Medium-induced modification of a parton shower

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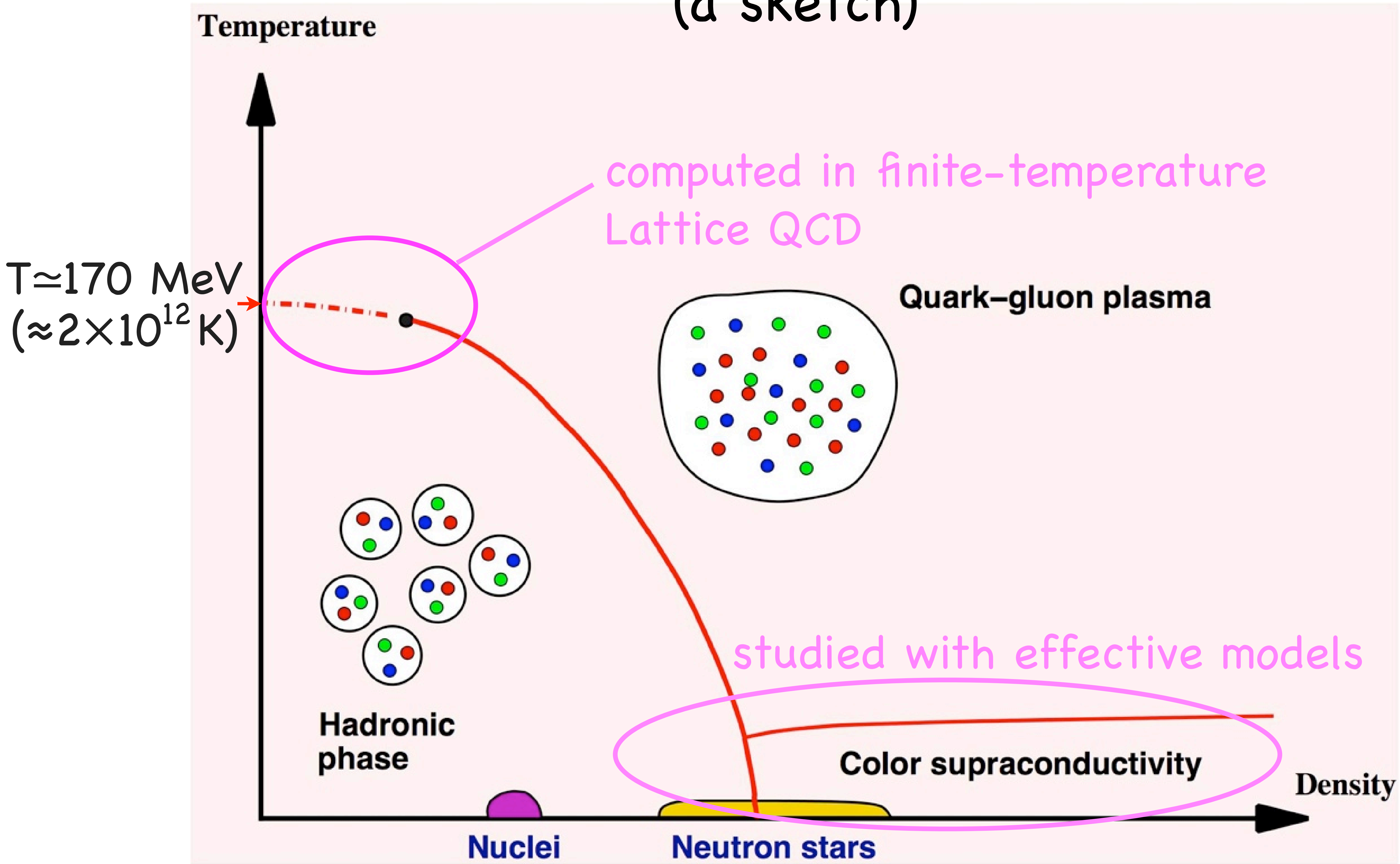
Medium-induced modification of jets

- "Jet" studies in high-energy nucleus-nucleus collisions
  - Motivation
  - Overview of some RHIC experimental results
  - Comments on existing models for RHIC phenomenology
- Jet physics in collisions of elementary particles

  Modified Leading Logarithmic Approximation (MLLA)
- Towards a "medium-modified MLLA"

Why heavy-ion collisions?
"phase diagram" of nuclear matter

(a sketch)



Why heavy-ion collisions?
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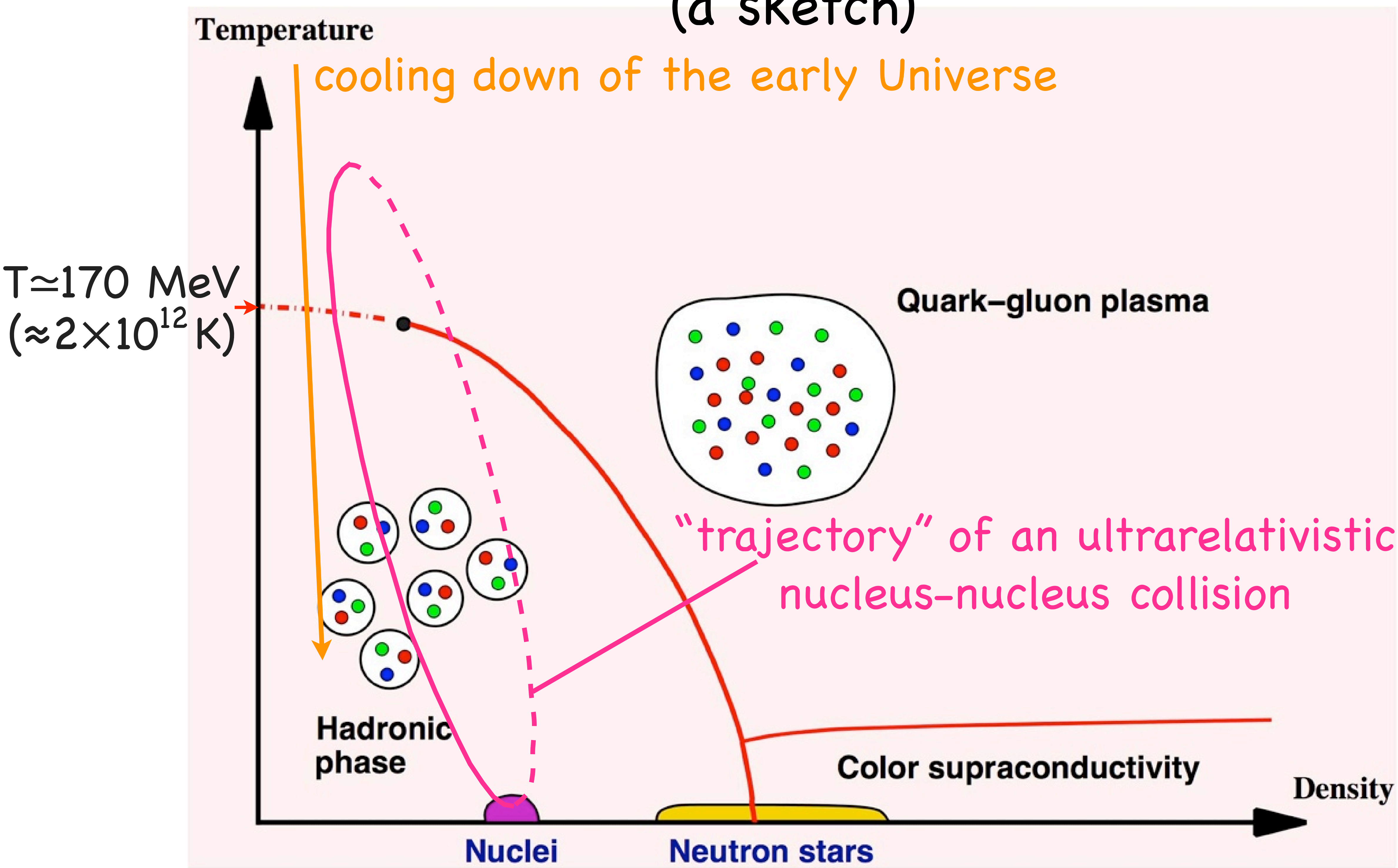
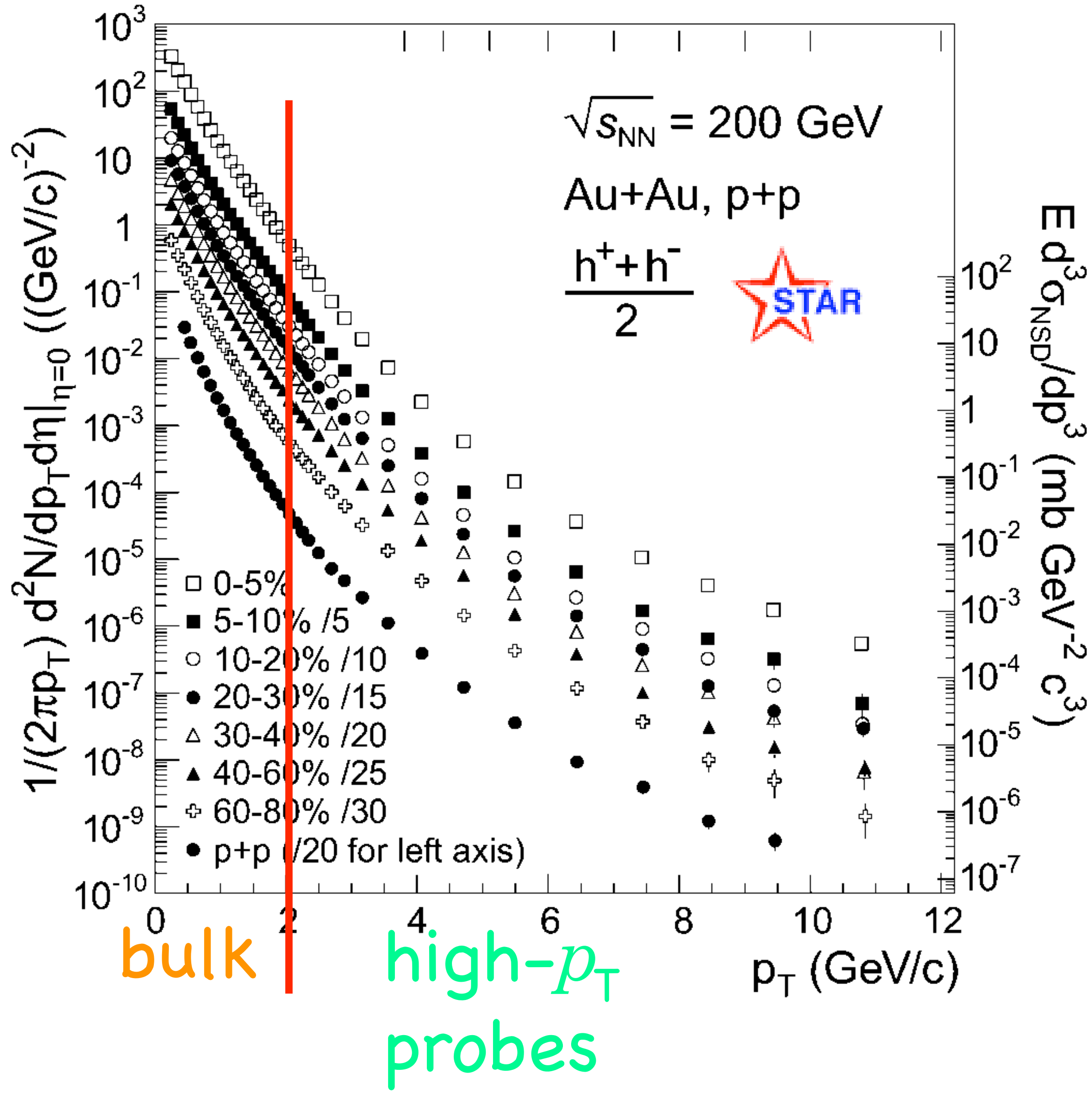


