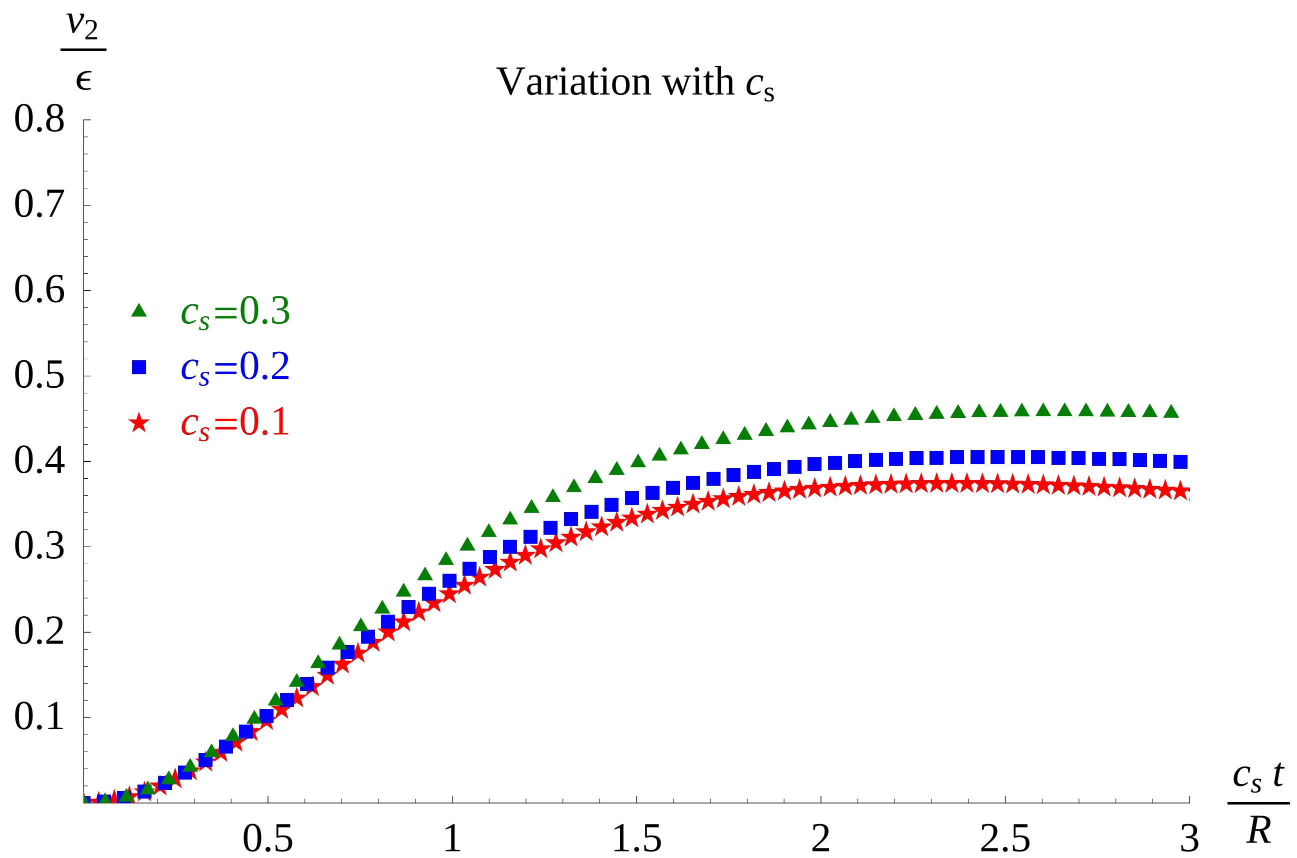

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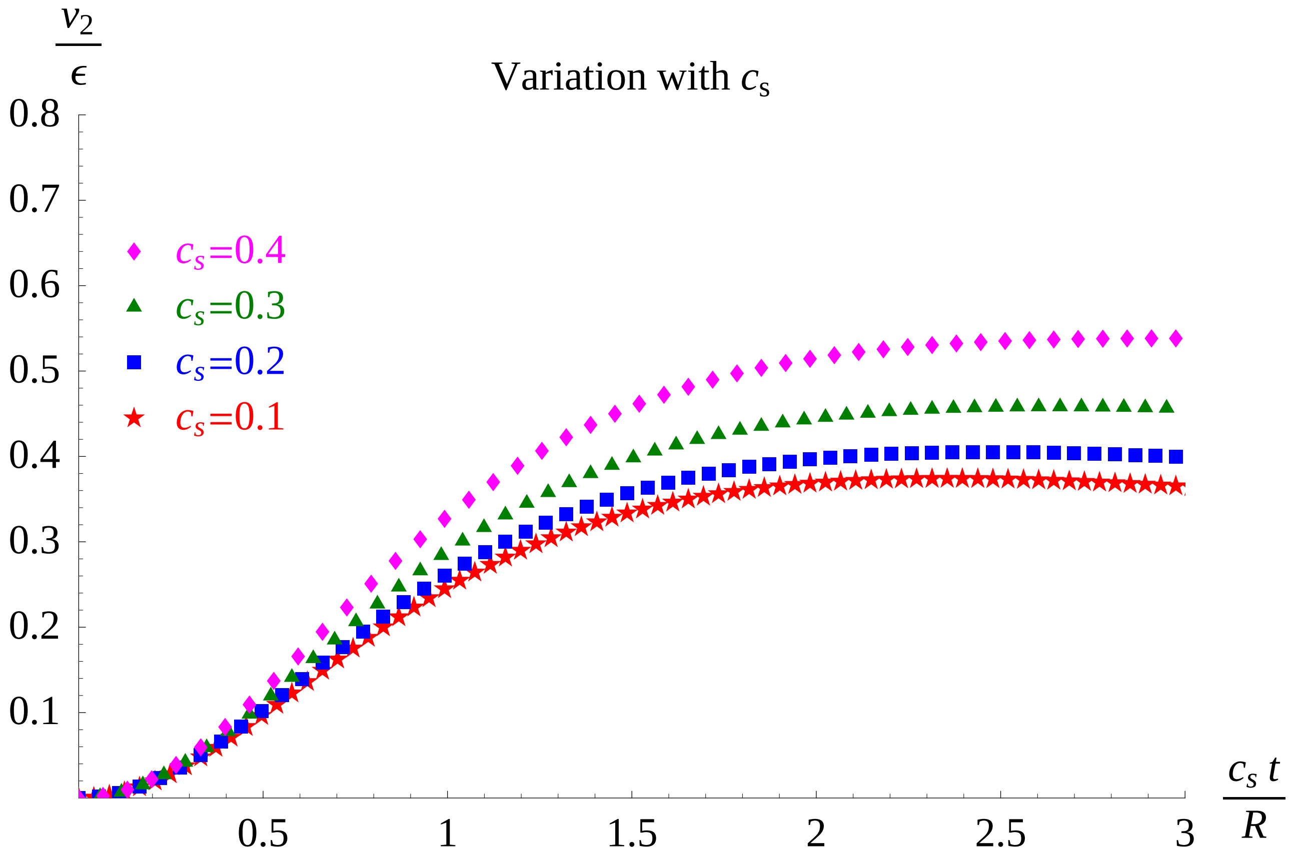

