



Hints of incomplete thermalization in RHIC data

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Saclay



RHIC Au–Au results: the fashionable view



RHIC Scientists Serve Up “Perfect” Liquid

New state of matter more remarkable than predicted -- raising many new questions

April 18, 2005

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RHIC Scientists Serve Up “Peri-

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Flüssige Quarkmaterie

Dieser Zustand verhält sich (...) nicht wie ein ideales Gas, sondern vielmehr wie eine fast ideale Flüssigkeit

Physik Journal, June 2005

RHIC Au–Au results: the fashionable view

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RHIC Sci-

Liquid universe hints at strings
New study in Action: June 2005

April 18, 2005

Researchers at RHIC have seen convincing new evidence for a quark-gluon plasma. But it looks more like a perfect liquid than a gas, which could have implications for string theory -- than predicted -- raising many questions.

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*... ein ideales
... ideale Flüssigkeit
... Physik Journal, June 2005*

RHIC Au–Au results: the fashionable view

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RHIC Science

New study
question

April 18, 2005

Liquid universe
physics in Action: June 2005
**Researchers at RHIC find
a quark-gluon plasma
thicker than a gas, which could**

Physics Today, May 2005

Flüssige Quarkmaterie
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A String-Theory Calculation of Viscosity
Applications of Viscosity

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RHIC Au–Au results: the fashionable view



Ideal fluid dynamics reproduce both p_t spectra and elliptic flow $v_2(p_t)$ of soft ($p_t \lesssim 2$ GeV/c) identified particles for minimum bias collisions, near central rapidity.

This agreement necessitates a soft equation of state, and very short thermalization times: $\tau_{\text{thermalization}} < 0.6$ fm/c.

⇒ **strongly interacting Quark-Gluon Plasma**



Ideal fluid dynamics in heavy-ion collisions

- A few reminders on fluid dynamics
- Ideal fluid dynamics in nucleus–nucleus collisions: theory
 - Overall scenario
 - General predictions of ideal fluid dynamics
 - Anisotropic flow
- Out-of-equilibrium scenario
 - Generic predictions
 - Reconciling data and theory (?)

R.S. Bhalerao, J.-P. Blaizot, N.B., J.-Y. Ollitrault, PLB 627 (2005) 49

Fluid dynamics: various types of flow

- **Thermodynamic equilibrium?** 🖱️ **Knudsen number** $Kn = \frac{\text{mean free path } \lambda}{\text{system size } L}$

 - $Kn \gg 1$: Free-streaming limit
 - $Kn \ll 1$: **Liquid** (hydro) limit

- **Viscous or Ideal?** 🖱️ **Reynolds number** $Re = \frac{\epsilon L v_{\text{fluid}}}{\eta}$

viscosity $\eta \sim \epsilon \lambda c_s$

 - $Re \gg 1$: Ideal (non-viscous) **flow**
 - $Re \leq 1$: Viscous **flow**

- **Compressible or Incompressible?** 🖱️ **Mach number** $Ma = \frac{v_{\text{fluid}}}{c_s}$

speed of sound c_s

 - $Ma \ll 1$: Incompressible **flow**
 - $Ma > 1$: Compressible (supersonic) **flow**

Fluid dynamics: various types of flow

Three numbers:

$$Kn = \frac{\lambda}{L}, \quad Re = \frac{\varepsilon L v_{\text{fluid}}}{\eta}, \quad Ma = \frac{v_{\text{fluid}}}{c_s}$$

⇒ an important relation:

$$Kn \times Re = \frac{\varepsilon \lambda v_{\text{fluid}}}{\eta} \sim \frac{v_{\text{fluid}}}{c_s} = Ma$$

In a heavy-ion collision, the **created matter** expands into the vacuum

⇒ **compressible flow**

“**Liquids are Ideal**” ⇔ **viscosity** ≡ departure from equilibrium

Ideal fluid picture of a heavy-ion collision

- ① Creation of a dense “gas” of particles
- ①. At some time τ_0 , the mean free path λ is much smaller than *all dimensions* in the system
 \Rightarrow thermalization (T_0), ideal fluid dynamics applies
- ②. The fluid expands: density decreases, λ increases (system size also)
- ③. At some time, the mean free path is of the same order as the system size: ideal fluid dynamics is no longer valid
“(kinetic) freeze-out”

Freeze-out usually parameterized in terms of a temperature $T_{f.o.}$

If λ varies smoothly with temperature, consistency requires $T_{f.o.} \ll T_0$

 analytical predictions

Ideal fluid dynamics: general predictions

Consistent **ideal fluid dynamics** picture requires $T_{\text{f.o.}} \ll T_0$

\Leftrightarrow

Ideal-fluid limit = $T_{\text{f.o.}} \rightarrow 0$ limit

☞ one can compute in a model-independent way

- the **spectrum** $E \frac{dN}{d^3\mathbf{p}} = C \int_{\Sigma} \exp\left(-\frac{p^\mu u_\mu(x)}{T_{\text{f.o.}}}\right) p^\mu d\sigma_\mu$
- and its **azimuthal anisotropies** (“**flow**”)

using saddle-point approximations **around the minimum of**

N.B. & J.-Y. Ollitrault, nucl-th/0506045

Ideal fluid dynamics: general predictions

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particle momentum \rightarrow

• and its **azimuthal anisotropies** (“flow”)

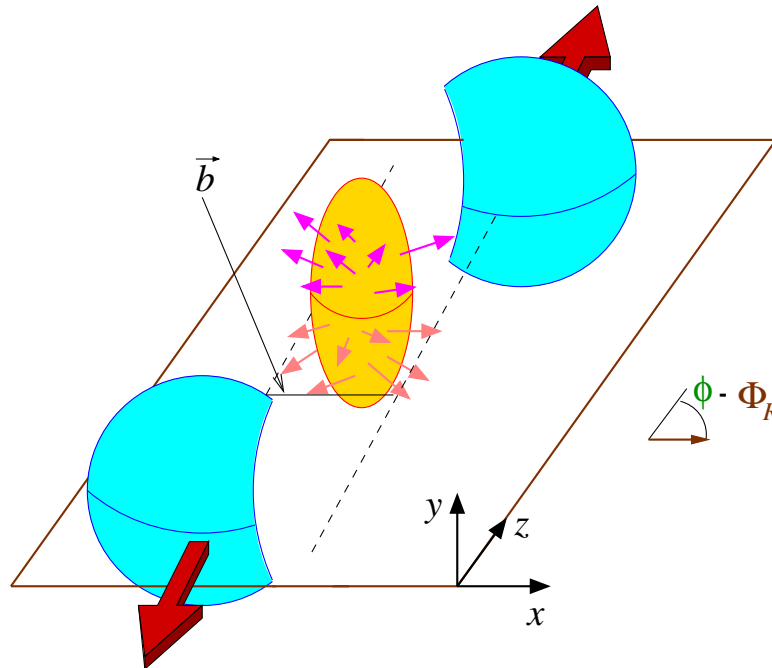
using saddle-point approximations around the minimum of

fluid velocity \rightarrow

N.B. & J.-Y. Ollitrault, nucl-th/0506045

Heavy-ion observable: Anisotropic flow

Non-central collision:



Initial anisotropy of the source
(in the transverse plane)

⇒ anisotropic pressure gradients,
larger along the impact parameter \vec{b}

⇒ anisotropic emission of particles:

anisotropic (collective) flow

$$E \frac{dN}{d^3\mathbf{p}} \propto \frac{dN}{p_t dp_t dy} \left[1 + \underset{\text{“directed”}}{2v_1} \cos(\phi - \Phi_R) + \underset{\text{“elliptic”}}{2v_2} \cos 2(\phi - \Phi_R) + \dots \right]$$

“Flow”: misleading terminology; does NOT imply fluid dynamics!

Non-central collisions: parameters

Initial conditions in **non-central** collisions, will be characterized by

- a parameter measuring the **shape** of the **overlap region**:

- spatial eccentricity $\epsilon = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$

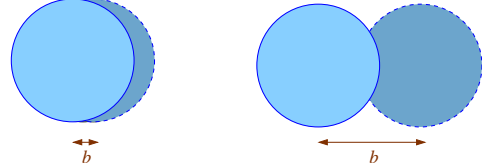
- two numbers measuring the **size** of the **overlap region**:

- “reduced” radius $\frac{1}{\bar{R}} = \sqrt{\frac{1}{\langle x^2 \rangle} + \frac{1}{\langle y^2 \rangle}}$

(**anisotropic flow** caused by pressure *gradients*)

- transverse area of the **collision zone** $S = 2\pi \sqrt{\langle x^2 \rangle \langle y^2 \rangle}$