Jet studies in heavy-ion collisions

The high-energy collision of two heavy nuclei (Au, Pb...) leads to the production of thousands of particles:

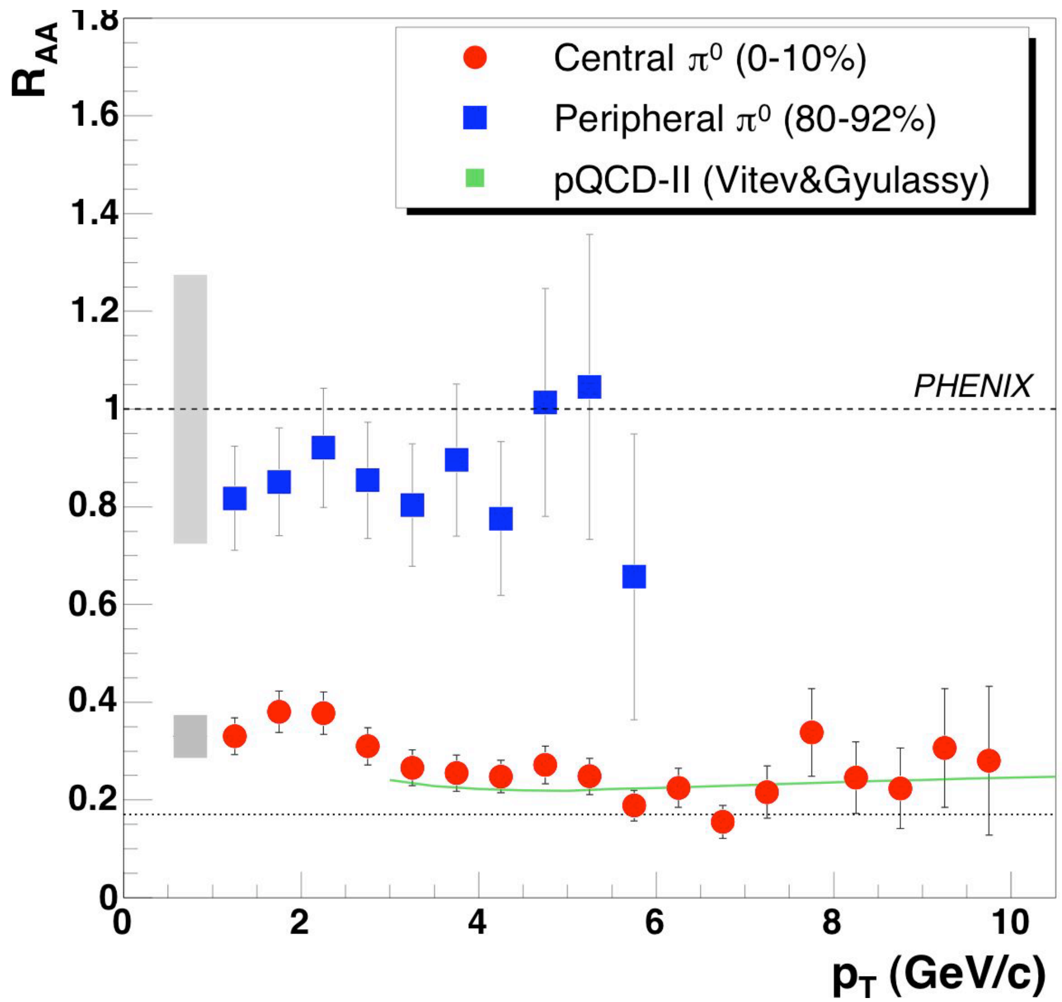


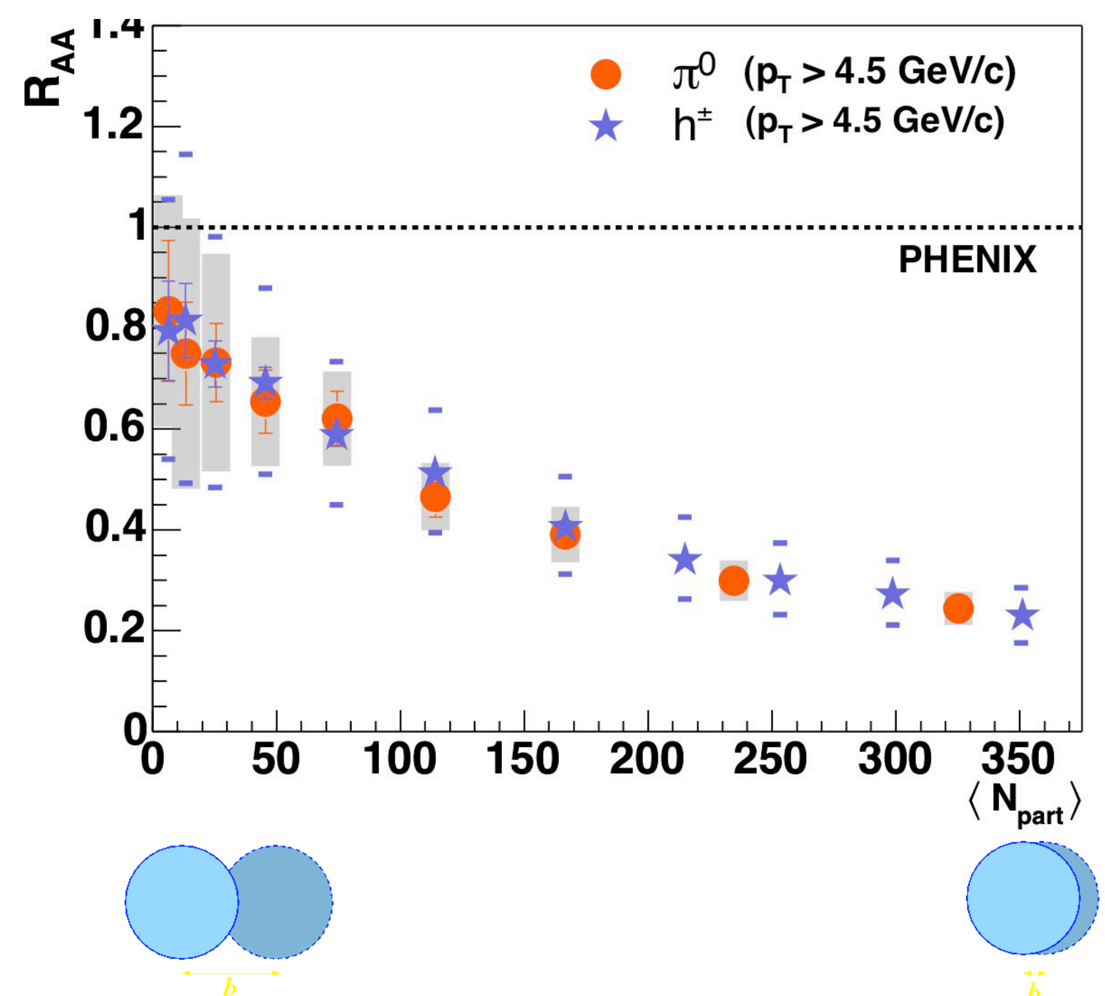
Particles with high momenta are rare, but their production mechanism is a priori better understood (perturbative QCD): can probe the bulk.