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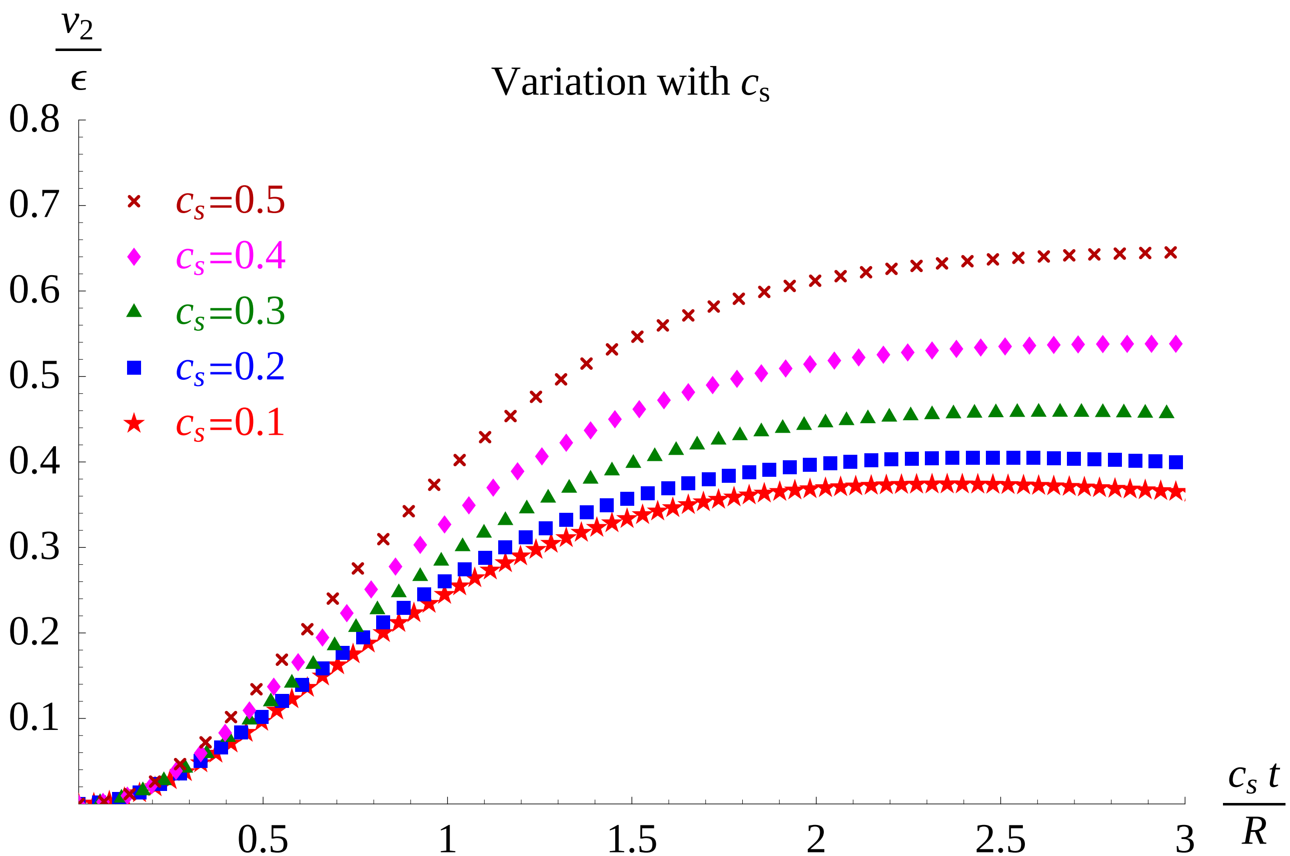

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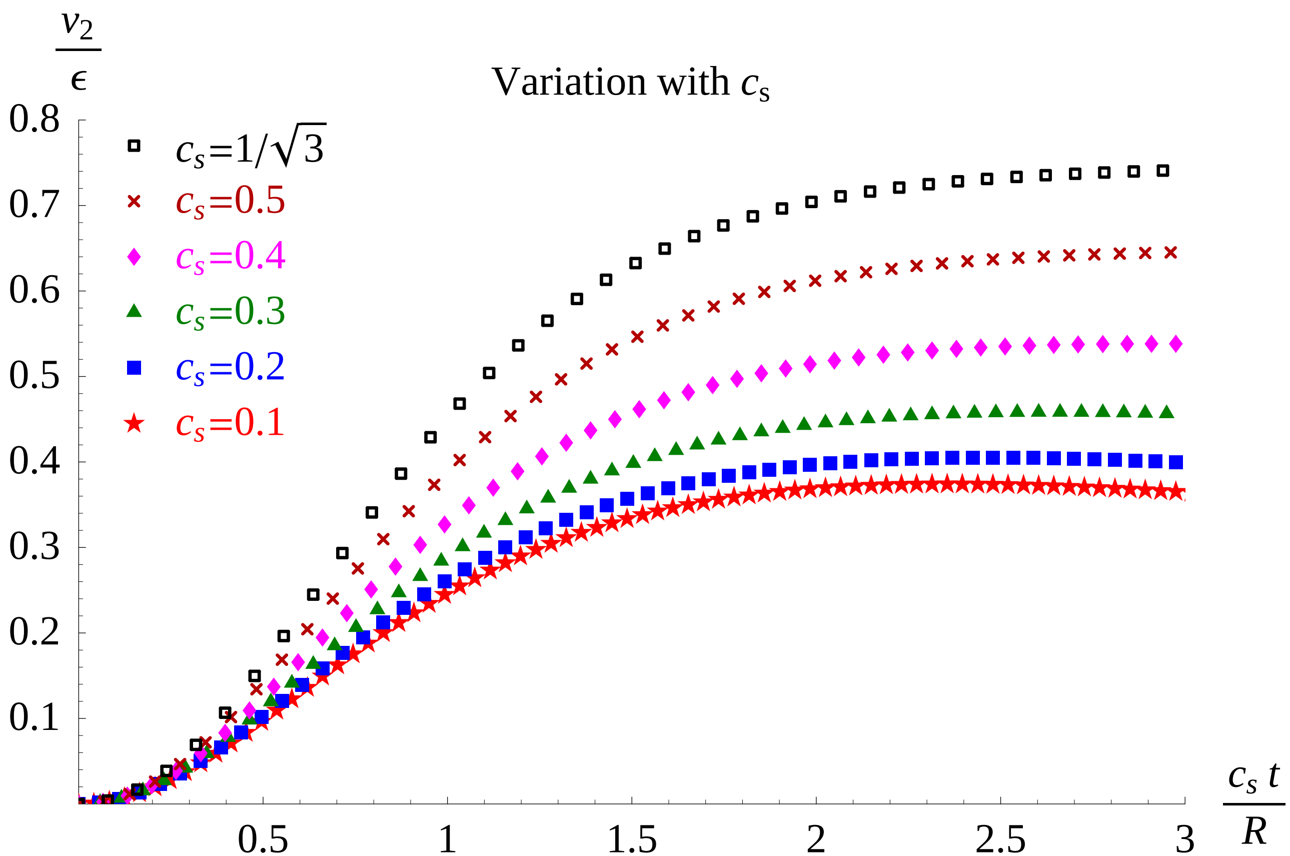
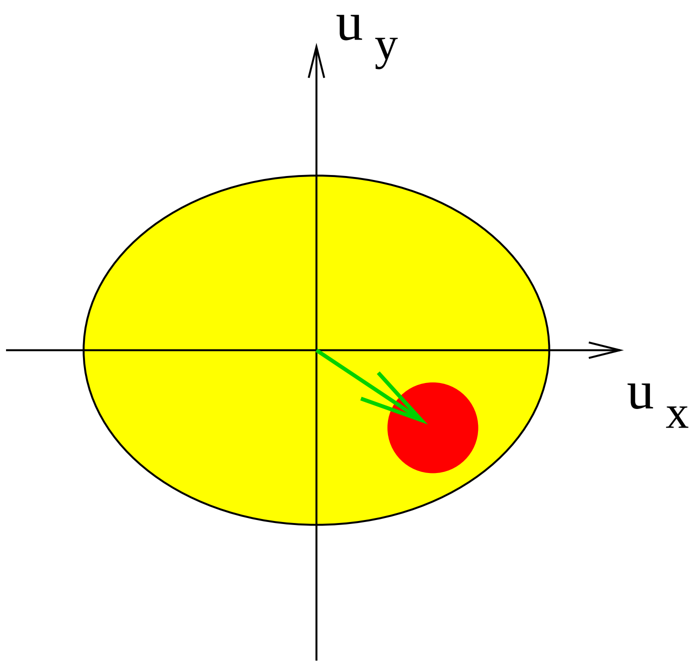


☞ one can increase v_2 by increasing c_s

Mass-ordering of the $v_2(p_T)$

"Slow particles" ($p_t/m < u_{\max}(\frac{\pi}{2})$) move together with the fluid



There is a point where the fluid velocity equals the particle velocity

☞ Integrand in the momentum spectrum is Gaussian, with width $(p^\mu u_\mu)^{1/2}_{\min} = \sqrt{m}$

→ saddle-point approximation!

- Similar spectra for different hadrons:

$$E\frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{p}} = c^h(m)\, f\left(\frac{p_t}{m}, y, \phi\right)$$