Anisotropic flow: predictions of hydro

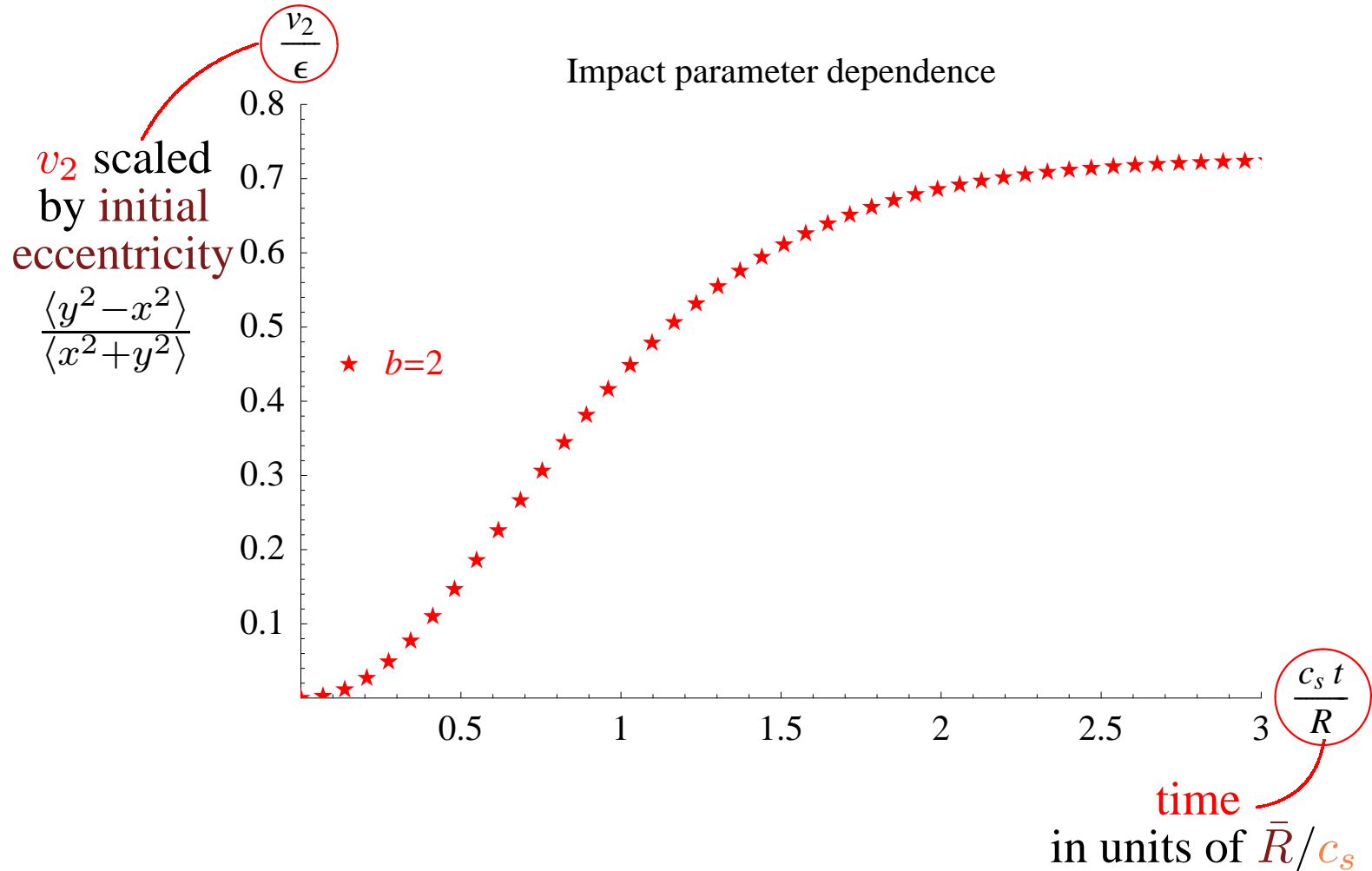
- Characteristic build-up time of v_2 is \bar{R}/c_s
typical system size \nearrow \bar{R} \nwarrow speed of sound c_s
- v_2/ϵ constant across different centralities
system eccentricity \nwarrow ϵ 
- v_2 roughly independent of the system size (Au–Au vs. Cu–Cu)
- v_2 increases with increasing speed of sound c_s
- Mass-ordering of the $v_2(p_T)$ of different particles
(the heavier the particle, the smaller its v_2 at a given momentum)
- Relationship between different harmonics: $\frac{v_4}{(v_2)^2} = \frac{1}{2}$

Dependence of v_2 on centrality

The natural time scale for v_2 is \bar{R}/c_s :