"Jets" in Au–Au collisions at RHIC

One-particle observable: nuclear modification factor $R_{AA} \equiv \frac{1}{N_{\rm coll}} \frac{\frac{{\rm d}^2 N_{AA}}{{\rm d}P_T\,{\rm d}y}}{\frac{{\rm d}^2 N_{pp}}{{\rm d}P_T\,{\rm d}y}}$

(=1 if AA collision is a superposition of independent NN collisions)

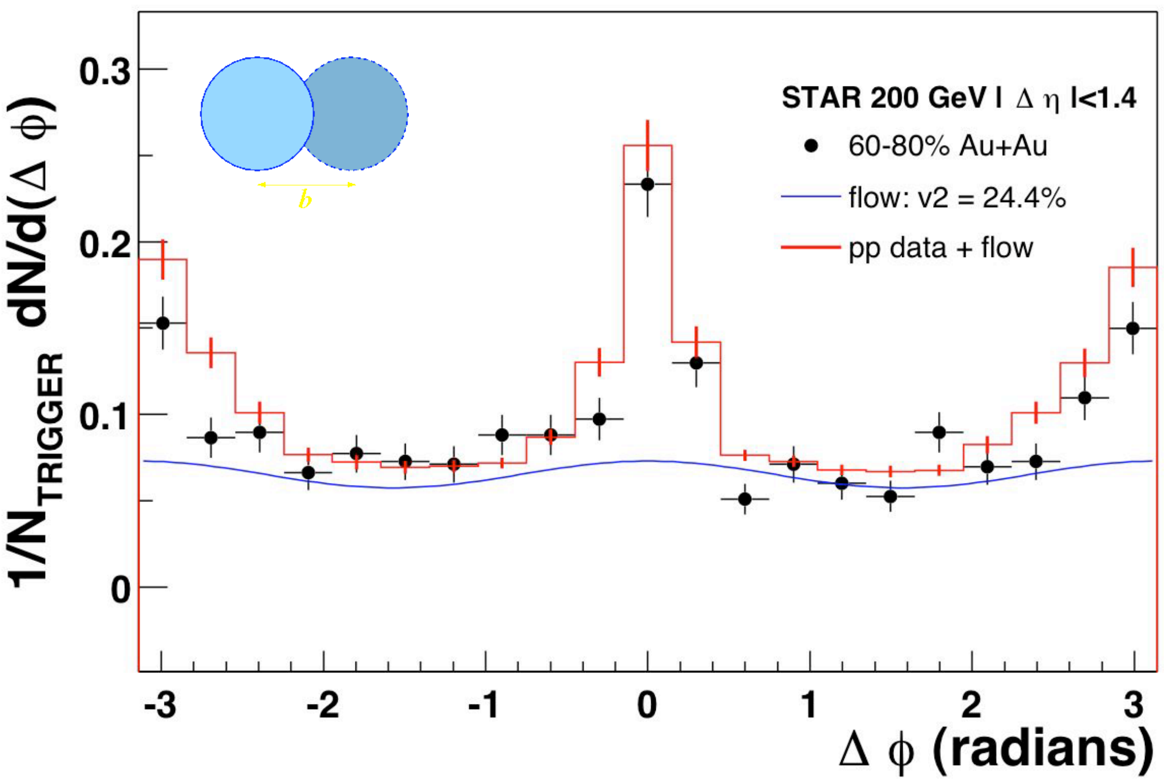


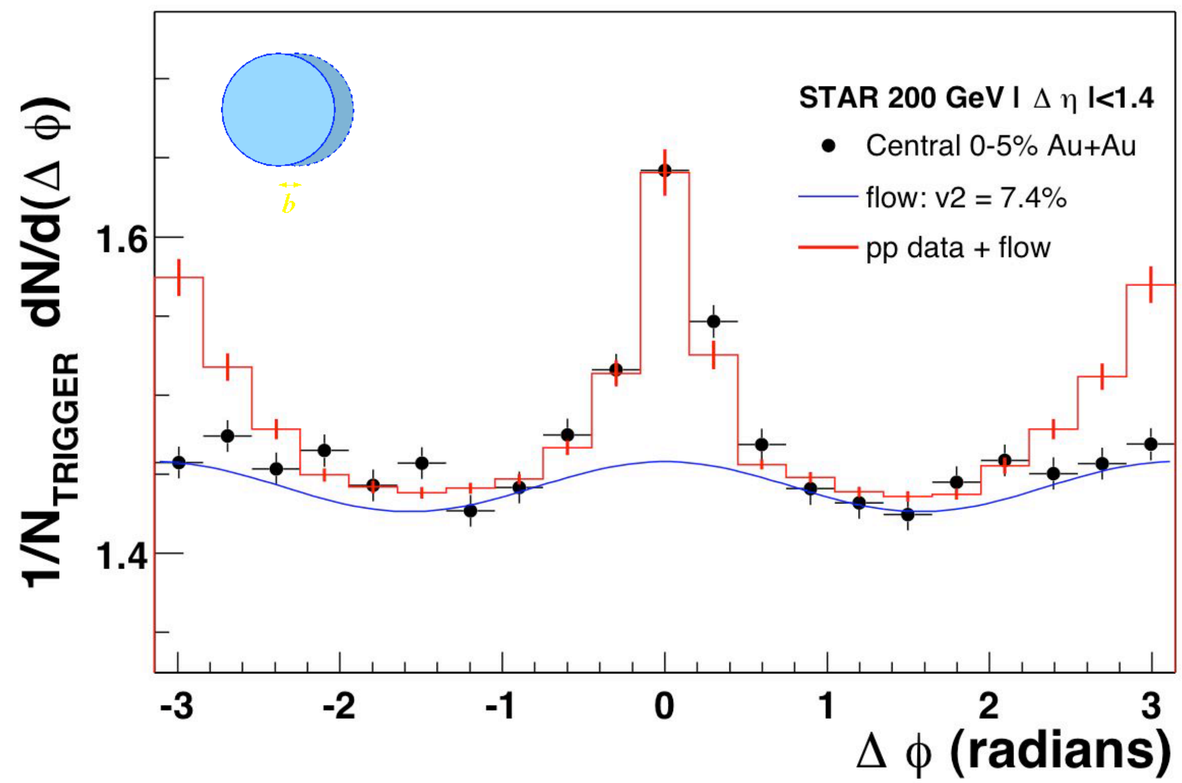


In central collisions, one misses 80% of the high transverse momentum hadrons!

“Jets” in Au-Au collisions at RHIC

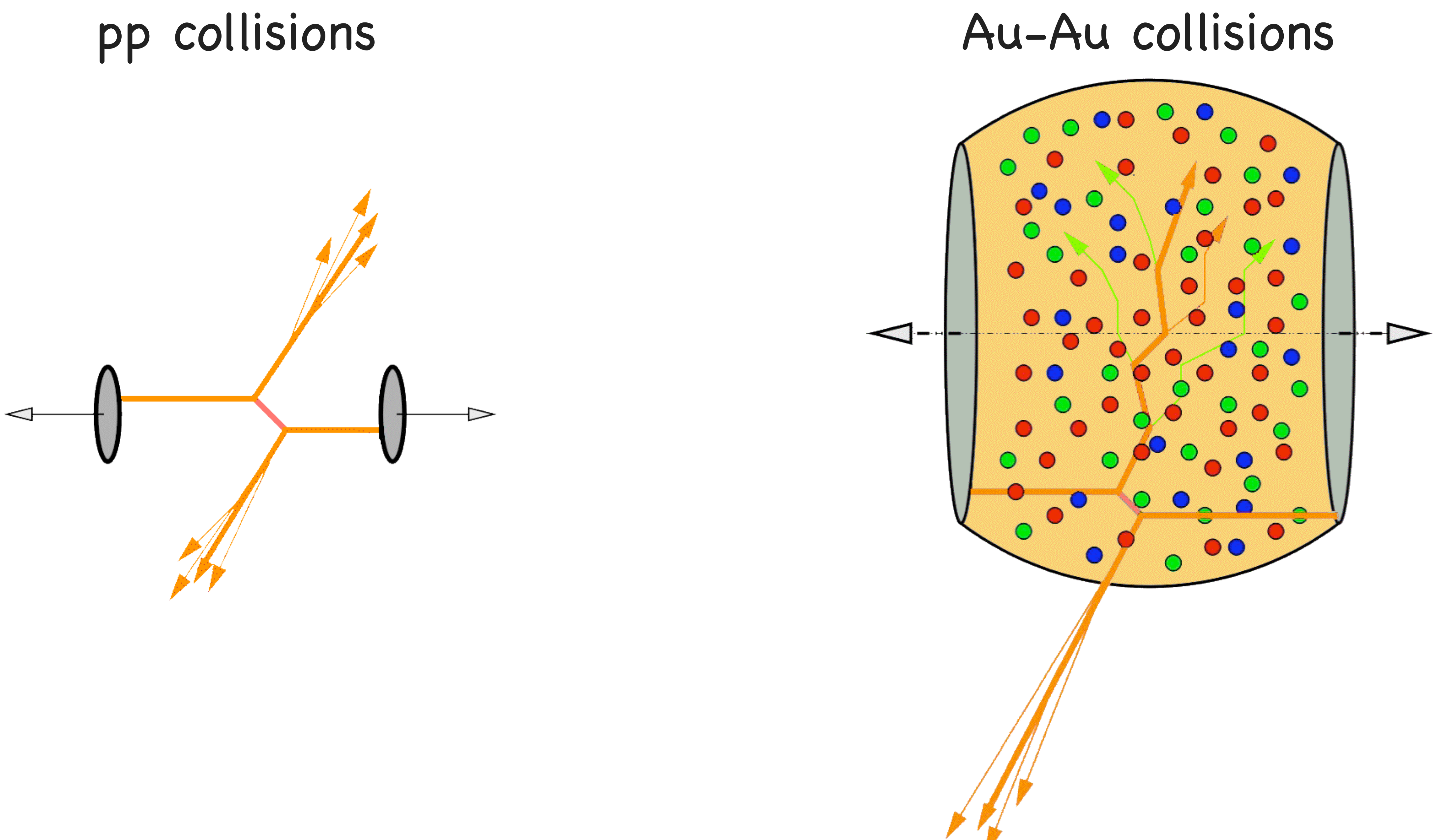
Study of azimuthal correlations between ① a reference, “trigger” particle (leading particle) with momentum $P_{T\,\mathrm{max}}$, and ② “associated particles” with momenta $P_{T\,\mathrm{cut}} < P_T < P_{T\,\mathrm{max}}$.





In central collisions, the “back jet” (= peak at 180° from the trigger particle) disappears

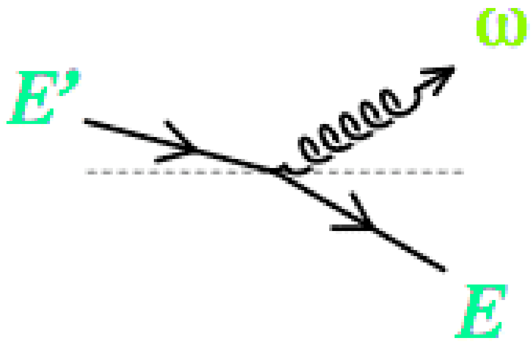
"Jets" in Au-Au collisions at RHIC: common lore



Only the fast partons created close to the edge of the medium can escape as jets; the others are "quenched".