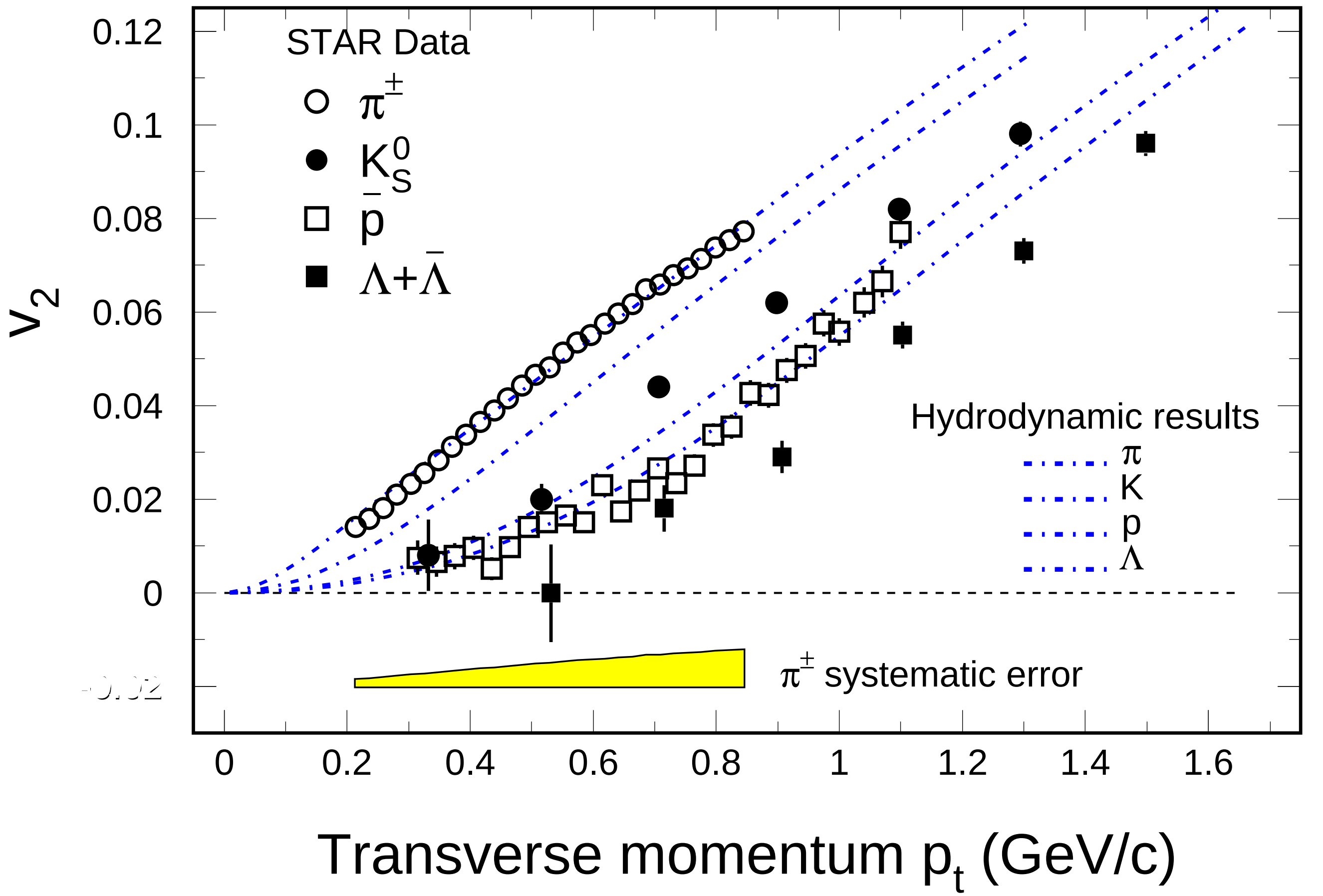
- $v_n\left(\frac{p_t}{m}, y\right)$ universal!

$\Rightarrow$ mass-ordering of $v_2(p_t, y)$

RHIC anisotropic flow data and ideal-fluid dynamics

$v_2(p_t)$ at midrapidity, minimum bias Au–Au collisions:

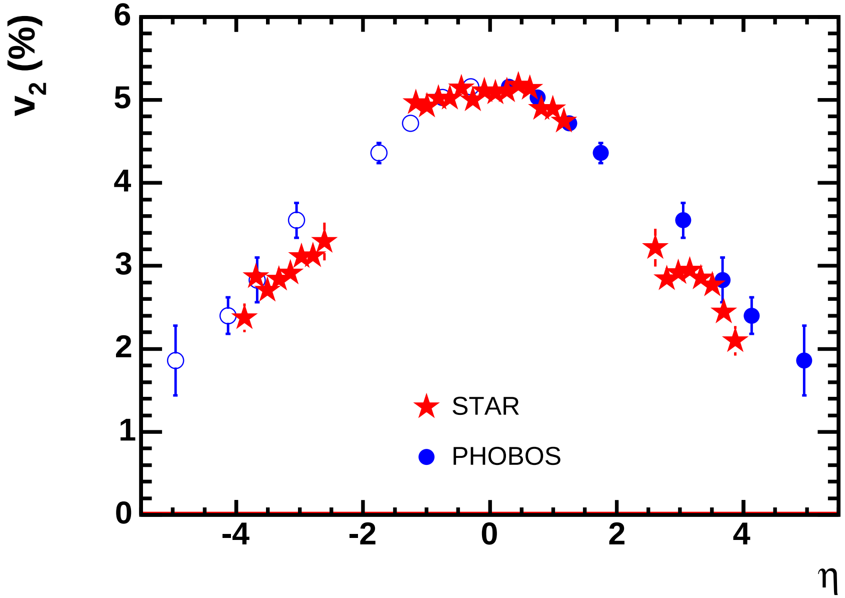
STAR Collaboration, PRC **72** (2005) 014904



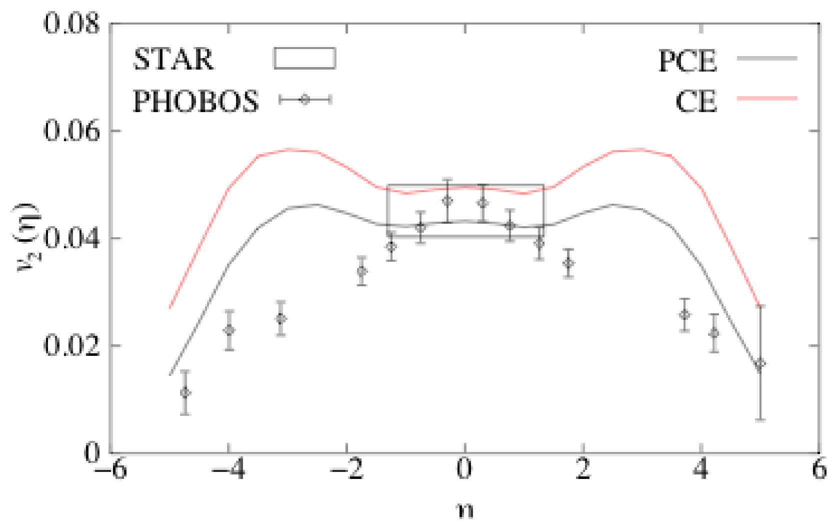
RHIC anisotropic flow data and ideal-fluid dynamics

(Pseudo)rapidity dependence of v_2

STAR Collaboration,
PRC **72** (2005) 014904



Hirano & Tsuda,
PRC **66** (2002) 054905

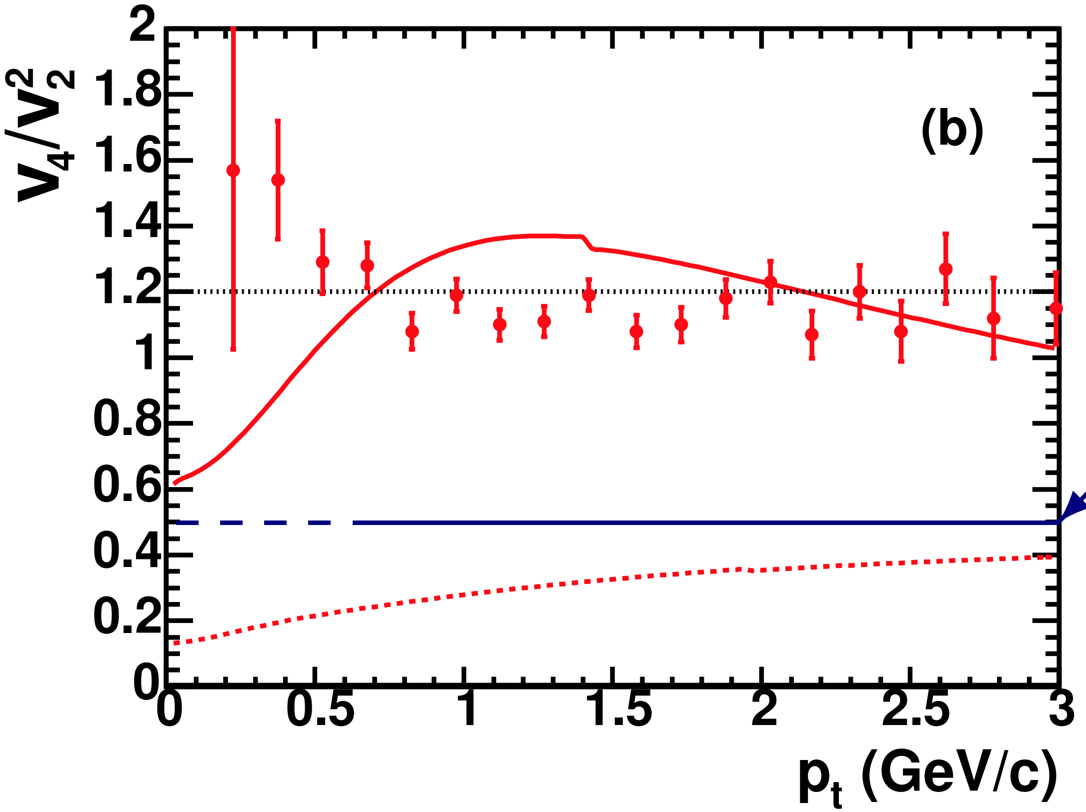


v_2(hydro) flatter than data

RHIC anisotropic flow data and ideal-fluid dynamics

Transverse momentum dependence of $\frac{v_4}{(v_2)^2}$

STAR Collaboration, Phys. Rev. C **72** (2005) 014904



ideal fluid prediction

What if collisions cannot ensure equilibrium?