massless particles

$$c_s^2 = \frac{1}{3}$$

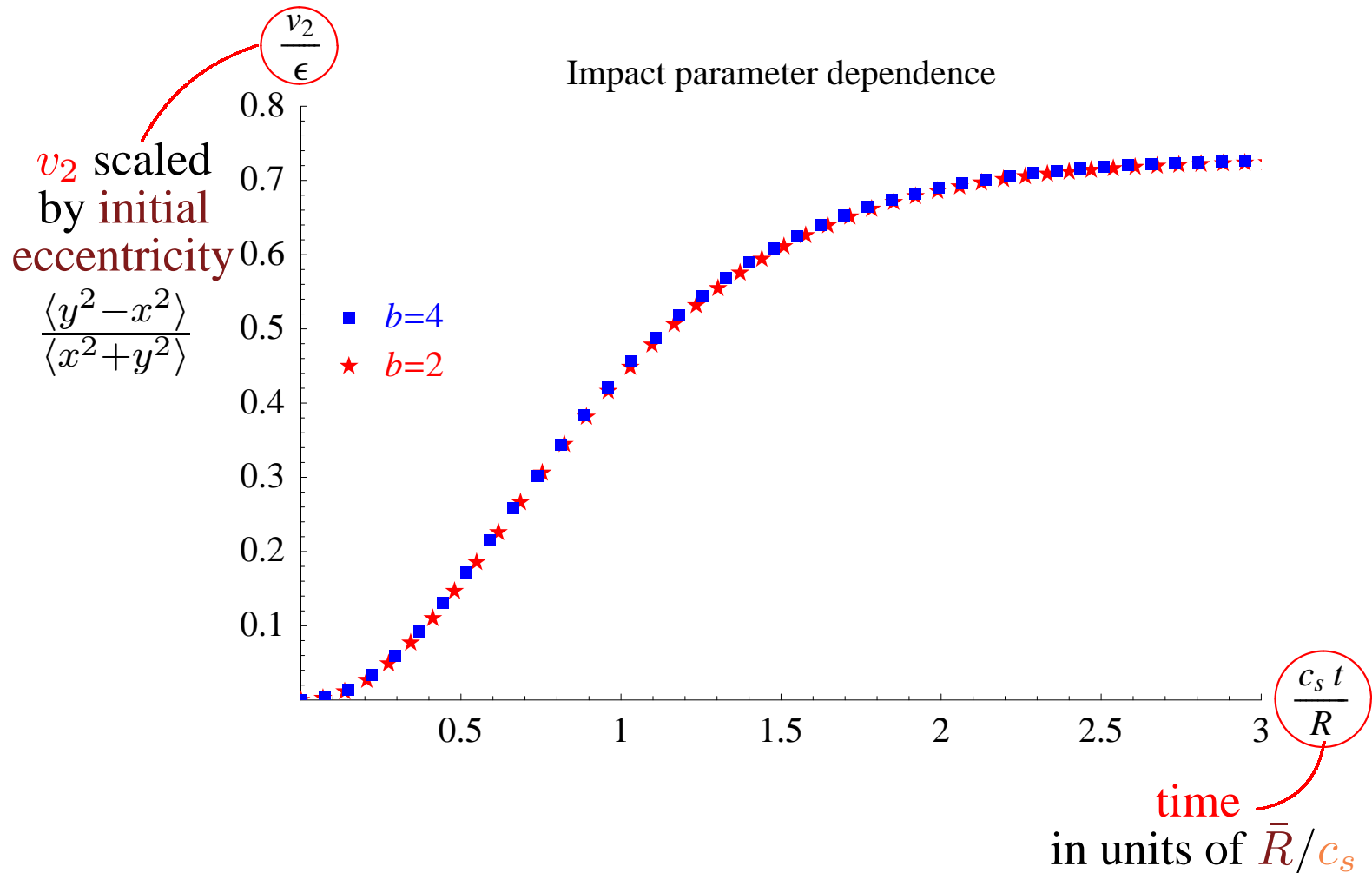


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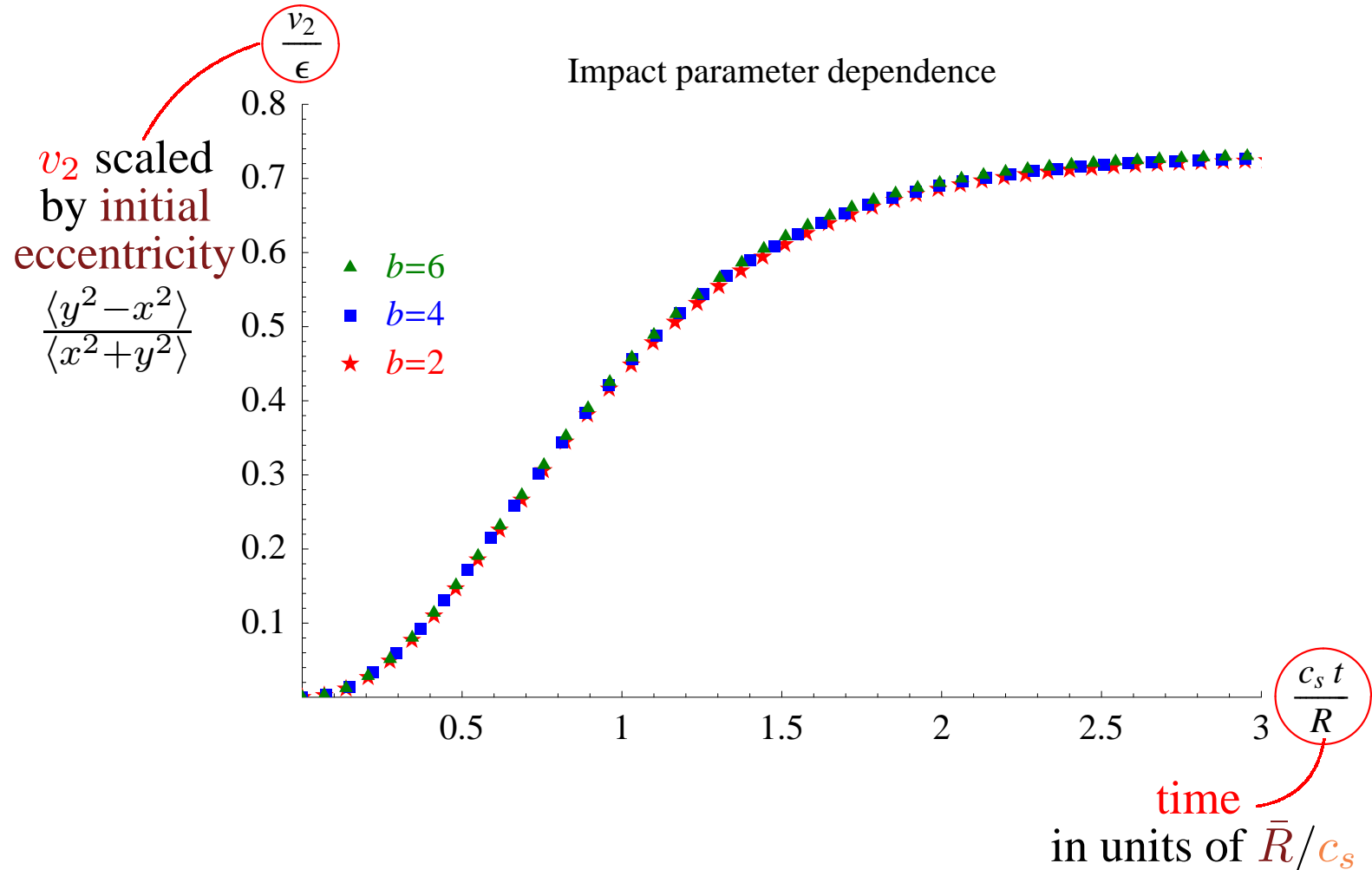


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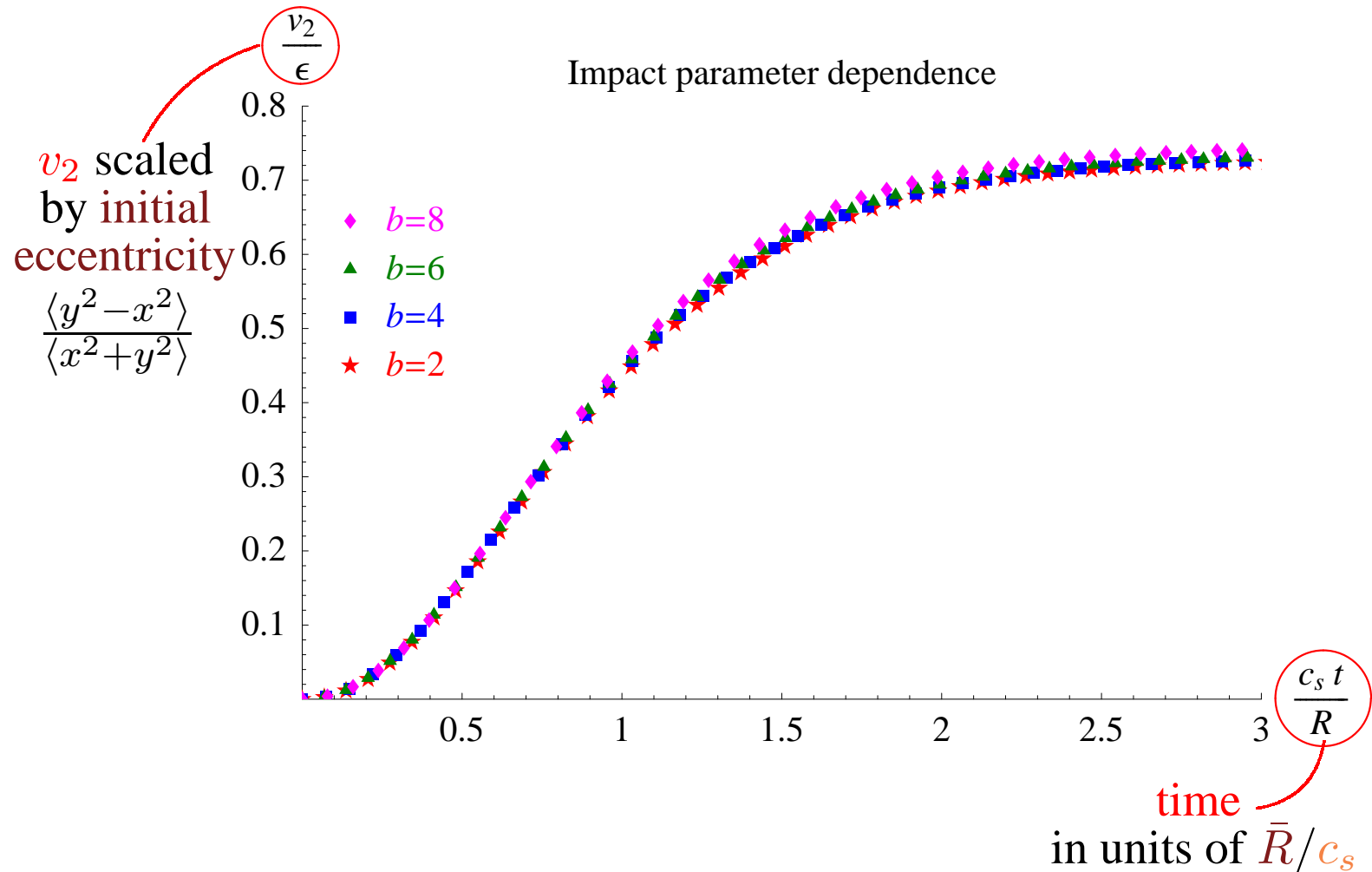


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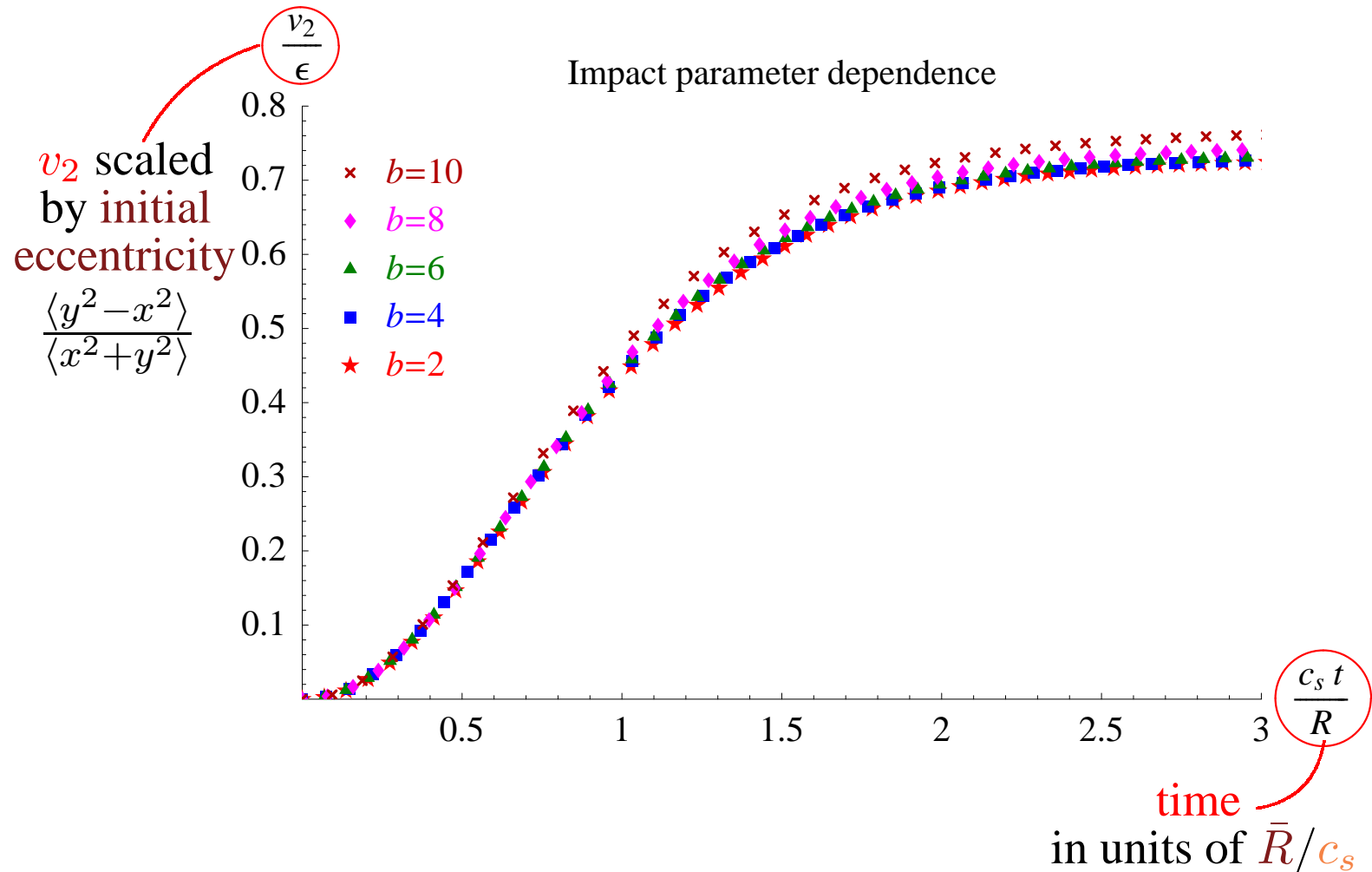