"Jets" in Au-Au collisions at RHIC: usual description

Jet quenching is usually modelled as the energy loss of a fast parton, which emits soft gluons when traversing the medium.



☞ radiated-energy spectrum per unit in-medium path length $\frac{\omega\,\mathrm{d}I}{\mathrm{d}\omega\,\mathrm{d}\ell}$

cf. BDMPS-Z-W, GLV...

Correctly reproduces the nuclear modification factor R_{AA}, but

- The formalism does not automatically ensure energy-momentum conservation (the parton can radiate more energy than it has initially!) ⇒ conservation is imposed a posteriori, globally ("quenching weights").

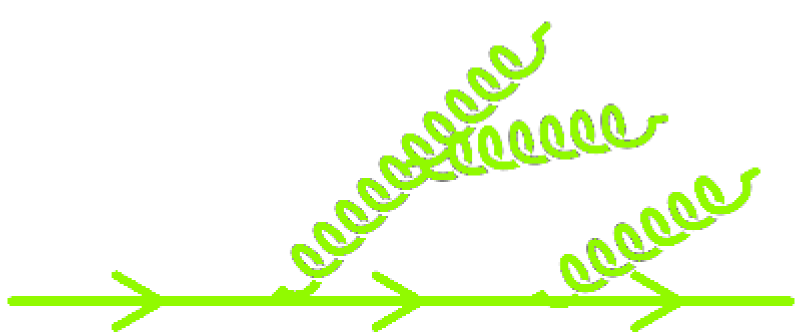
- The formalism deals differently with the leading parton (for which the medium-enhanced radiation is considered) and the subleading ones ⇒ cannot address intra-jet correlations.

MLLA: some theory...

Main ingredients:

- Resummation of double- and single-logarithms in $\ln\frac{1}{x}$ and $\ln\frac{E_{\rm jet}}{\Lambda_{\rm eff}}$;
- Takes into account the running of α_s along the parton shower evolution;
- Probabilistic interpretation (results from intra-jet colour coherence):
  - independent successive branchings $g\to gg$, $g\to q\bar{q}$, $q\to qg$;
  - with angular ordering of the sequential parton decays:

    at each step in the evolution, the angle between father and offspring partons decreases.



- Includes in a systematic way next-to-leading-order corrections. $\mathcal{O}(\sqrt{\alpha_s})$!

MLLA: some theory...

Central object: generating functional $Z_i[Q, \Theta; u(k)]$

☞ generates the various cross-sections ($\to ggg$, $\to ggq\bar{q}$ ...) for a jet initiated by a parton i ($= g, q, \bar{q}$) with energy Q in a cone of angle Θ

$$Z_i[Q, \Theta; u(k)] = \mathrm{e}^{-w_i(Q,\Theta)} u(Q) + \sum_j \int^{\Theta} \frac{\mathrm{d}\Theta'}{\Theta'} \int_0^1 \mathrm{d}z\, \mathrm{e}^{w_i(Q,\Theta')-w_i(Q,\Theta)} \frac{\alpha_s(k_\perp)}{2\pi} \times P_{ji}(z)\, Z_j[zQ, \Theta'; u]\, Z_k[(1-z)Q, \Theta'; u]$$

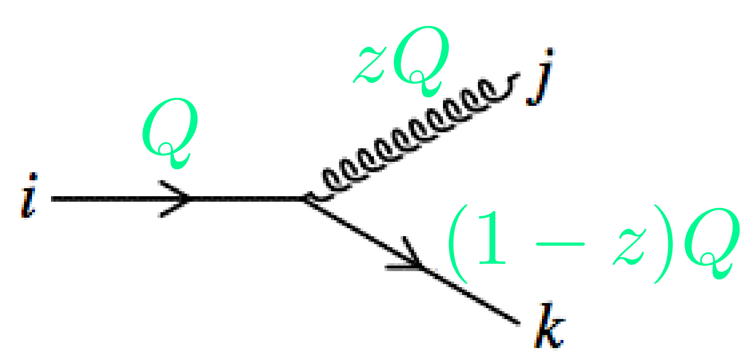
probability to have no branching with angle < Θ

between Θ and Θ'

angular ordering

LO splitting function $i \to jk$

$k_\perp \approx z(1-z)Q$



MLLA: some theory…

Some remarks:

- One only considers partons with energy $Q \geq \Lambda_{\rm eff} \simeq \Lambda_{\rm QCD}$

$\Lambda_{\rm eff}$ infrared cutoff: parameter of the model.

$$\Rightarrow \text{ in fact, } Z_i[Q,\Theta;\Lambda_{\rm eff};u(k)]$$

- For $Q = \Lambda_{\rm eff}$, no further parton splitting:

$$Z_i[Q,\Theta;\Lambda_{\rm eff};u(k)]\Big|_{Q=\Lambda_{\rm eff}} = u(k\!=\!Q)$$

- Physics should not depend on the choice of $\Lambda_{\rm eff}$!

$$\frac{\partial}{\partial Q} Z_i[Q,\Theta;\Lambda_{\rm eff};u(k)]\Big|_{Q=\Lambda_{\rm eff}} = 0$$

- Actually, Z_i only depends on the combination $Q\sin\Theta$

Hereafter, I shall use $\tau \equiv \ln \dfrac{Q\sin\Theta}{\Lambda_{\rm eff}}$

MLLA: some theory...

The parton distribution inside a jet with "energy" τ is given by

$$\bar{D}_i(x,\tau) \equiv Q \frac{\delta}{\delta u(xQ)} Z_i[\tau; u(k)]\Big|_{u\equiv 1}$$

The evolution of the distribution obeys

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\left[x\bar{D}_i(x,\tau)\right] = \sum_j \int_0^1 \mathrm{d}z\, \frac{\alpha_s}{2\pi} P_{ji}(z) \frac{x}{z} \bar{D}_i\left(\frac{x}{z}, \tau'\right)$$

with $\tau' \equiv \tau + \ln z$

To solve the evolution equation, one considers the Mellin moments

$$D_i(\nu,\tau) \equiv \int_0^1 \mathrm{d}x\, x^{\nu-1}\left[x\bar{D}_i(x,\tau)\right]$$

$\Rightarrow$ differential equations for $D_g(\nu,\tau)$ and $D_q(\nu,\tau)$

MLLA: some theory...

Differential equations for $D_g(\nu, \tau)$ and $D_q(\nu, \tau)$... which can be solved

☞ $D^+(\nu, \tau, \Lambda_{\rm eff})$ (linear combination of D_g and D_q), linear combination of the confluent hypergeometric functions Φ and Ψ.

One comes back to x-space by inverse Mellin transform

$$\bar{D}(x, \tau, \Lambda_{\rm eff}) = \int_{\epsilon - {\rm i}\infty}^{\epsilon + {\rm i}\infty} \frac{{\rm d}\nu}{2\pi{\rm i}}\, x^{-\nu} D^+(\nu, \tau, \Lambda_{\rm eff})$$

$$\vdots$$

Assuming* $\Lambda_{\rm eff} \ll Q$, one obtains the limiting spectrum $\bar{D}^{\rm lim}(x, \tau, \Lambda_{\rm eff})$

* This assumption, which can be relaxed, allows one to derive an analytical expression for the single-parton distribution.

MLLA: limiting spectrum

$$\bar{D}^{\lim}(x,\tau,\Lambda_{\rm eff}) = \frac{4N_c\tau}{bB(B+1)}\int_{\epsilon-{\rm i}\infty}^{\epsilon+{\rm i}\infty}\frac{{\rm d}\nu}{2\pi{\rm i}}\,x^{-\nu}\Phi(-A+B+1,B+2;-\nu\tau)$$

with

$$A \equiv \frac{4N_c}{b\nu}, \qquad B \equiv \frac{a}{b}, \qquad a \equiv \frac{11}{3}N_c + \frac{2N_f}{3N_c^2}, \qquad b \equiv \frac{11}{3}N_c - \frac{2}{3}N_f$$

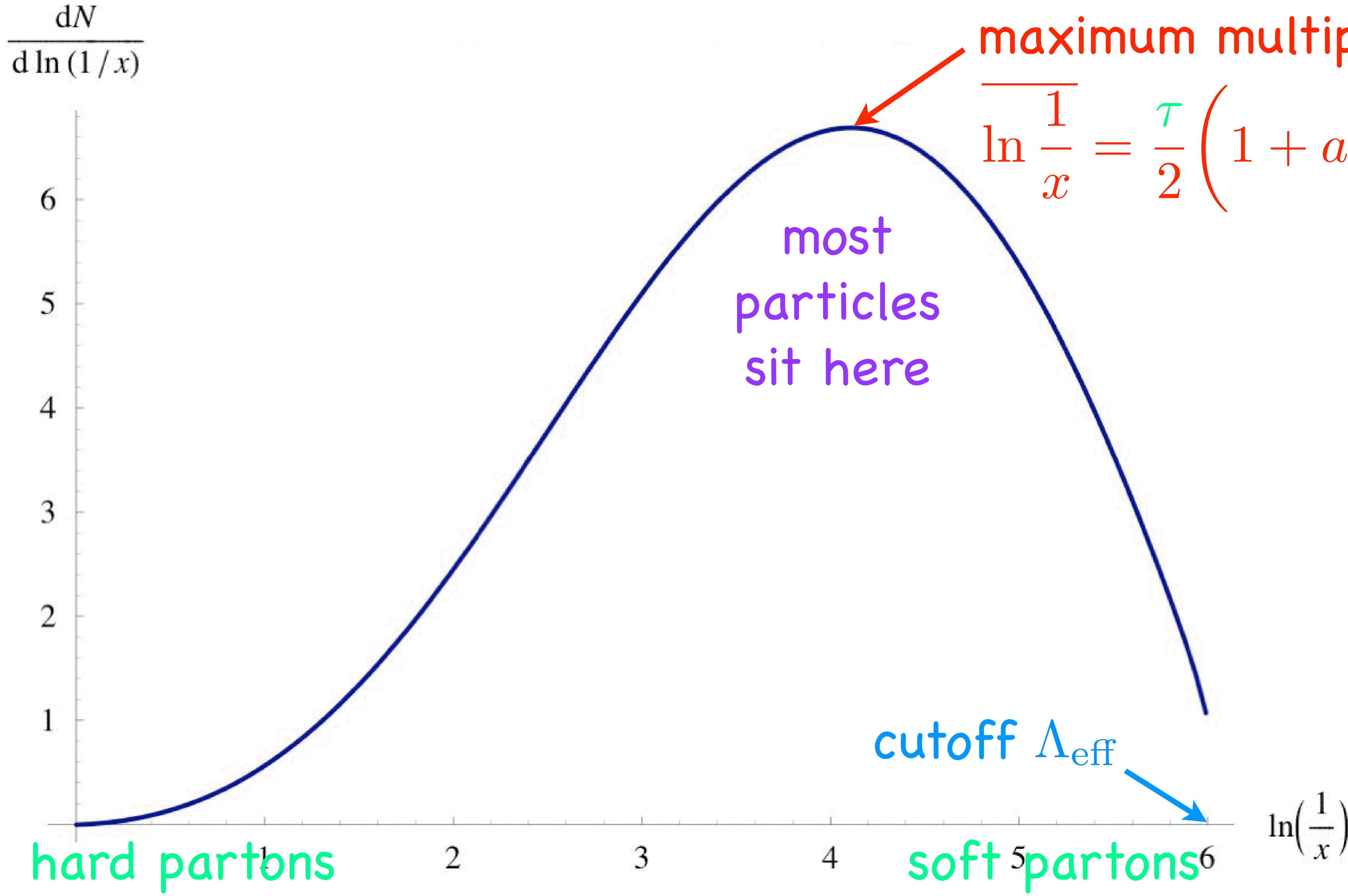
(these coefficients follow from the prefactors of the leading-order splitting functions) and