Out-of-equilibrium scenario

Anisotropic flow: out-of-equilibrium scenario

An exact computation of the dependence of v_2, v_4 on the number of collisions per particle Kn^{-1} requires some cascade model…

…but we can guess the general tendency! $\frac{\bar{R}}{\lambda}$

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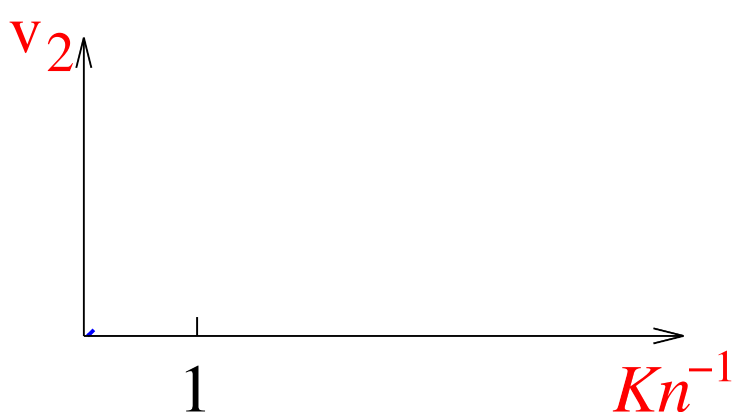
- in the absence of rescatterings ($Kn^{-1} = 0$), no flow develops



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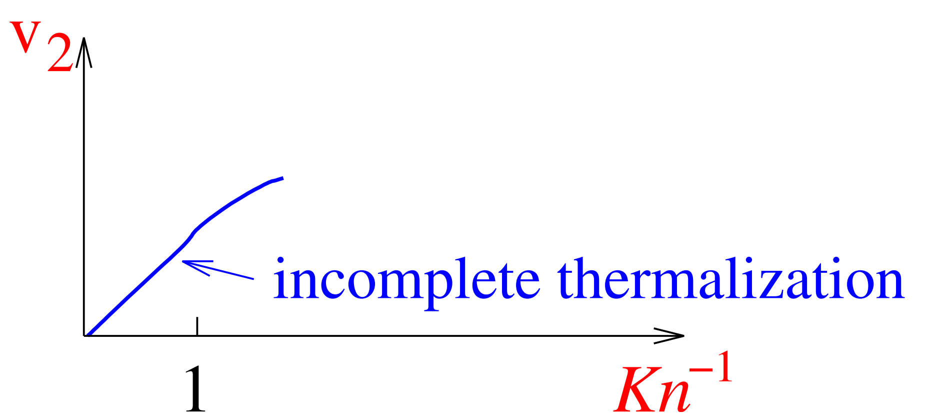
- the more collisions, the larger the anisotropic flow



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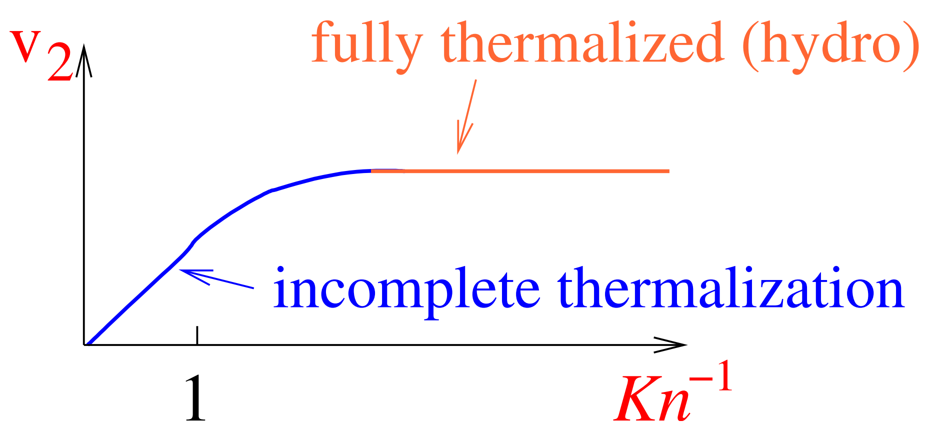
An exact computation of the dependence of v_2, v_4 on the number of collisions per particle Kn^{-1} requires some cascade model…

$$Kn^{-1} \rightarrow \frac{\bar{R}}{\lambda}$$

…but we can guess the general tendency!

- in the absence of rescatterings ($Kn^{-1} = 0$), no flow develops
- the more collisions, the larger the anisotropic flow
- for a given number of collisions, the system thermalizes: further collisions no longer increase v_2



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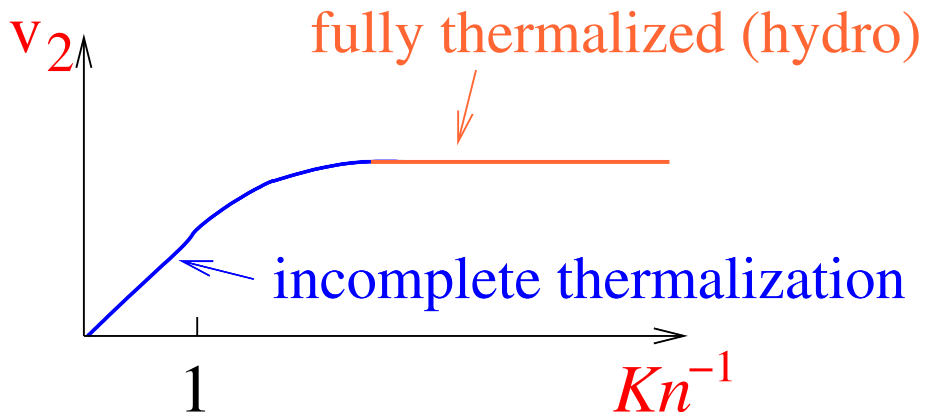
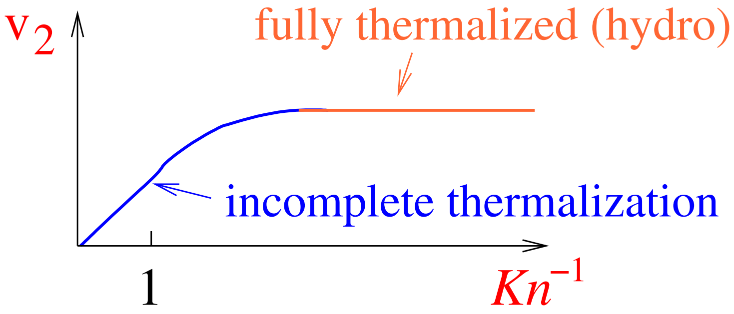
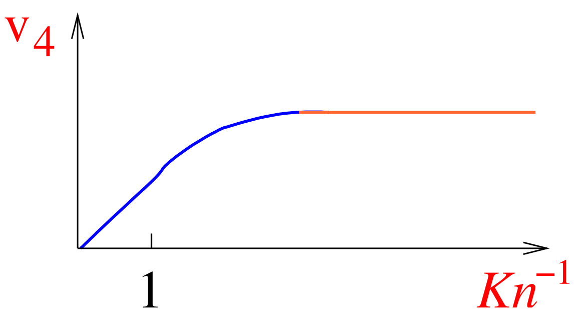
- in the absence of rescatterings ($Kn^{-1} = 0$), no flow develops
- the more collisions, the larger the anisotropic flow
- for a given number of collisions, the system thermalizes: further collisions no longer increase v_2 should be quantified!