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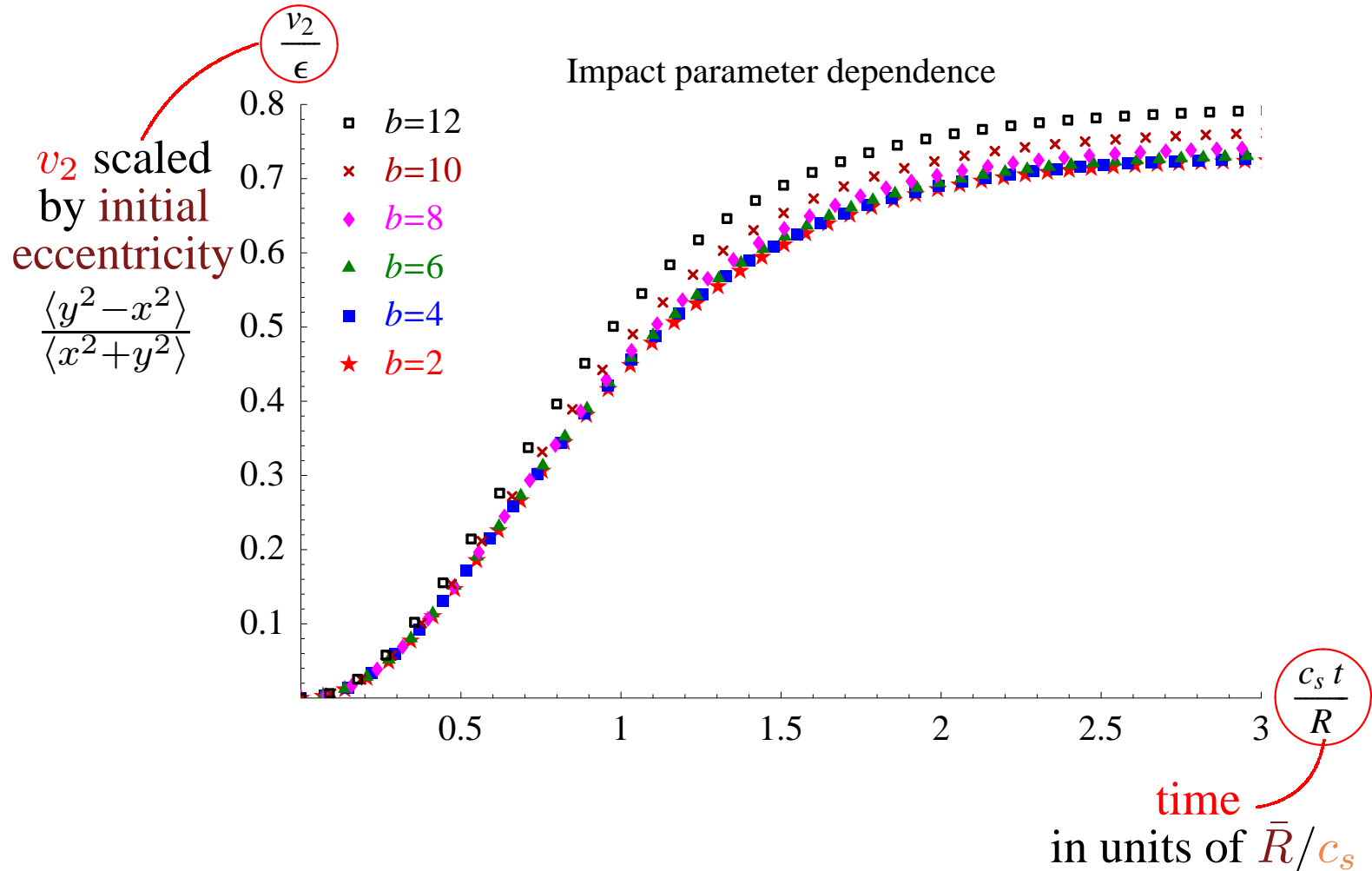


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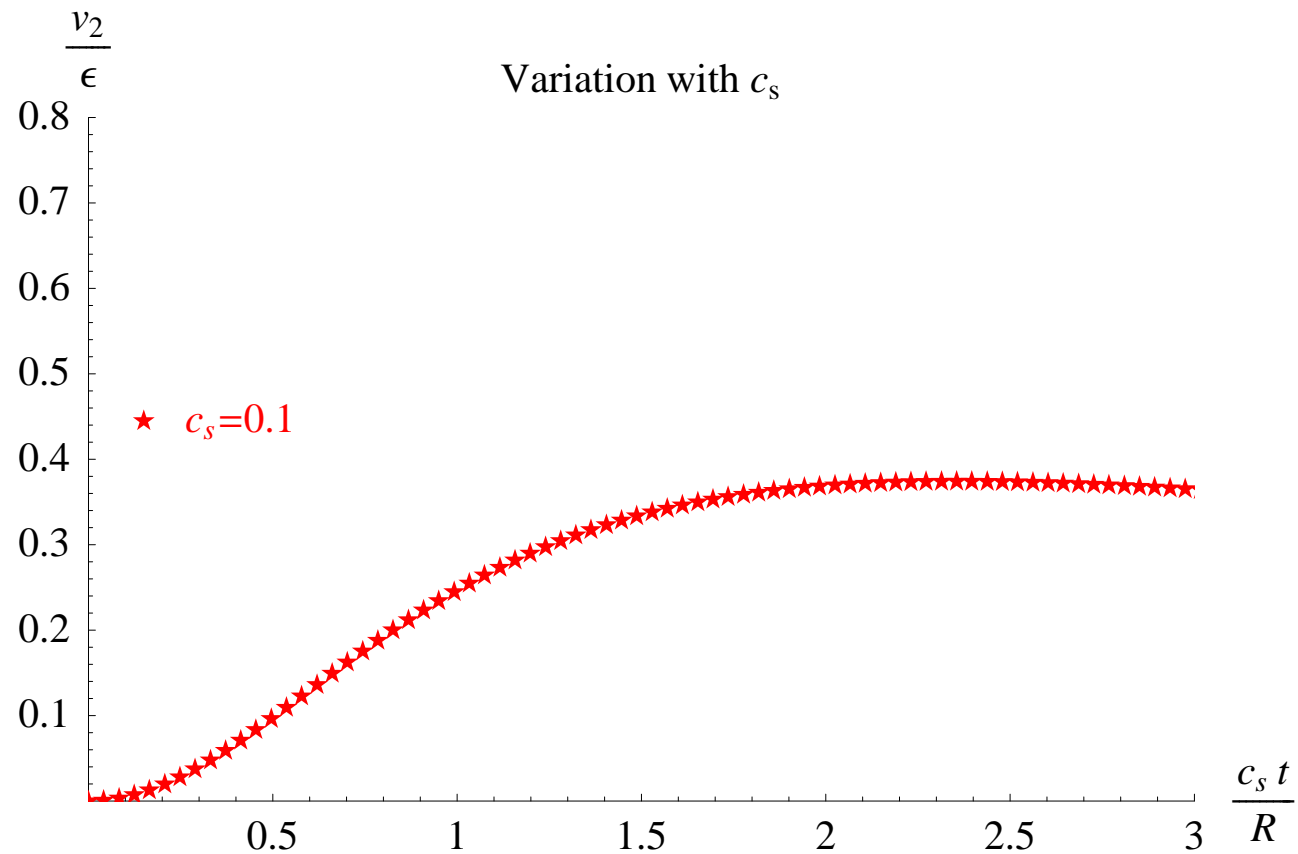
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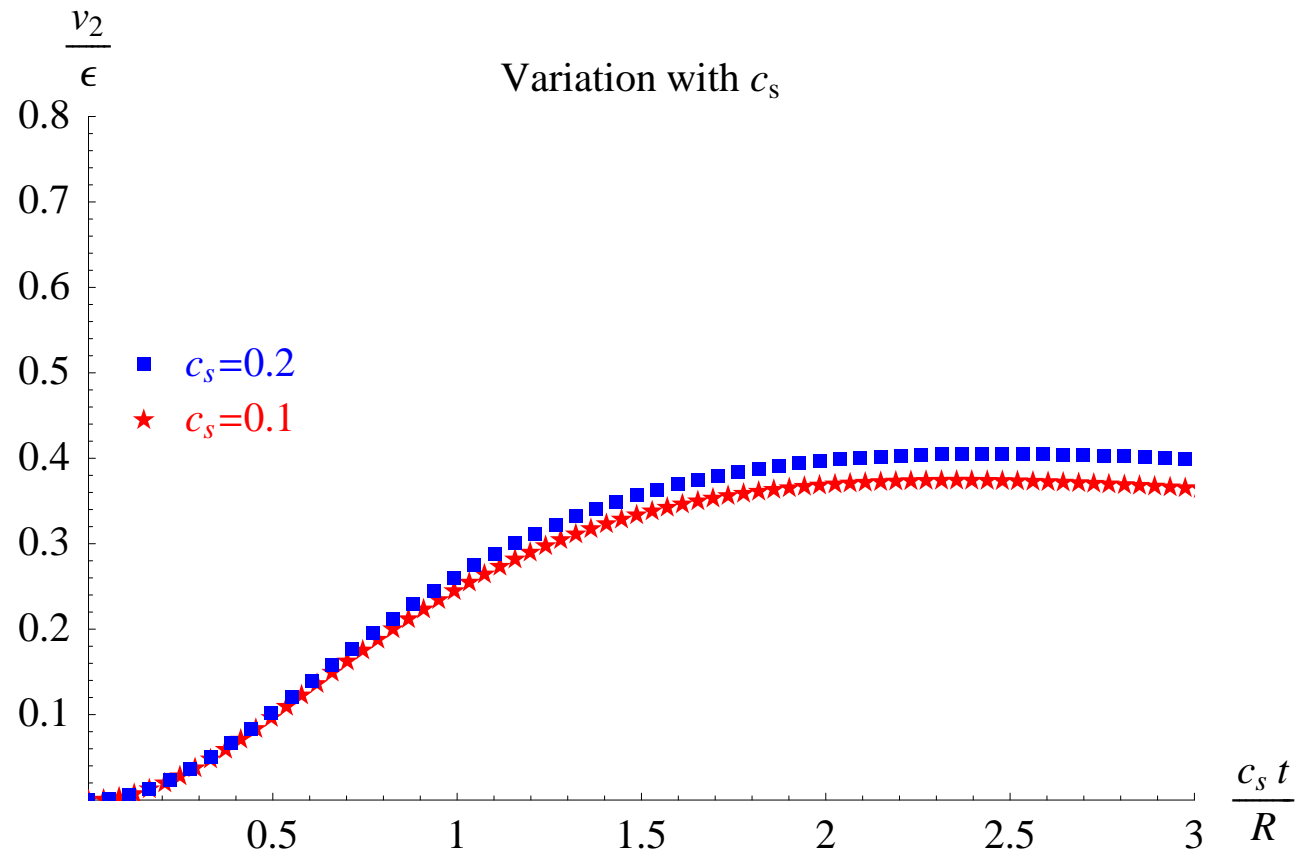
$$c_s^2 = \frac{1}{3}$$