$$\tau \equiv \ln\frac{Q\sin\Theta}{\Lambda_{\rm eff}}$$

Impressive expression... which can be dealt with!

MLLA: limiting spectrum

For a 100 GeV parton:



maximum multiplicity for

$$\overline{\ln\frac{1}{x}} = \frac{\tau}{2}\left(1 + a\sqrt{\frac{\alpha_s}{8\pi N_c}}\right)$$

☞ "hump-backed plateau"

Note: the hump is dominated by the singular parts $\frac{1}{z}, \frac{1}{1-z}$ of the $P_{ji}(z)$

MLLA: some theory...

Modified Leading Logarithmic Approximation:

- Successive independent parton splittings, with a constraint on the emission angles

⇒ limiting spectrum $\bar{D}^{\lim}(x, \tau, \Lambda_{\text{eff}})$

- The spectrum is exact in the asymptotic $\tau \to \infty$ limit and includes in a systematic way corrections to subleading order

$$\mathcal{O}(\sqrt{\alpha_s})$$

- What about hadronization? ($\bar{D}^{\lim}(x, \tau, \Lambda_{\text{eff}})$ is a parton spectrum)

☞ Local parton-hadron duality (LPHD)

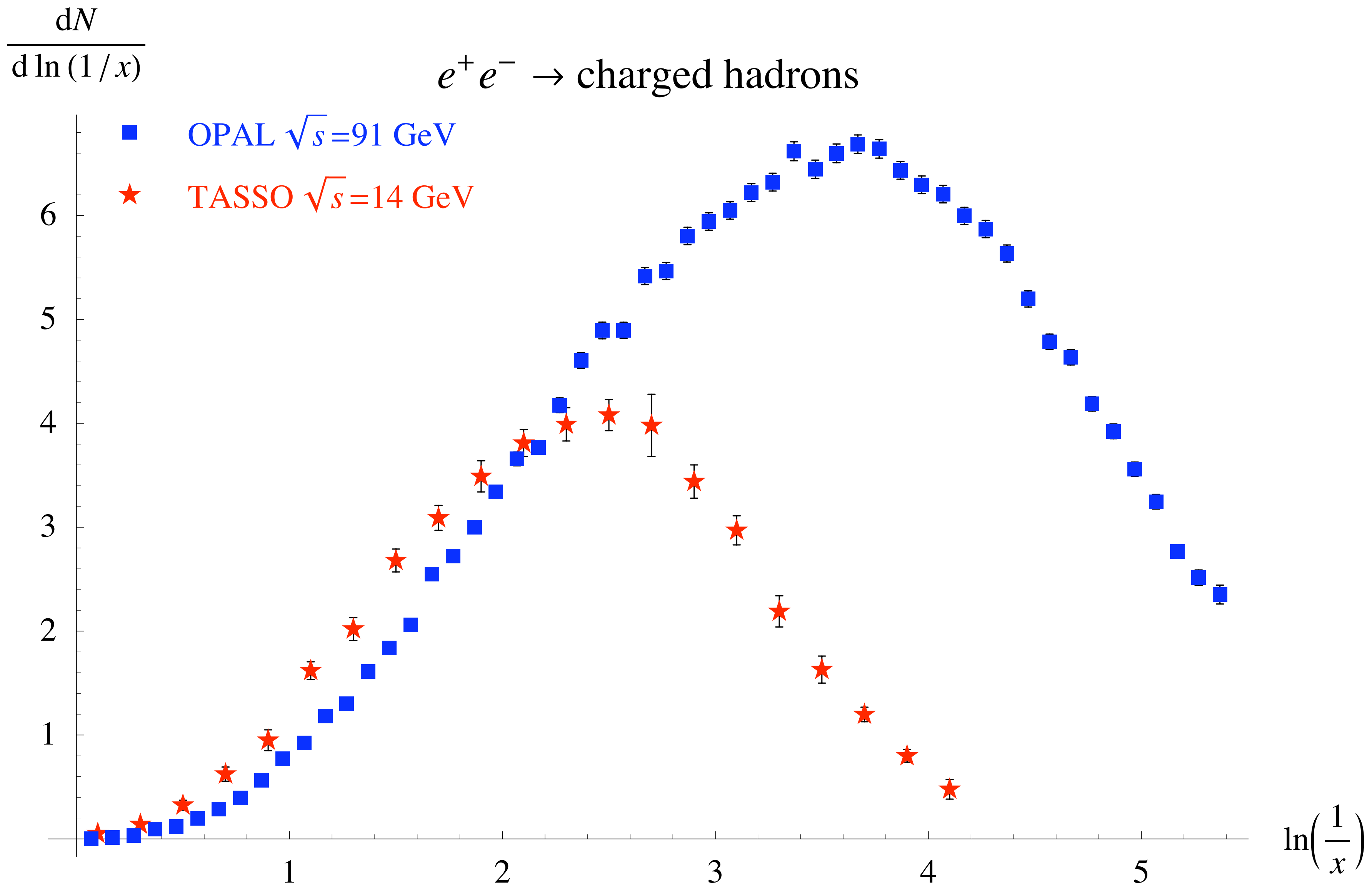
$$\bar{D}^h(x, \tau, \Lambda_{\text{eff}}) = K^h \bar{D}^{\lim}(x, \tau, \Lambda_{\text{eff}})$$

⇒ two parameters Λ_{eff} and K^h

(Actually, one can refine the description using different K^h for different hadrons, and stopping the shower evolution at different scales...)

MLLA vs. e^+e^- data

Longitudinal distribution of hadrons inside a jet:



MLLA vs. e^+e^- data

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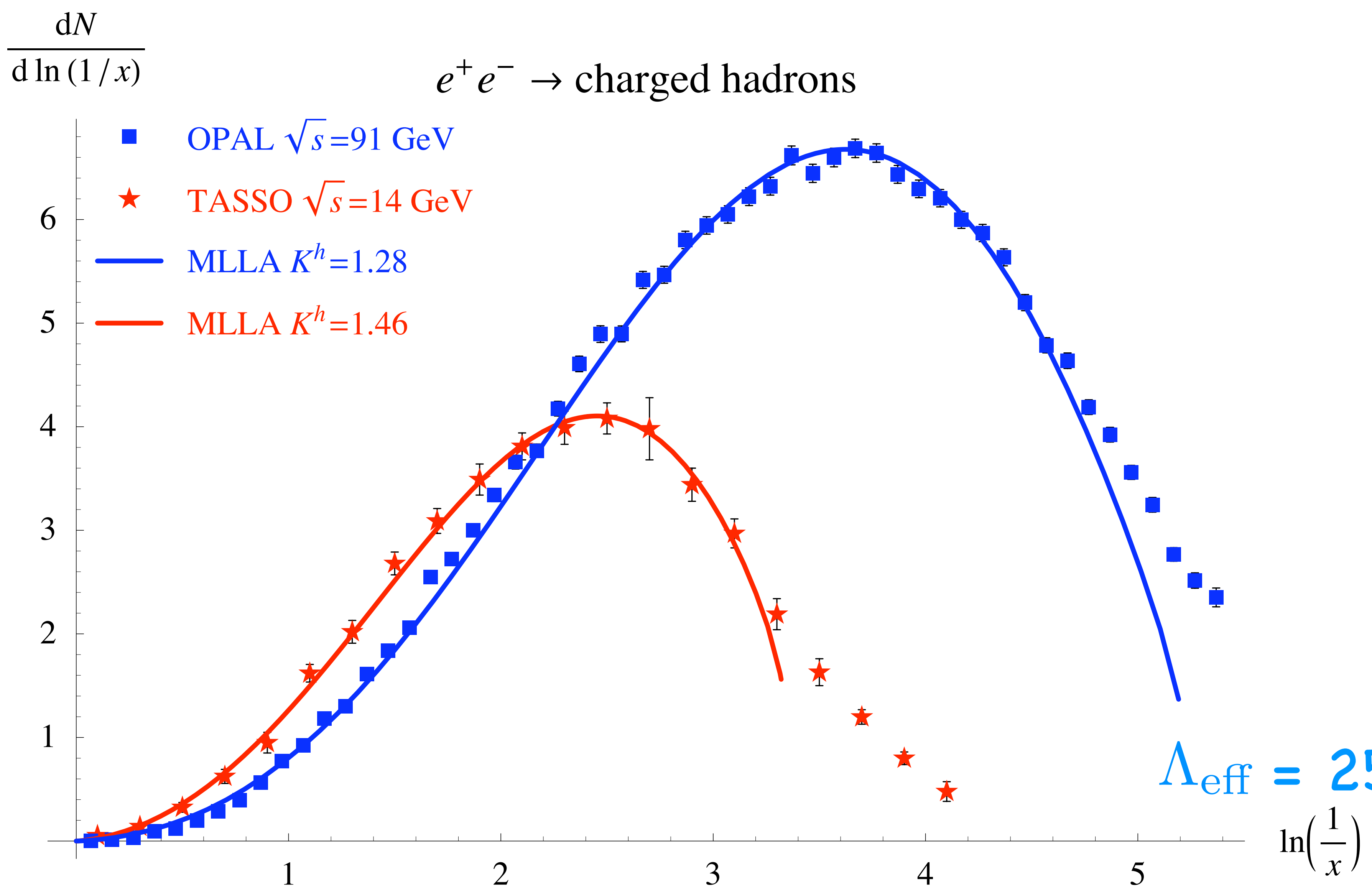


Good description of data also in $p\bar{p}$ collisions (CDF):

MLLA is reliable! (also at large x...)