Anisotropic flow: out-of-equilibrium scenario

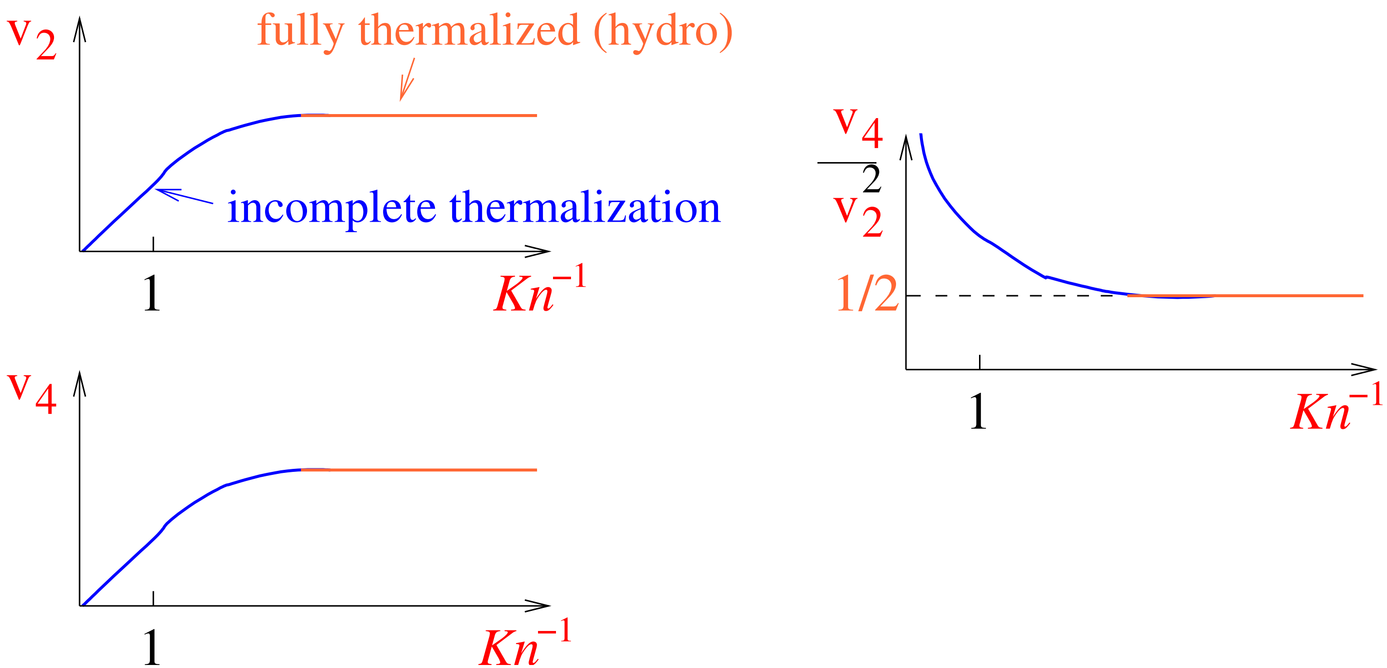
v_n proportional to the number of collisions Kn^{-1}





Anisotropic flow: out-of-equilibrium scenario

v_n proportional to the number of collisions Kn^{-1} $\Rightarrow$ $\dfrac{v_4}{(v_2)^2} \propto \dfrac{1}{Kn^{-1}}$



☞ in the out-of-equilibrium scenario $\dfrac{v_4}{(v_2)^2} > \dfrac{1}{2}$

STAR (PRC **72** (2005) 014904) & PHENIX (QM'05) find $\dfrac{v_4}{(v_2)^2} \approx 1$–$1.5$

Out-of-equilibrium scenario: a control parameter

The natural time (resp. length) scale for v_2 is $\bar{R}/c_s$ (resp. $\bar{R}$)

$\Rightarrow$ mean number of collisions per particle to build up v_2:

$$Kn^{-1} \simeq \frac{\bar{R}}{\lambda}$$

In the out-of-equilibrium scenario, v_2 depends on Kn^{-1}, hence on

- the system size $\bar{R}$

  ☞ breakdown of the scale-invariance of hydrodynamics

R.S. Bhalerao, J.-P. Blaizot, N.B., J.-Y. Ollitrault, PLB **627** (2005) 49

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σ interaction cross section, $n(\tau)$ particle density

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  - ☞ breakdown of the scale-invariance of hydrodynamics

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$$Kn^{-1} \simeq \frac{\bar{R}}{\lambda} = \bar{R}\,\sigma\,n\left(\frac{\bar{R}}{c_s}\right) \simeq \frac{c_s}{c}\frac{\sigma}{S}\frac{\mathrm{d}N}{\mathrm{d}y}$$

σ interaction cross section, $n(\tau)$ particle density, S transverse surface

In the out-of-equilibrium scenario, v_2 depends on Kn^{-1}, hence on

- the system size $\bar{R}$

  ☞ breakdown of the scale-invariance of hydrodynamics

- the "control parameter" $\dfrac{1}{S}\dfrac{\mathrm{d}N}{\mathrm{d}y}$

R.S. Bhalerao, J.-P. Blaizot, N.B., J.-Y. Ollitrault, PLB **627** (2005) 49

Out-of-equilibrium scenario: a control parameter

Number of collisions per particle to build up v_2: $Kn^{-1} \simeq \frac{c_s}{c}\frac{\sigma}{S}\frac{\mathrm{d}N}{\mathrm{d}y}$

☞ the variation (out-of-equilibrium scenario) or independence (ideal liquid paradigm) of v_2 with Kn^{-1} can be checked using its

- centrality dependence (using the universality of v_2/ϵ)
- beam-energy dependence
- system-size dependence $\rightarrow$ importance of lighter systems
- rapidity dependence
- transverse momentum dependence

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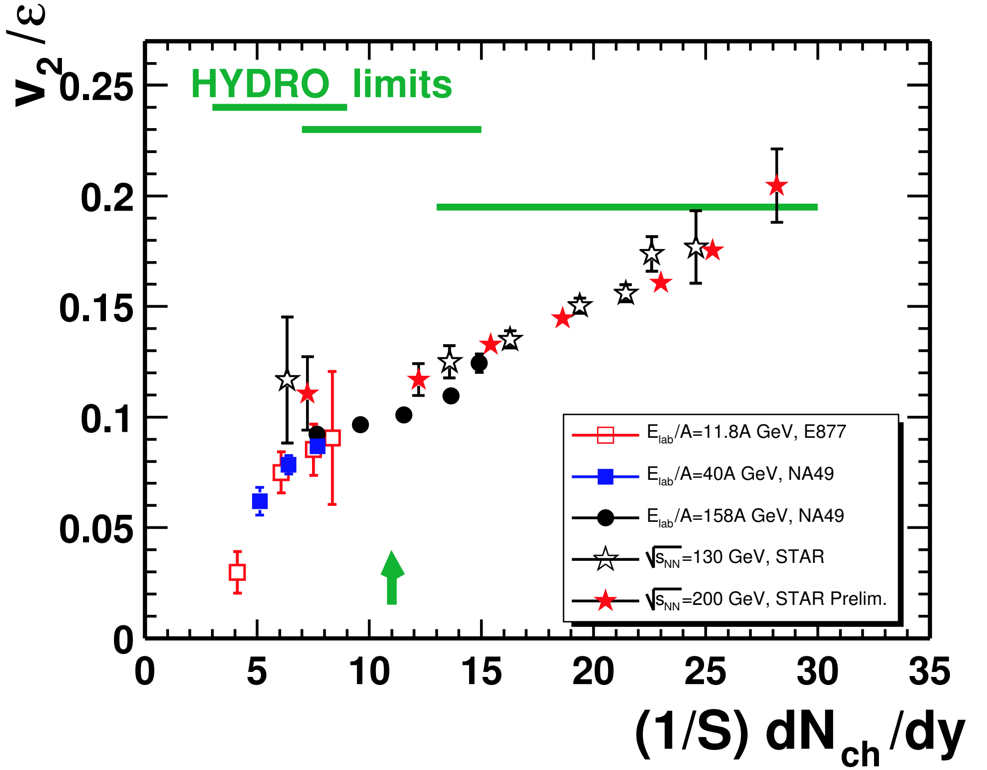
As p_t increases, σ decreases, and so does Kn^{-1}

$\Rightarrow$ equilibrium is less and less likely

"Breakdown of hydro at $p_t \gtrsim 2$ GeV" Teaney, PRC **68** (2003) 034913

RHIC anisotropic flow data and incomplete equilibration

Centrality and beam-energy dependence:



NA49 Collaboration, PRC **68** (2003) 034903

RHIC anisotropic flow data and incomplete equilibration

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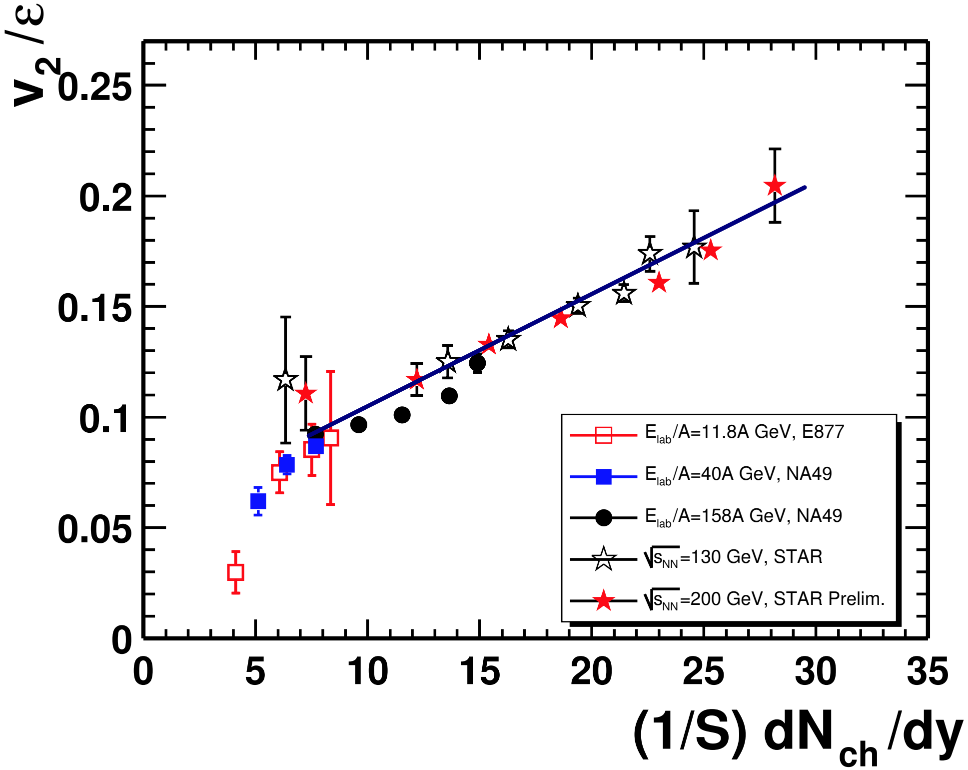