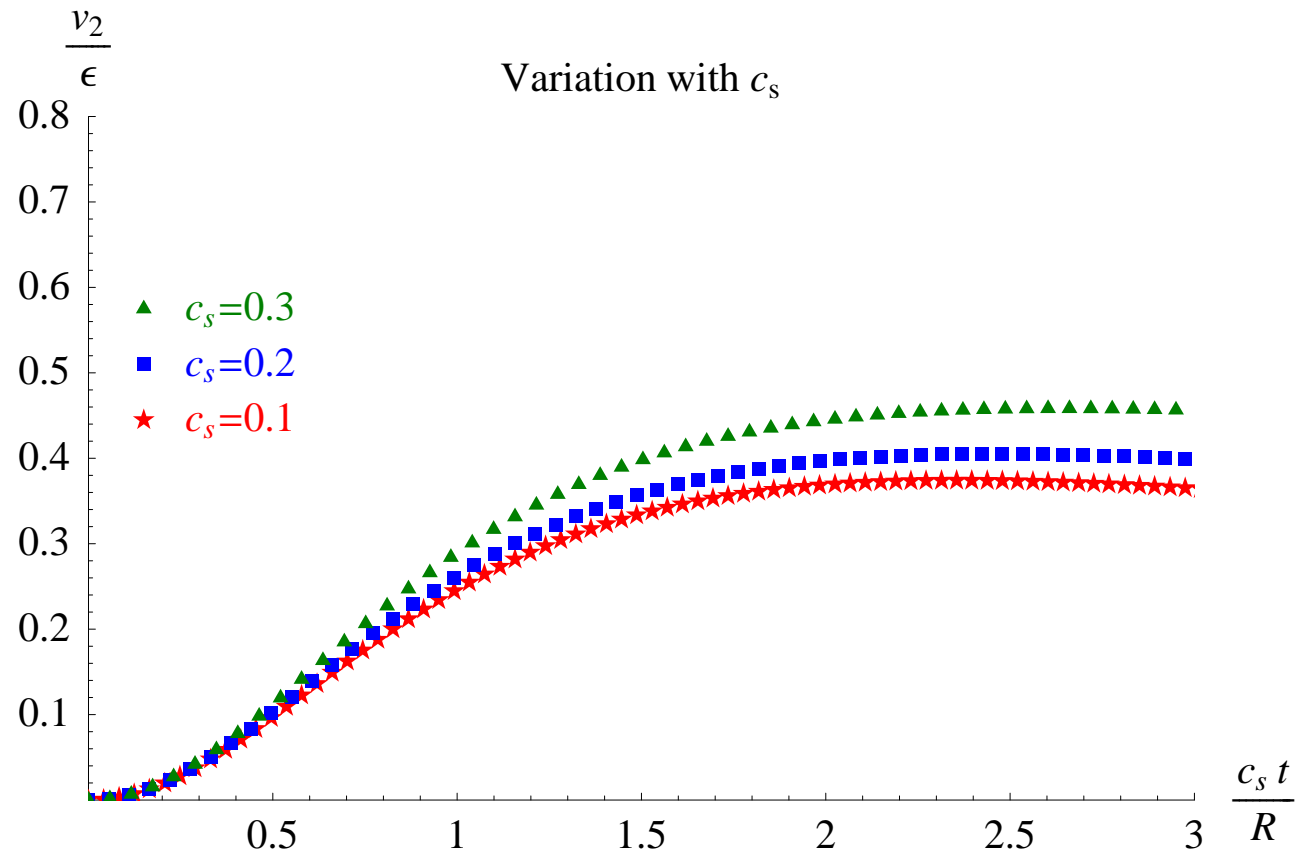
Dependence of v_2 on the speed of sound



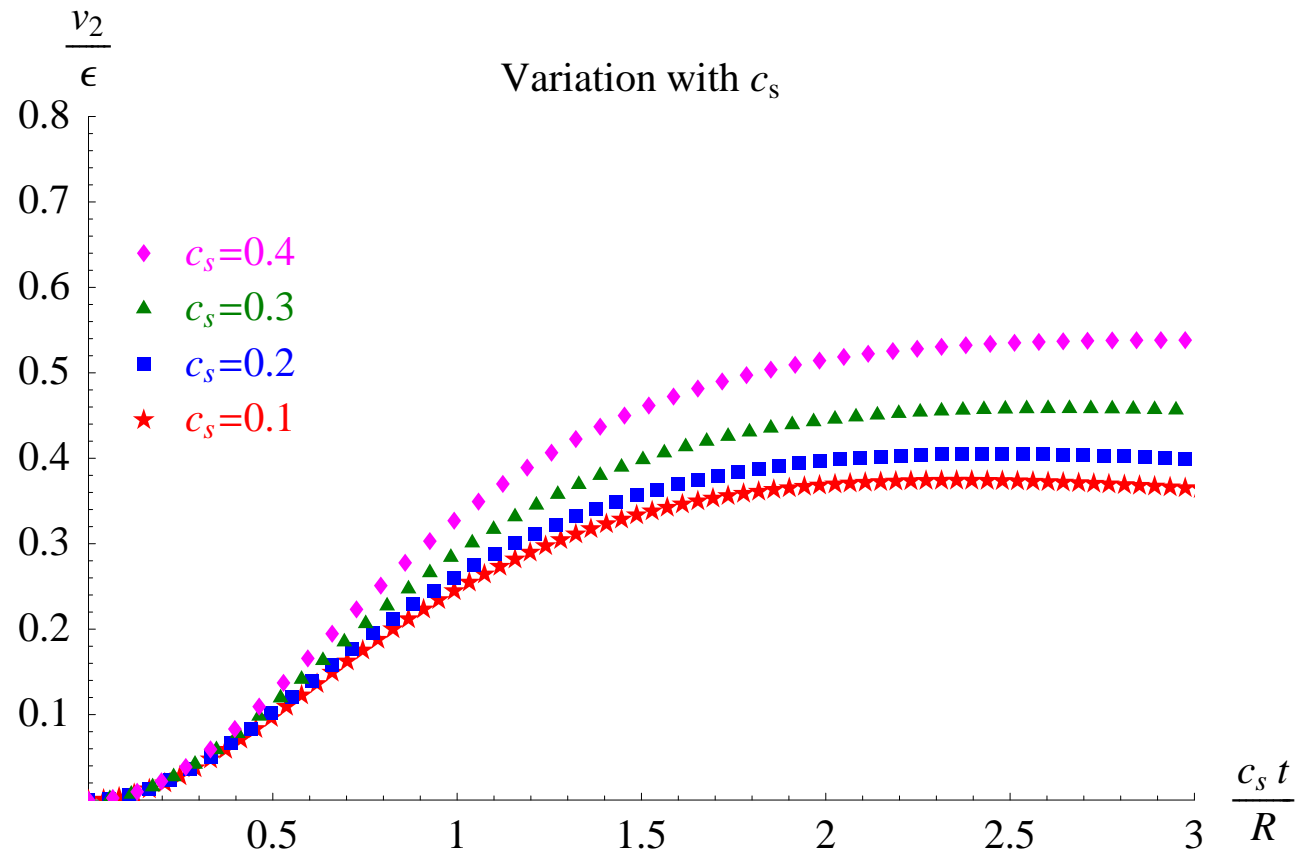
Dependence of v_2 on the speed of sound



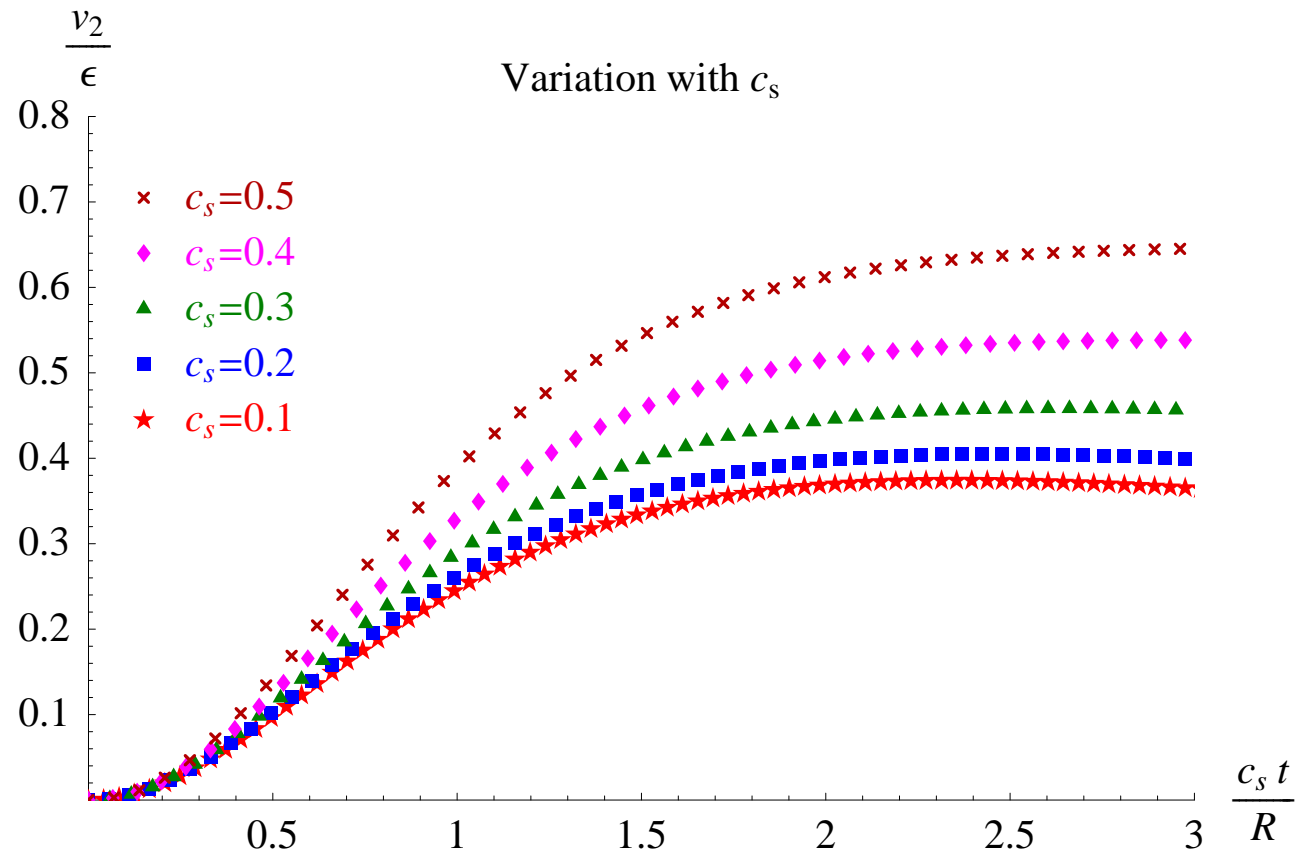
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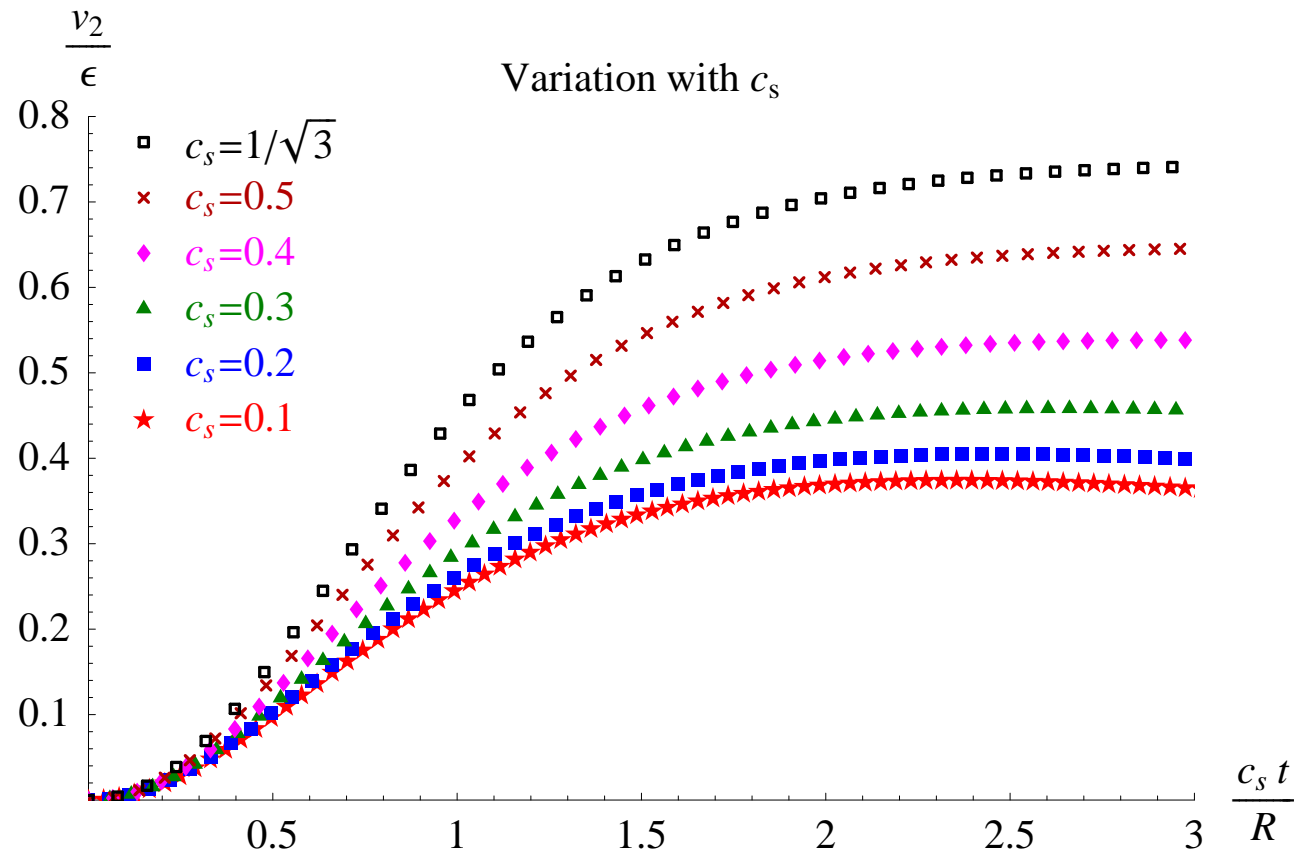
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Dependence of v_2 on the speed of sound