Modeling the medium influence: a suggestion

- The hump of the limiting spectrum is mostly due to the singular parts of the splitting functions.
- In medium, the emission of a soft gluons by a fast parton increases.

☞ One can model medium-induced effects by modifying the parton splitting functions $P_{ji}(z)$...

(see e.g. Guo & Wang, PRL **85** (2000) 3591)

... and especially their singular parts:

$$P_{qq}(z) = \frac{4}{3}\left[\frac{2(1+f_{\rm med})}{(1-z)_+} - (1+z)\right]$$

$f_{\rm med} > 0 \quad \Rightarrow$ Bremsstrahlung increases

Modeling the medium influence

$f_{\rm med}$ = the influence of the medium on the parton cascade evolution

$\Rightarrow$ should account for

- geometry (the in-medium path length depends on the origin and orientation of the fast parton);
- dilution with time of the expanding medium;

(These effects are taken into account in the standard approaches)

- dependence with the parton virtuality:

a parton with energy E, virtuality Q, travels $\frac{E}{Q}\frac{1}{Q}$ before splitting

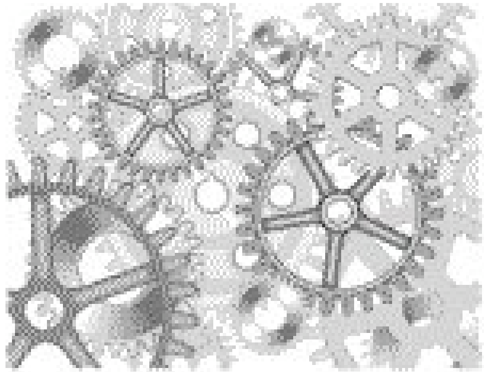
$\Rightarrow$ $f_{\rm med}$ decreases with increasing Q.

In the following, $f_{\rm med}$ will be taken as constant!

(makes analytical calculations possible + not unreasonable in RHIC regime)

Parton cascade
in the presence of a medium

One writes the evolution equation of the single parton distribution within a jet with modified splitting functions...

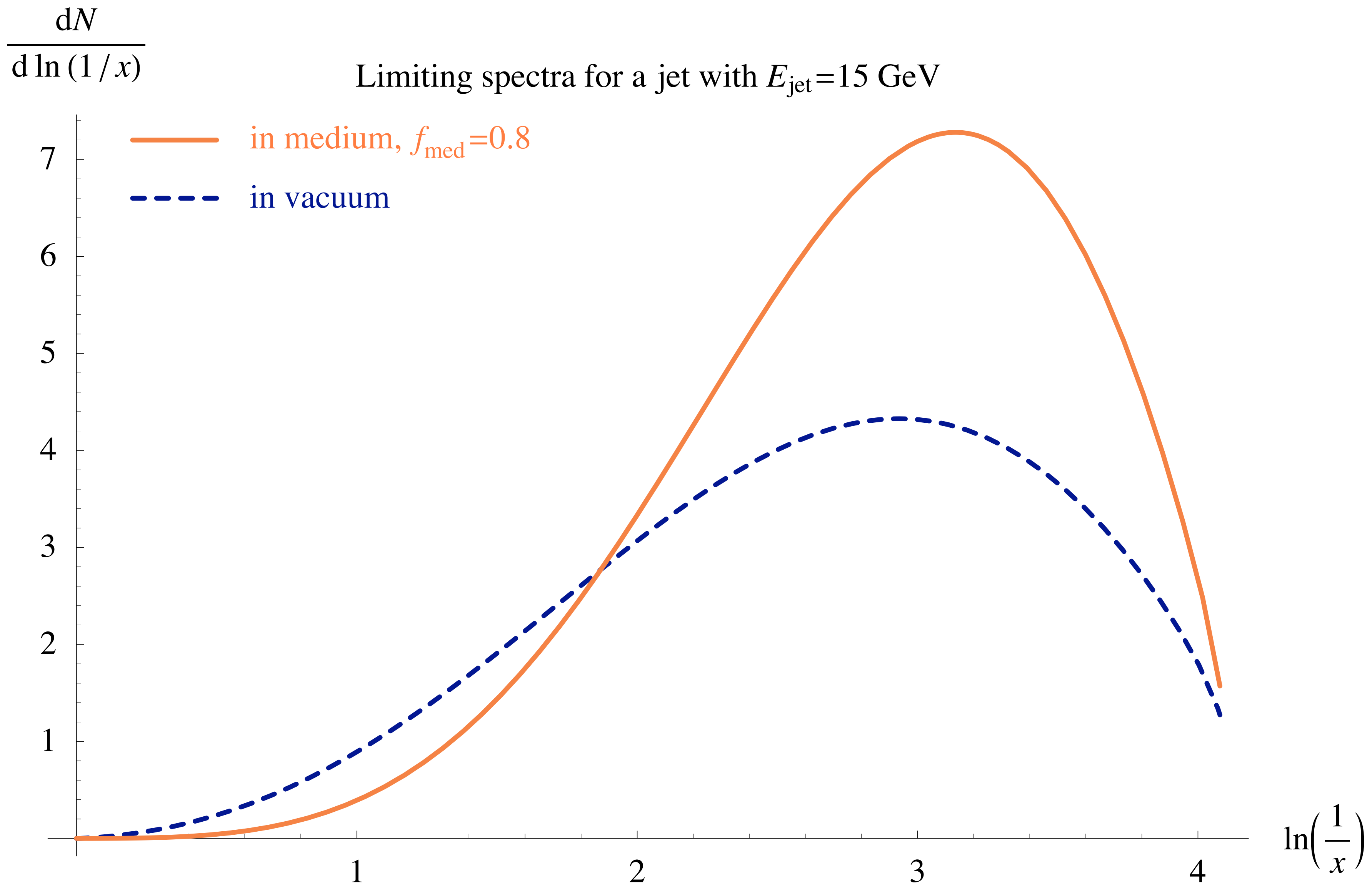


$$\bar{D}^{\rm lim}(x,\tau) = \frac{4N_c\tau(1+f_{\rm med})}{b\hat{B}(\hat{B}+1)}\int_{\epsilon-{\rm i}\infty}^{\epsilon+{\rm i}\infty}\frac{{\rm d}\nu}{2\pi{\rm i}}\,x^{-\nu}\,\Phi(-\hat{A}+\hat{B}+1,\hat{B}+2;-\nu\tau)$$

$$\hat{A} \equiv \frac{4N_c(1+f_{\rm med})}{b\nu}, \qquad \hat{B} \equiv \frac{\hat{a}}{b}, \qquad \hat{a} \equiv \frac{11+12f_{\rm med}}{3}N_c + \frac{2N_f}{3N_c^2}$$

Hadronization takes place in vacuum: K^h unchanged

Parton distribution
in the presence of a medium



f_{med} fixed to reproduce R_{AA} (see below)

Partons are redistributed from high p_T (large x) to low p_T (small x)

Medium-induced modification of the associated multiplicity

Ideal case: photon + jet

☞ the photon gives the jet energy $E_{\rm jet}$

- Count how many jet particles have a momentum larger than some given cut $P_{\rm cut}$ after propagating through the medium:

$$\mathcal{N}(P_T \geq P_{\rm cut})_{\rm medium}$$

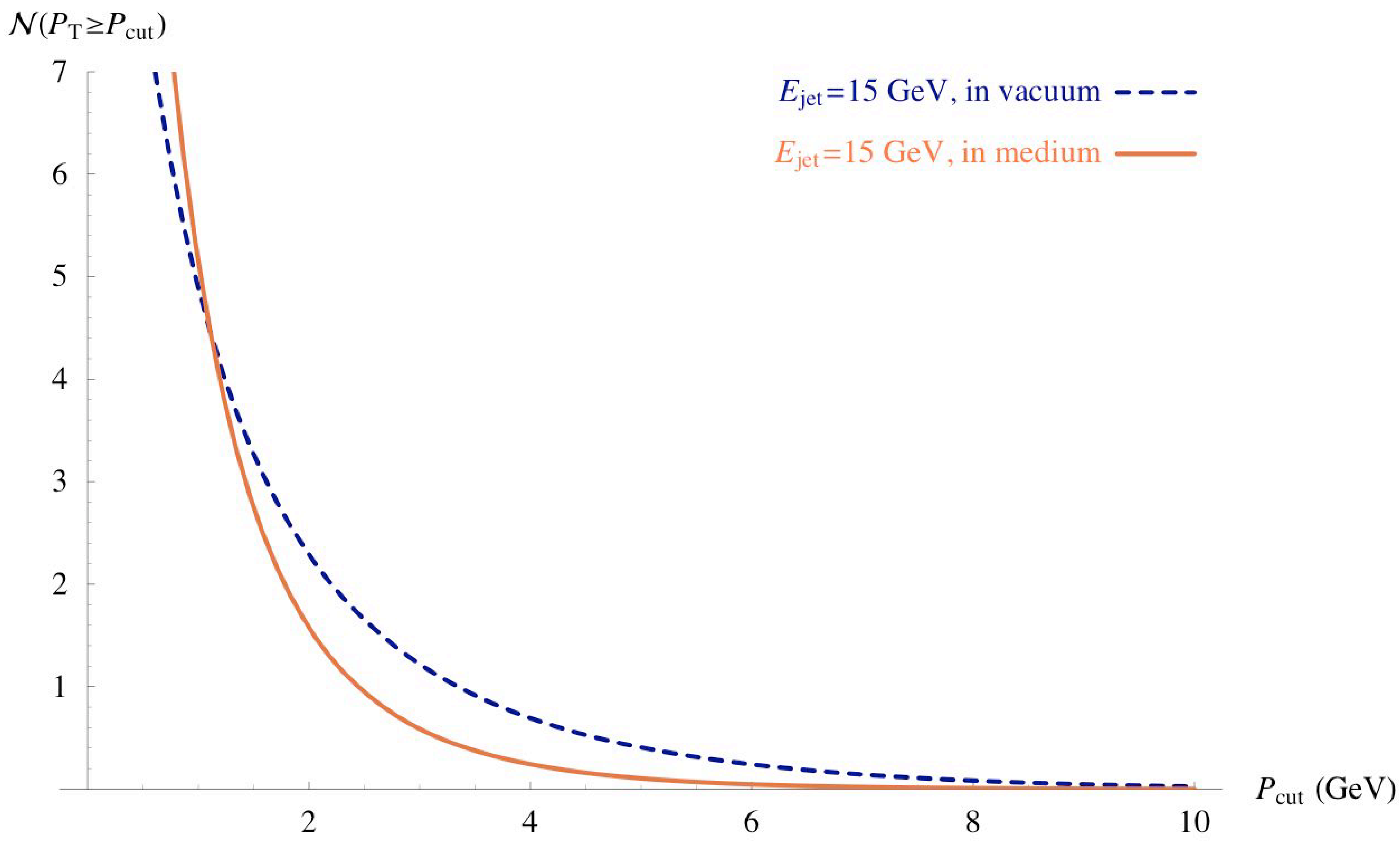
- For a jet in vacuum with the energy $E_{\rm jet}$, the spectrum is known

⇒ one knows (measurement / in vacuum MLLA)

$$\mathcal{N}(P_T \geq P_{\rm cut})_{\rm vacuum}$$

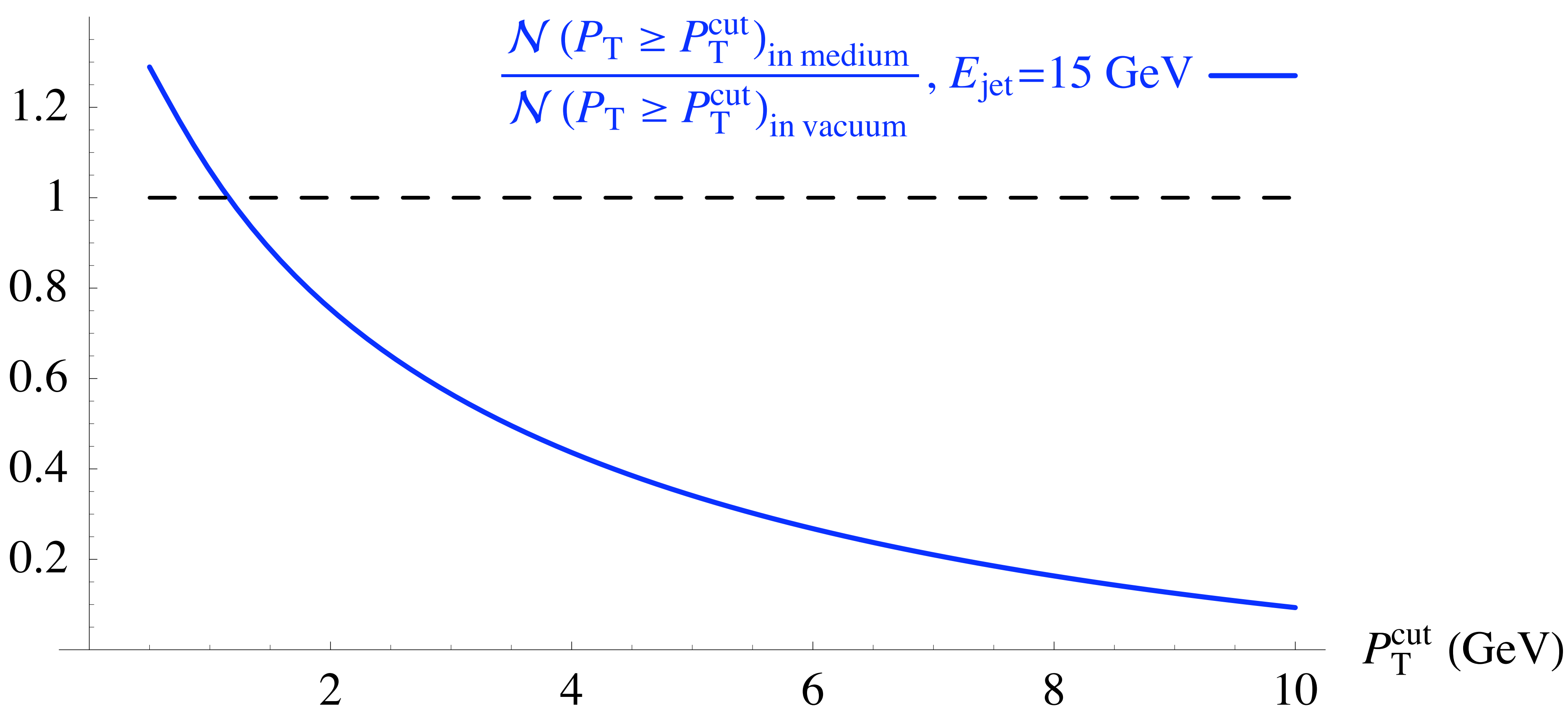
- Compare $\mathcal{N}(P_T \geq P_{\rm cut})_{\rm medium}$ with $\mathcal{N}(P_T \geq P_{\rm cut})_{\rm vacuum}$

Medium-induced modification of the associated multiplicity



There are less high-P_T particles in the presence of a medium.

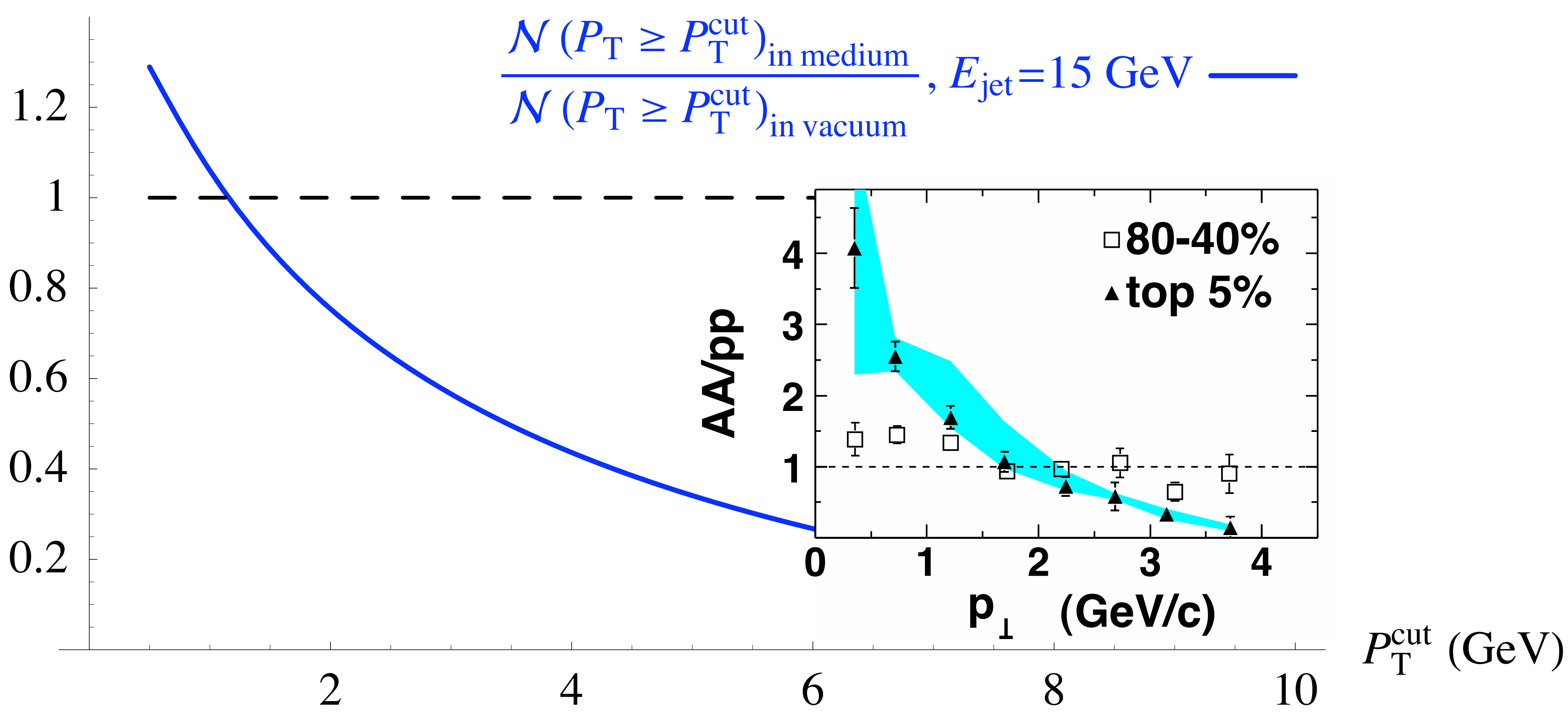
Medium-induced modification of the associated multiplicity



In the presence of a medium, less particles for $P_T \gtrsim 1.5$ GeV

(particle excess for $P_T \lesssim 1.5$ GeV!)