NA49 Collaboration, PRC **68** (2003) 034903

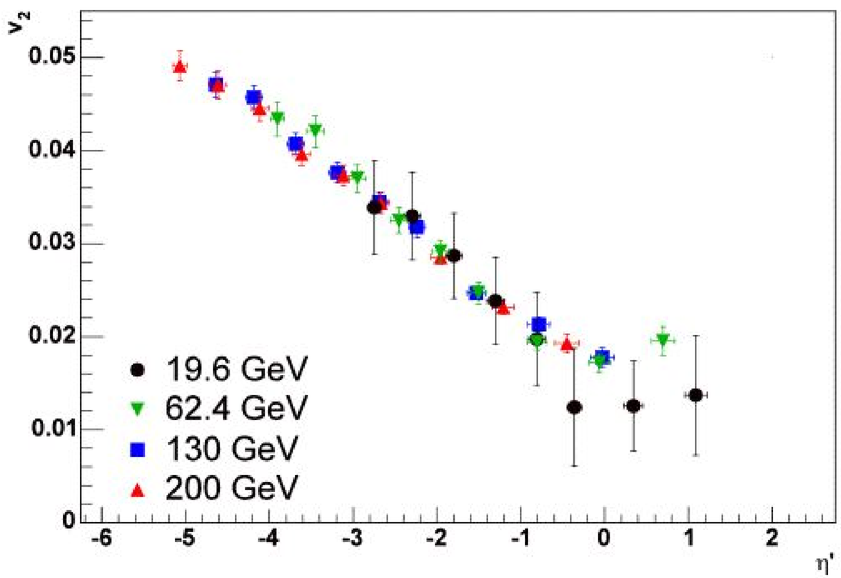
Scaling law seems to work for RHIC data (+ matching with SPS)

$v_2(Kn^{-1})$ increases steadily (no hint at hydro saturation in the data)

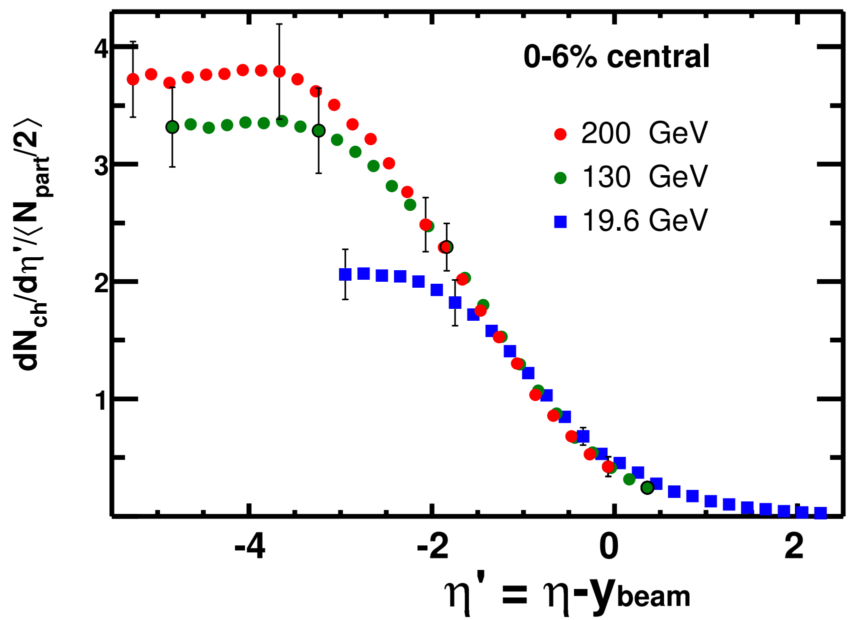
RHIC anisotropic flow data and incomplete equilibration

(Pseudo)rapidity dependence of v_2

Steve Manly (PHOBOS Coll.) QM'05



PHOBOS Collaboration PRL **91** (2003) 052303



☞ $v_2(\eta)$ and $\dfrac{\mathrm{d}N}{\mathrm{d}y}$ approximately proportional $\Leftrightarrow$ $v_2 \propto Kn^{-1}$

Hirano, PRC **65** (2002) 011901

Incomplete equilibration: predictions for Cu–Cu flow

- The matching between central SPS and peripheral RHIC suggests that we can even compare systems with different densities, i.e., different σ (and c_s)

  ☞ compare Au–Au at $b = 8$ fm with Cu–Cu at $b = 5.5$ fm (similar centrality)

  - If hydro holds, v_2 should scale like ϵ: $v_2(\mathrm{Cu}) = 0.69\, v_2(\mathrm{Au})$
  - If thermalization is incomplete, $\dfrac{v_2}{\epsilon} \propto \dfrac{1}{S}\dfrac{\mathrm{d}N}{\mathrm{d}y} \propto Kn^{-1}$, i.e.

$$v_2(\mathrm{Cu}) = 0.34\, v_2(\mathrm{Au})$$

- Cu–Cu further from equilibrium than Au–Au $\Rightarrow$ $\dfrac{v_4}{(v_2)^2} > 1.2$
- First results presented at QM'05 are too preliminary…

Out-of-equilibrium scenario: predictions for LHC

Measuring anisotropic flow at LHC, one should find

- $\dfrac{v_2}{\epsilon}$ larger than at RHIC

  ☞ getting closer to equilibrium when flow develops

- $\dfrac{v_4(p_t)}{(v_2(p_t))^2}$ smaller than at RHIC

  ☞ closer to the ideal-fluid dynamics value $\dfrac{1}{2}$

- Try different systems! (Pb–Pb vs. smaller nuclei)

Ideal liquid dynamics

vs.

out-of-equilibrium scenario

Anisotropic flow and thermalization at RHIC

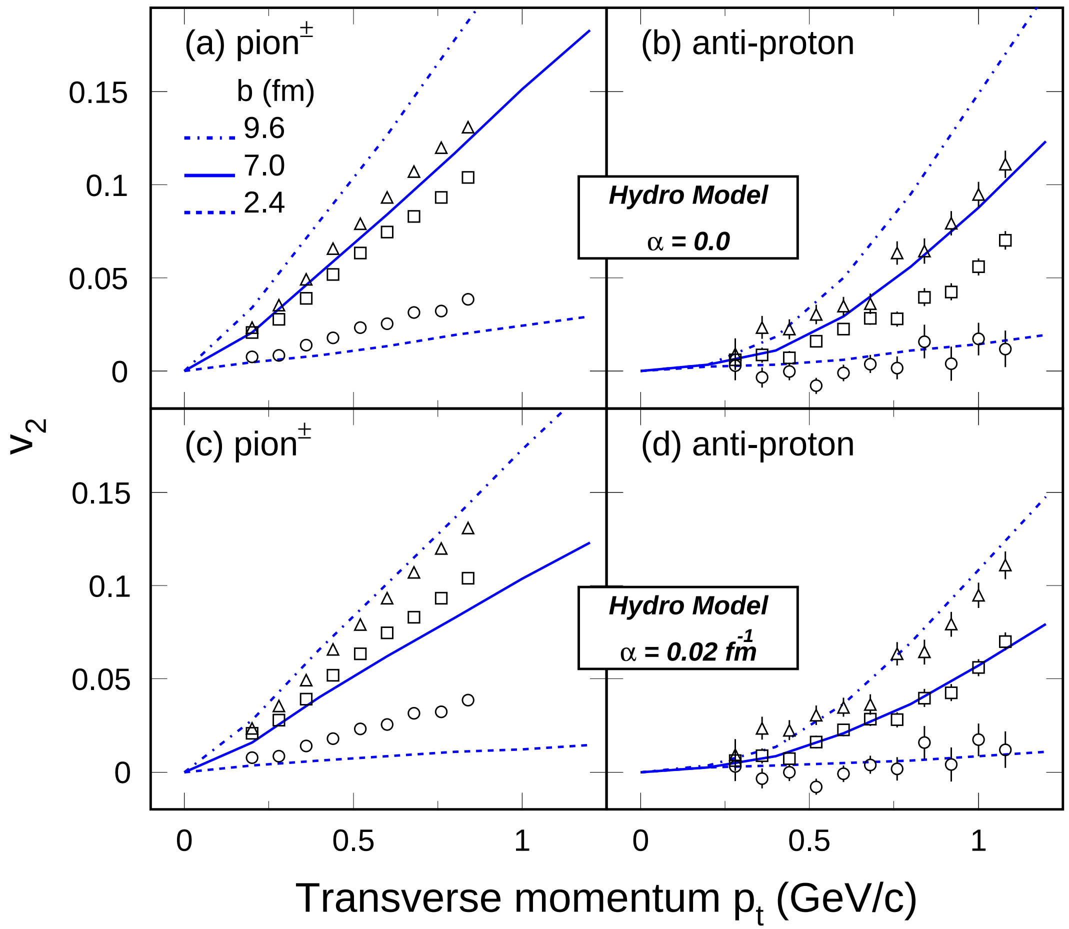
- On the theoretical side...
  - bottom-up approaches cannot accommodate short thermalization times
  - new mechanisms are emerging, but their outcome is unclear