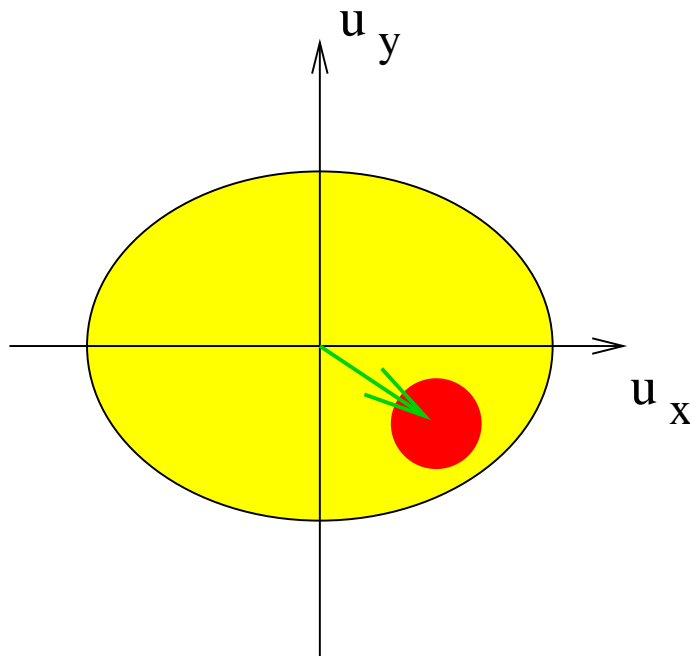
Dependence of v_2 on the speed of sound



 one can increase v_2 by increasing c_s

Mass-ordering of the $v_2(p_T)$

“Slow particles” ($p_t/m < u_{\max}$) move together with the fluid



There is a point where the fluid velocity equals the particle velocity

☞ Integrand in the momentum spectrum is Gaussian, with width $(p^\mu u_\mu)_{\min}^{1/2} = \sqrt{m}$
 ↪ saddle-point approximation!