Medium-induced modification of the associated multiplicity



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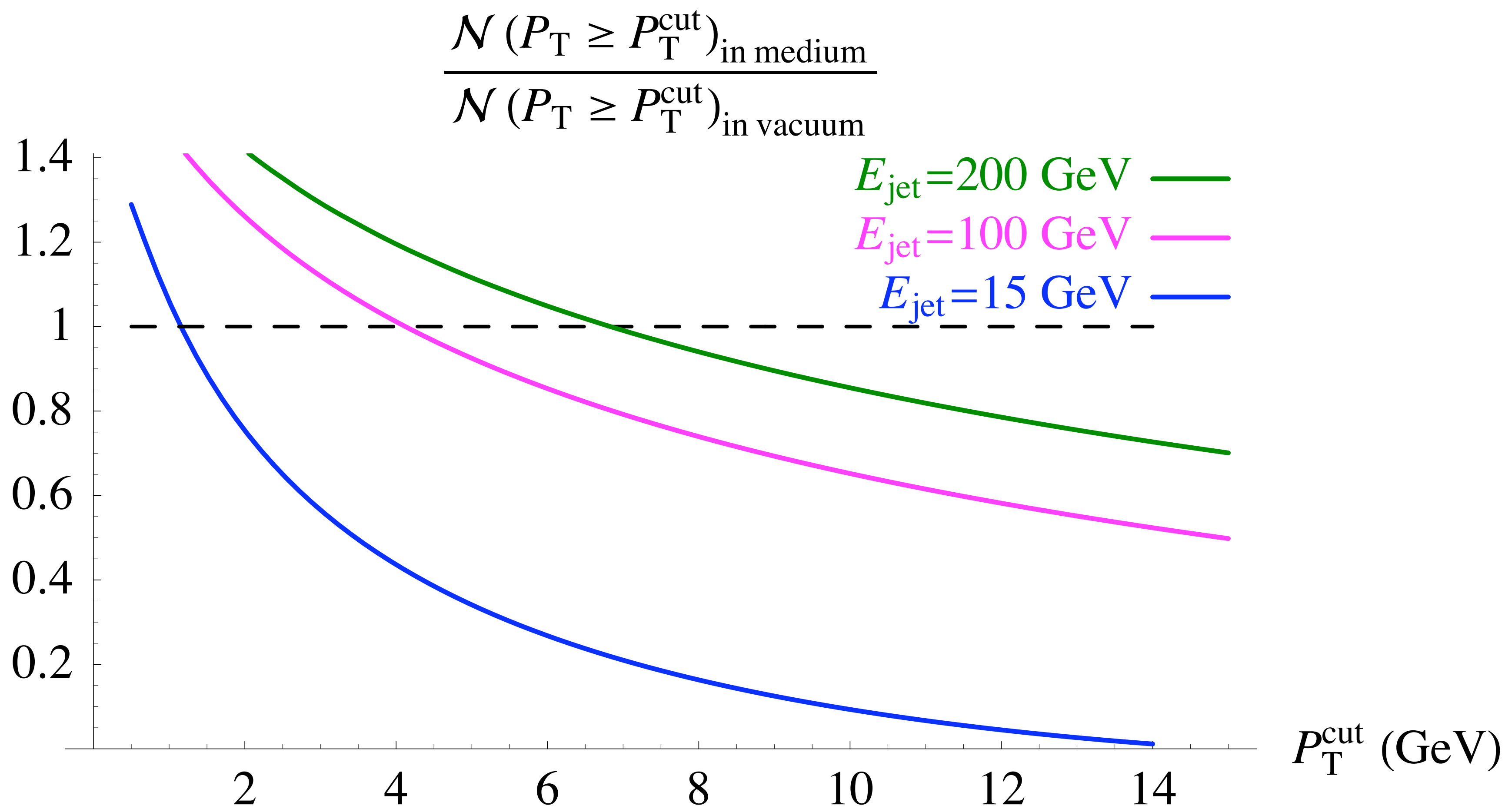
(particle excess for $P_T \lesssim 1.5$ GeV!)

cf. STAR PRL **95** (2005) 152301

Medium-induced modification of the associated multiplicity



Measurement more promising at LHC:
The additional soft jet multiplicity can more easily be seen above the event "background".

Hadron spectra

What if the jet energy is unknown?

The measured hadron spectrum is the convolution of

- A parton spectrum $\propto 1/(p_T)^n$ (with possibly a p_T-dependent power, to account for experimental biases)

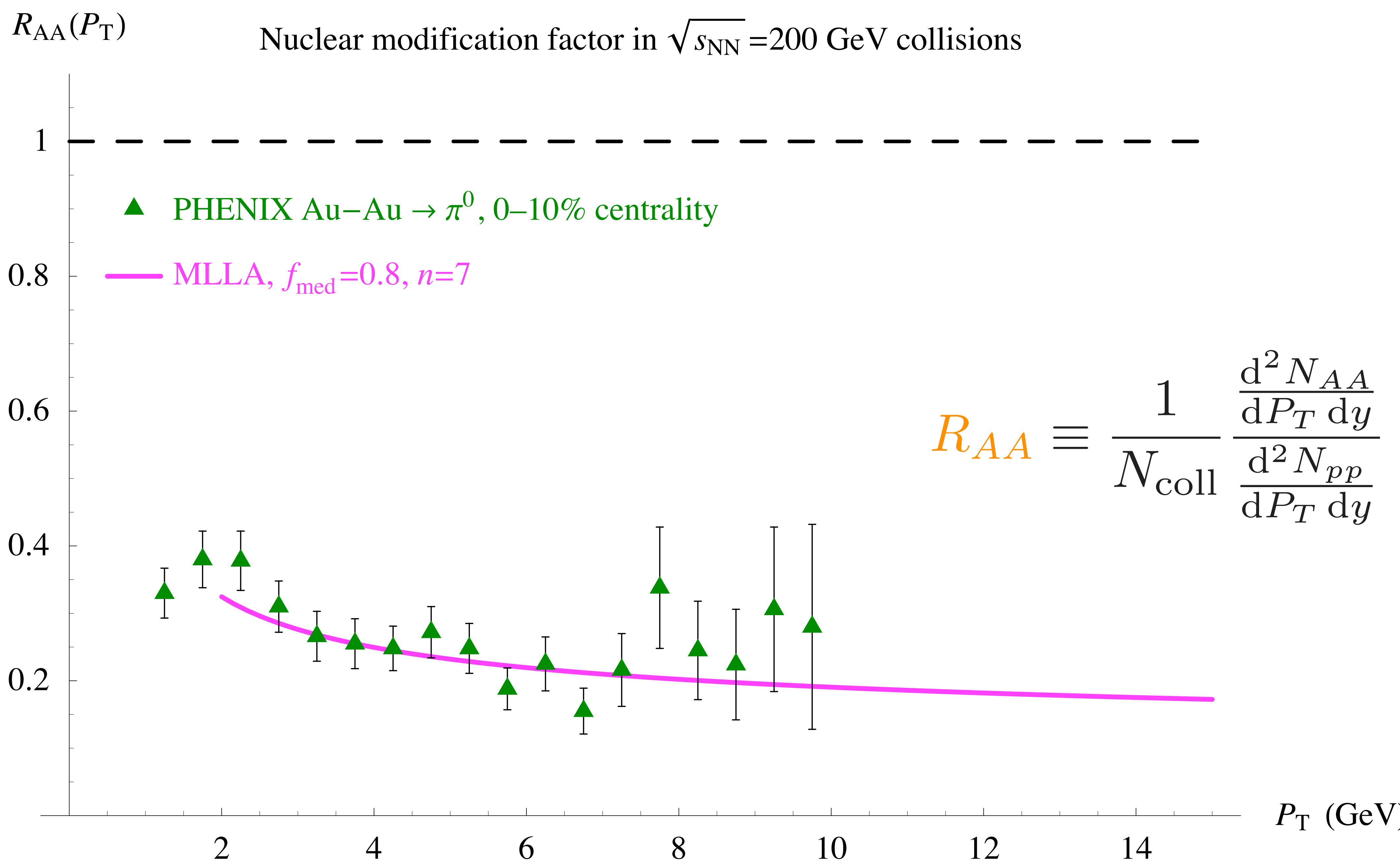
- The "fragmentation function" $\bar{D}^h(x,\tau)$

$$\frac{\mathrm{d}N}{\mathrm{d}P_T} \propto \int \frac{\mathrm{d}x}{x^2} \frac{1}{(p_T)^n} \bar{D}^h(x,\tau = p_T) = \int \frac{\mathrm{d}x}{x^2} \frac{x^n}{(P_T)^n} \bar{D}^h\left(x,\tau = \frac{P_T}{x}\right)$$

which can be computed within MLLA for both a jet in vacuum and a jet propagating through a medium.

☞ gives the nuclear modification factor R_{AA}

Nuclear modification factor



$$R_{AA} \equiv \frac{1}{N_{\rm coll}} \frac{\frac{{\rm d}^2 N_{AA}}{{\rm d}P_T\,{\rm d}y}}{\frac{{\rm d}^2 N_{pp}}{{\rm d}P_T\,{\rm d}y}}$$

MLLA parton shower in a medium

MLLA analytical description of the particle distribution with a jet.

Formalism generalized to the propagation in a medium

N.B. & U.A.Wiedemann, hep-ph/0506218 & 0509364

- Consistent treatment of parton splittings
  - energy-momentum conservation
  - all branchings treated on an equal footing
- Phenomenological consequences
  - Distortion of the hump-backed plateau
  - Large P_T range available at LHC will test the dependence of parton energy loss on virtuality
  - Multiplicity above a trigger cutoff

MLLA parton shower in a medium

Future studies...

- The second derivative of the generating functional $Z_i[Q,\Theta;u(k)]$ gives the two-particle cross-section ☞ two-particle correlations
- Implementation in a Monte-Carlo
  - Analytical results make a useful reference
  - Geometry, $f_{\rm med}(Q)$...
- Jet broadening(?)
- Jet hadrochemistry

  S.Sapeta & U.A.Wiedemann, arXiv:0707.3494 [hep-ph]
- ...