- What do the data tell us?
  Conflicting interpretations of Au–Au measurements!
  - early thermalization, which allows one to use hydrodynamics
    ☞ $v_2(p_t)$, spectra
  - incomplete thermalization, which manifests itself by the breakdown of several scaling laws of hydro
    ☞ $v_2(y)$, $v_2(E_{\rm beam})$, $v_4/(v_2)^2$

Rescuing
ideal fluid dynamics

The use of hydrodynamics at RHIC may have been too simplistic…

$v_2(p_t)$ for various centralities STAR Collaboration, PRC **72** (2005) 014904



Rescuing ideal fluid dynamics

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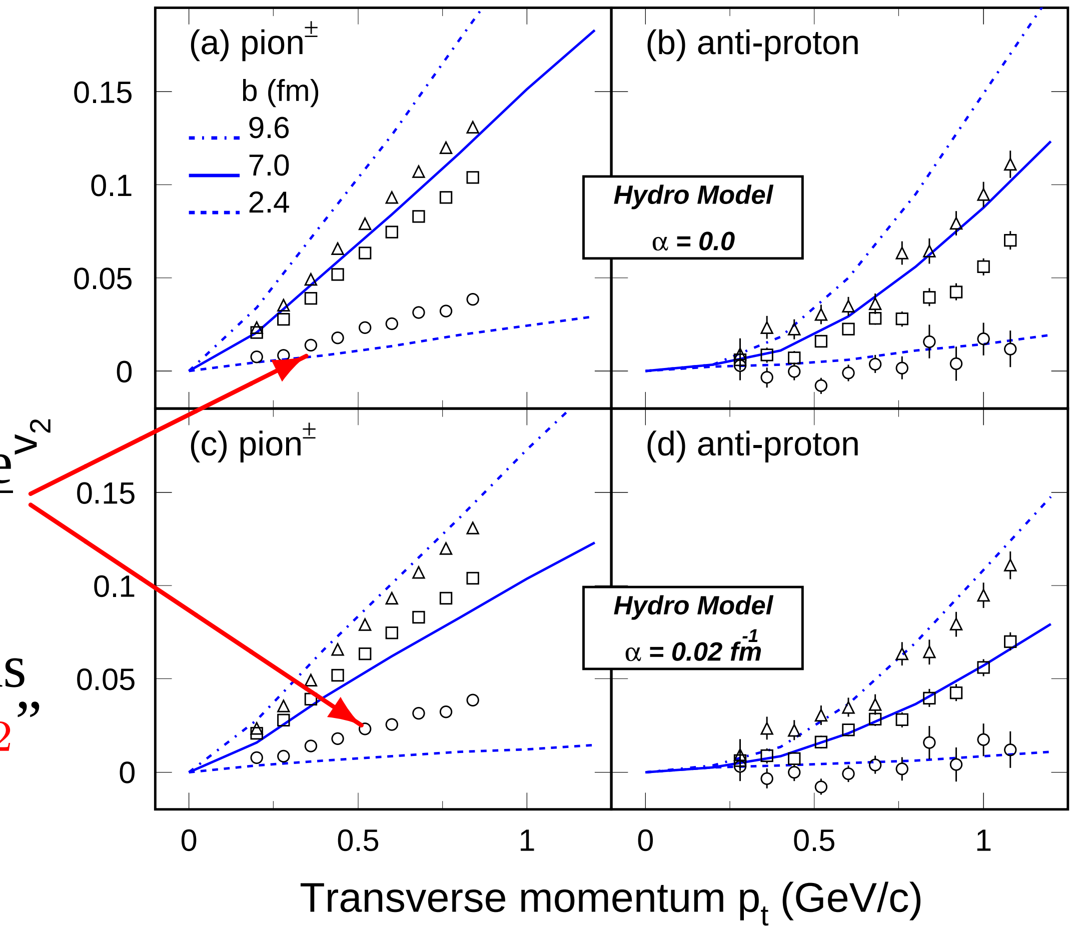
data above ideal fluid

$\neq$

"hydro yields maximum v_2"



Rescuing
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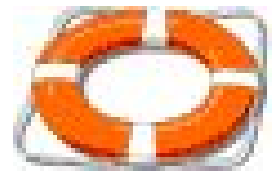


In hydrodynamical fits, the speed of sound is constrained by p_t spectra, which require a soft equation of state

$\longrightarrow$ with a hard equation of state, the energy per particle is too high

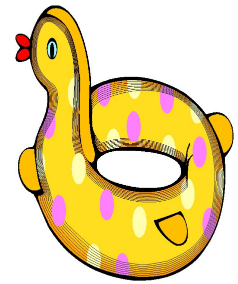
All relies on the assumption that the energy per particle is related to the density, i.e., that chemical equilibrium is maintained

- chemical equilibrium is more fragile than kinetic equilibrium
- the only experimental indication of chemical equilibrium is in the particle-abundance ratios (cf. however $e^+e^-\ldots$)

If there is no chemical equilibrium, energy per particle and density are independent variables, as in ordinary thermodynamics

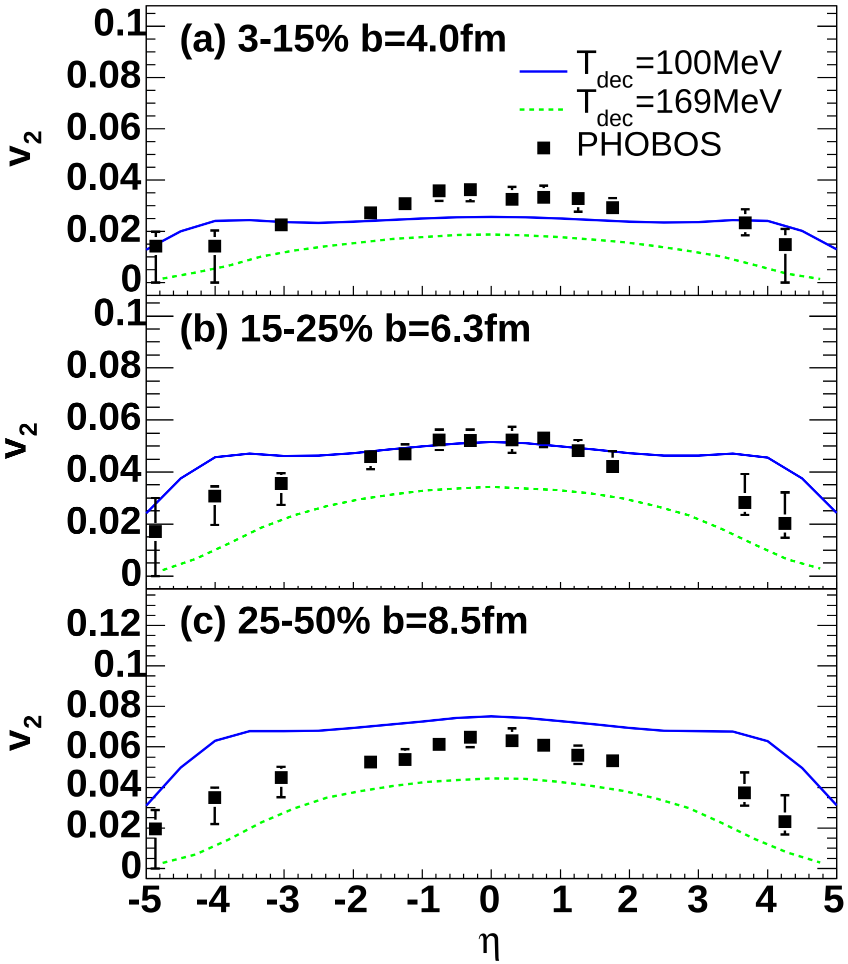
☞ there is no constraint on the equation of state from p_t spectra:
one can consider a larger c_s to increase v_2 in central collisions

Rescuing ideal fluid dynamics



T. Hirano, U. Heinz, D. Kharzeev, R. Lacey, Y. Nara, **nucl-th/0511046**

- Assume a quickly equilibrated, perfect liquid
- Vary the initial conditions of its hydro evolution
  - quasi-Gaussian in rapidity
  - CGC
- $\Rightarrow$ $v_2(\eta)$ is not reproduced