• Similar spectra for different hadrons:

$$E \frac{dN}{d^3\mathbf{p}} = c^h(m) f\left(\frac{p_t}{m}, y, \phi\right)$$

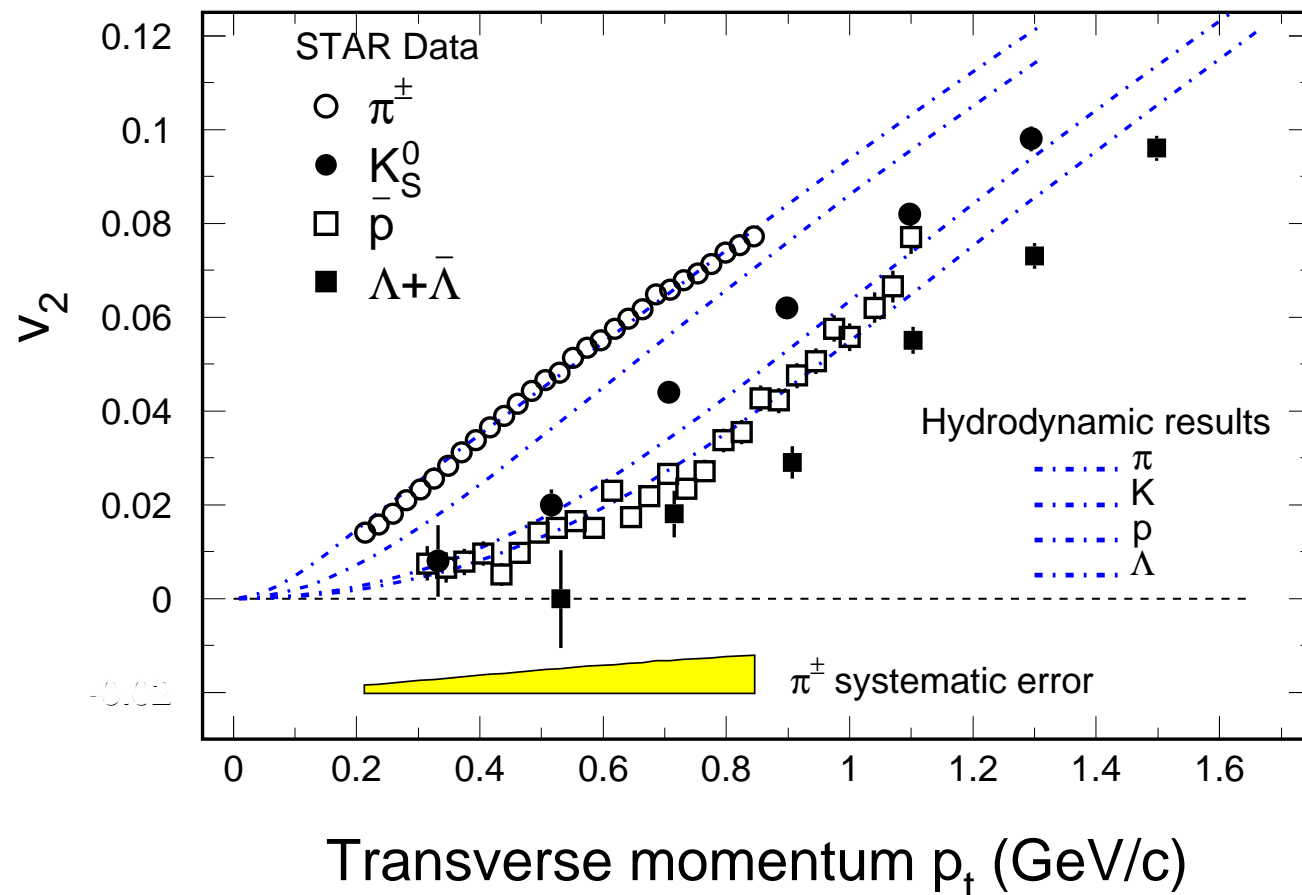
• $v_n\left(\frac{p_t}{m}, y\right)$ universal!

⇒ mass-ordering of $v_2(p_t, y)$

RHIC anisotropic flow data and ideal-fluid dynamics

$v_2(p_t)$ at midrapidity, minimum bias Au–Au collisions:

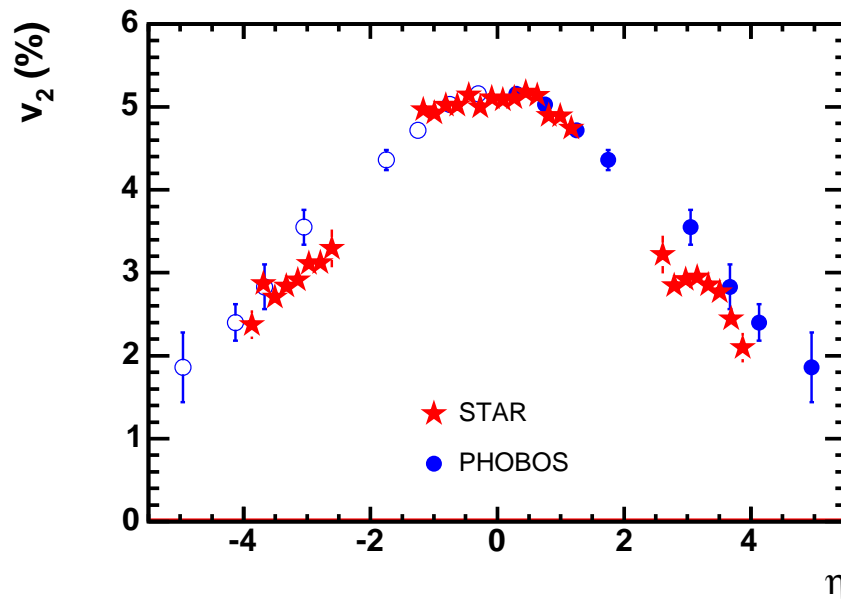
STAR Collaboration, PRC 72 (2005) 014904