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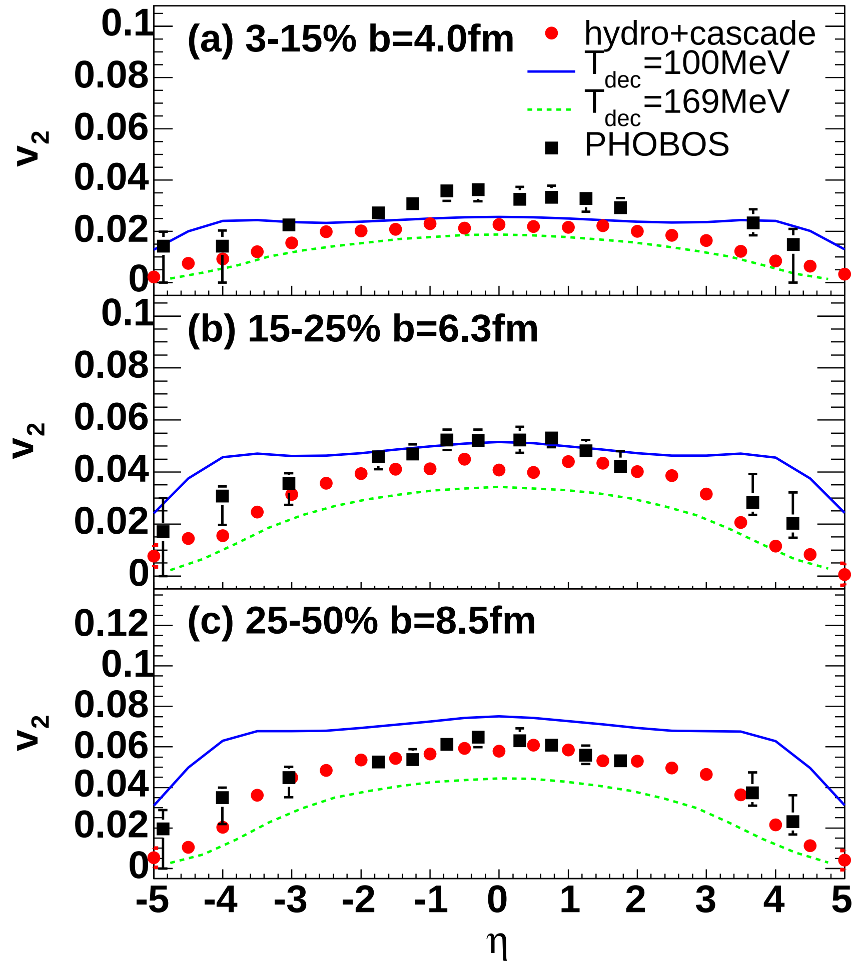
- Invoke "hadronic dissipation" to explain the discrepancy between hydro and data

  → modeled by hadronic cascade



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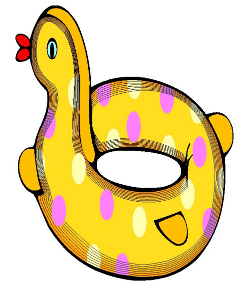

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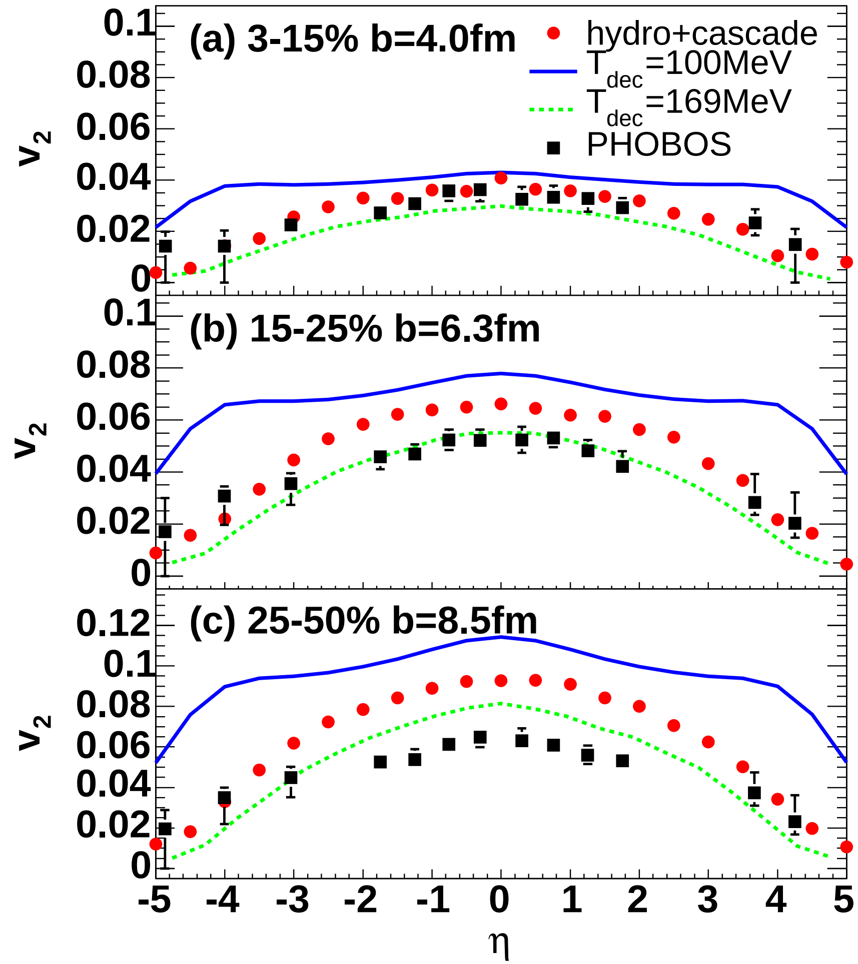


Hints of incomplete thermalization in RHIC data

A wealth of experimental measurements on anisotropic flow is already available from RHIC, and much more will come.

But these data still await a consistent quantitative interpretation:

a model which aims at describing v_2 should simultaneously explain v_4!
(not to mention fancier effects: Mach cone...)

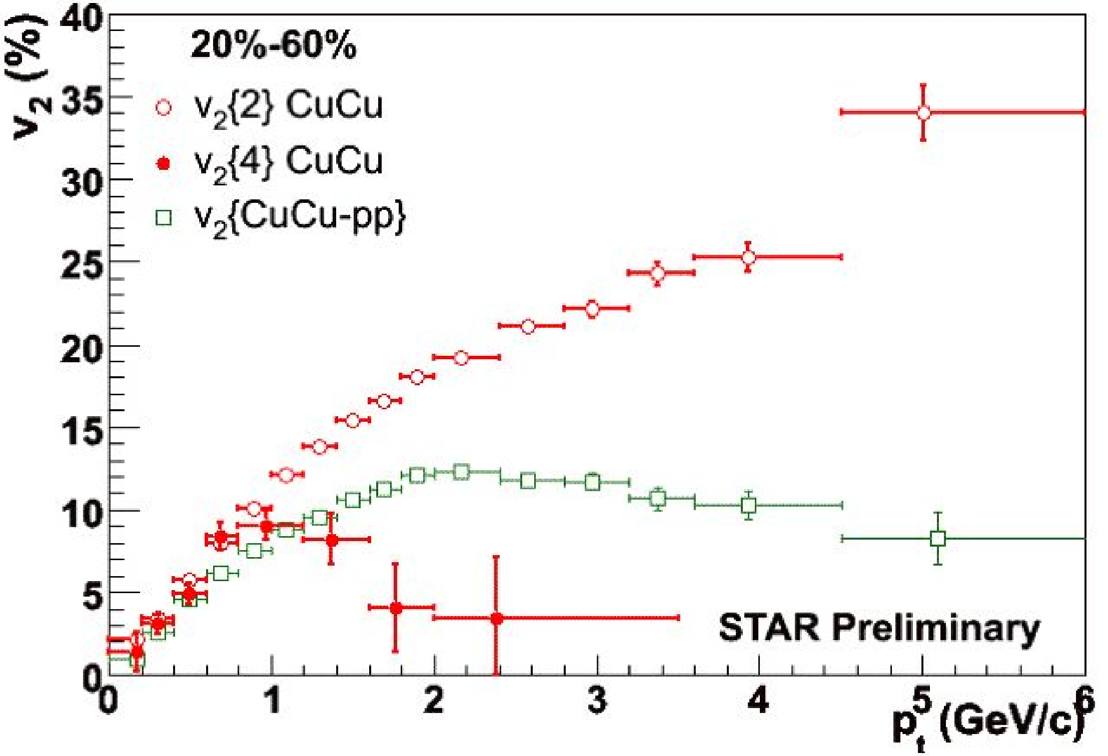
Assuming that thermalization is incomplete at the time when flow develops gives a qualitative explanation of several features observed in the data (which do not come out naturally in a perfect-liquid picture).

Hopefully, this qualitative agreement may turn into a quantitative one, yielding information on the created medium: viscosity, other transport coefficients.

Extra slide

Cu–Cu collisions at RHIC: anisotropic flow

Gang Wang (STAR Collaboration) @ QM’05:



Measurements with different methods give very different v_2 values (not a surprise. . .)

Wait and see!