RHIC anisotropic flow data and ideal-fluid dynamics

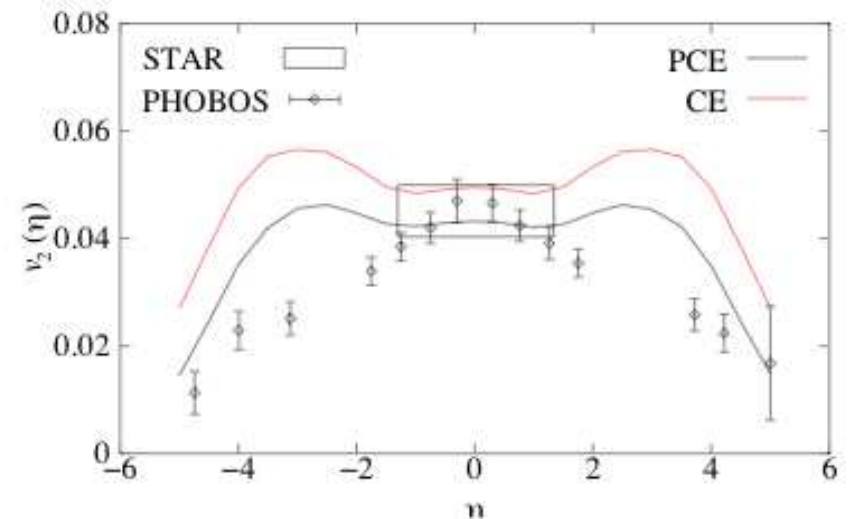
(Pseudo)rapidity dependence of v_2

STAR Collaboration,
PRC 72 (2005) 014904



v_2 (hydro) flatter than data

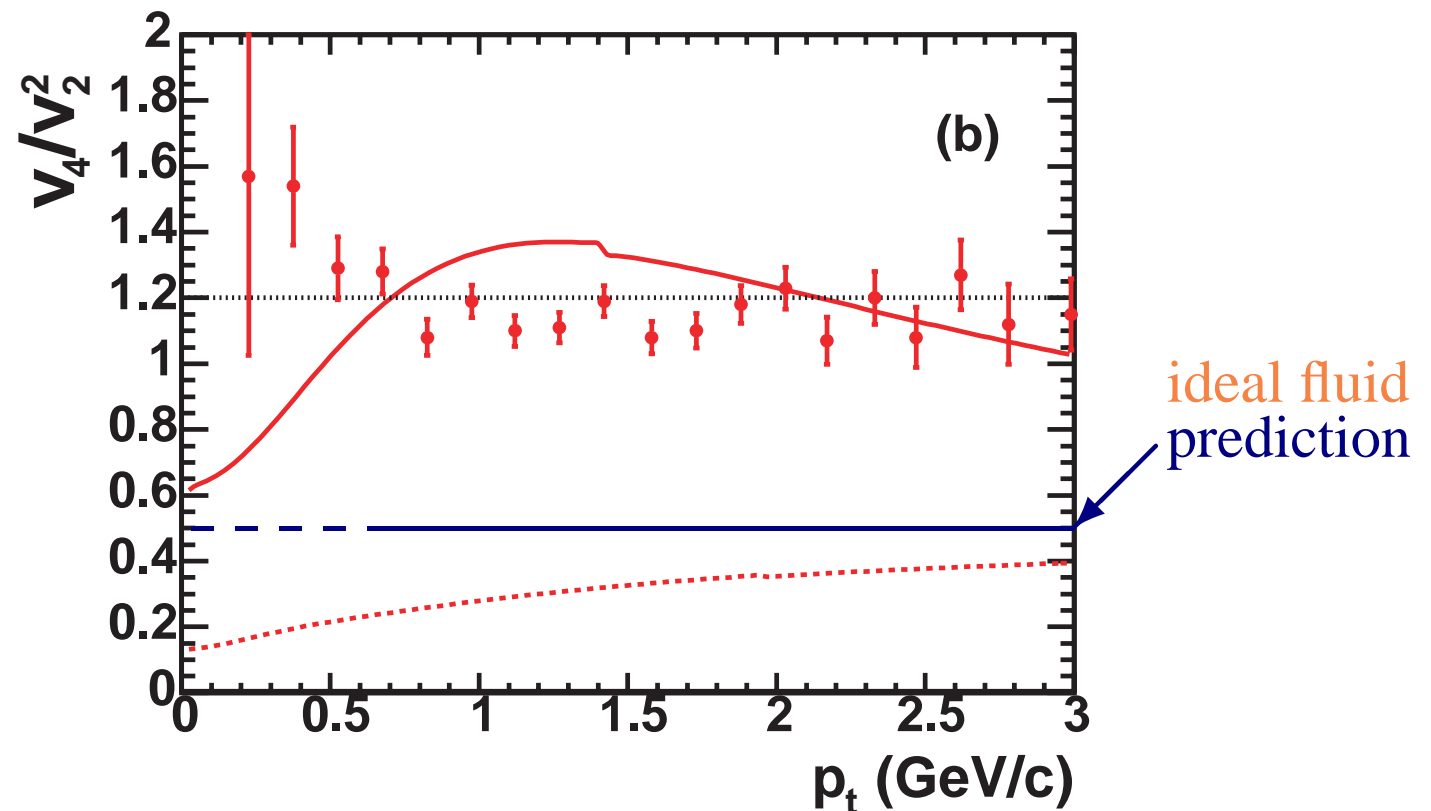
Hirano & Tsuda,
PRC 66 (2002) 054905



RHIC anisotropic flow data and ideal-fluid dynamics

Transverse momentum dependence of $\frac{v_4}{(v_2)^2}$

STAR Collaboration, Phys. Rev. C 72 (2005) 014904





**What if collisions cannot ensure
equilibrium?
Out-of-equilibrium scenario**

Anisotropic flow: out-of-equilibrium scenario

An exact computation of the dependence of v_2 , v_4 on the number of collisions per particle (Kn^{-1}) requires some cascade model...

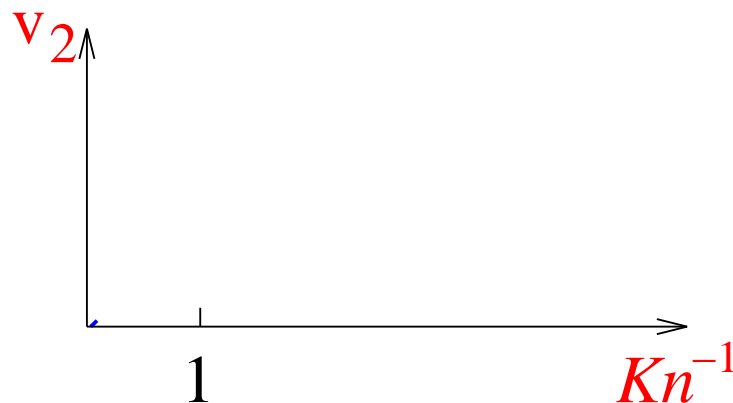
...but we can guess the general tendency! $\rightarrow \frac{\bar{R}}{\lambda}$

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- in the absence of rescatterings ($Kn^{-1} = 0$), no **flow** develops

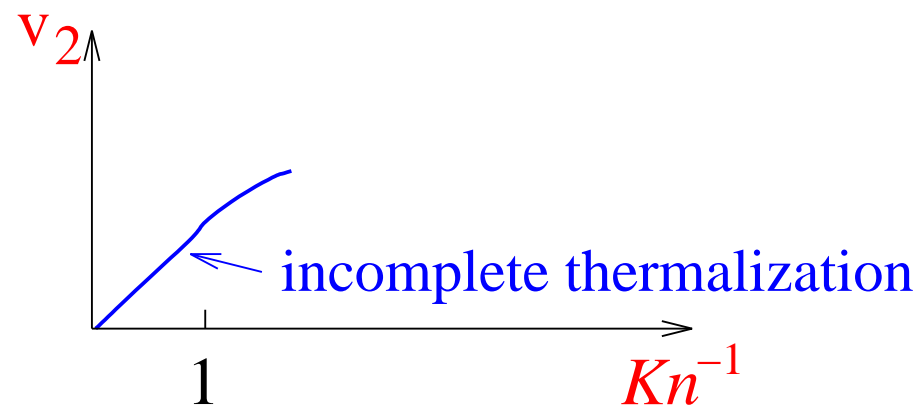


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- the more collisions, the larger the **anisotropic flow**

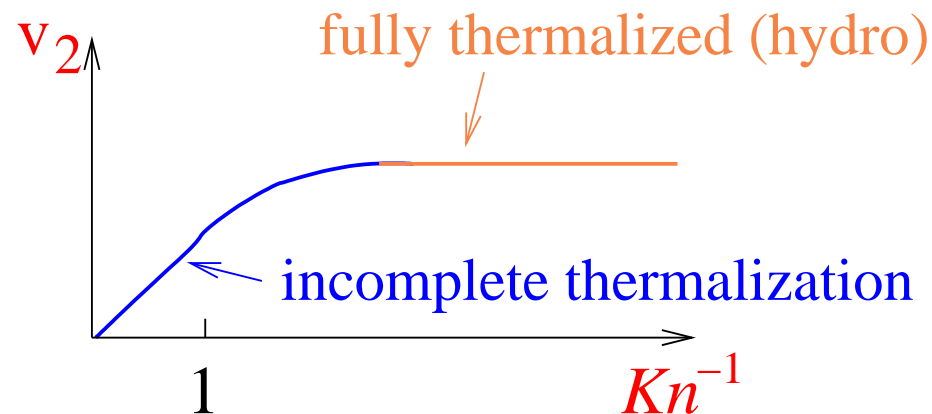


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- the more collisions, the larger the **anisotropic flow**
- for a given number of collisions, the **system** thermalizes: further collisions no longer increase v_2

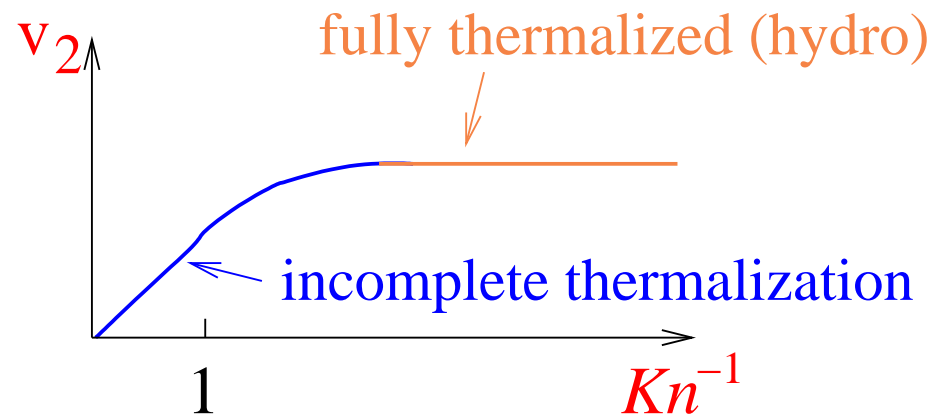


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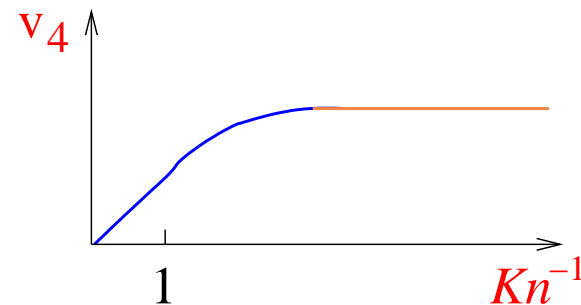
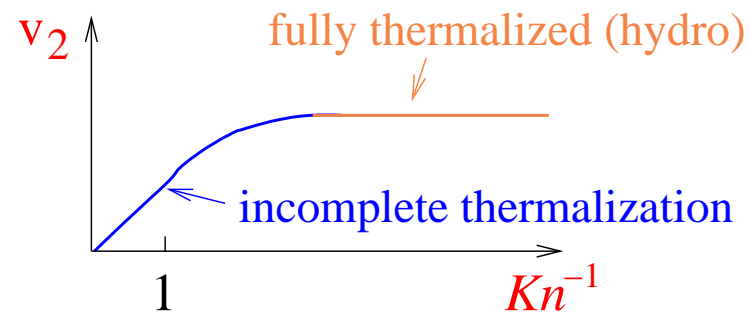
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- in the absence of rescatterings ($Kn^{-1} = 0$), no **flow** develops
- the more collisions, the larger the **anisotropic flow**
- for a **given number of collisions**, the **system** thermalizes: further collisions no longer increase v_2 **should be quantified!**



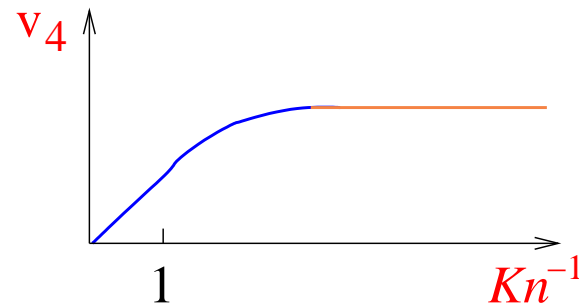
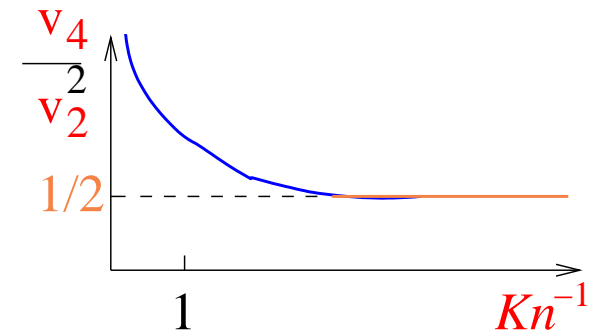
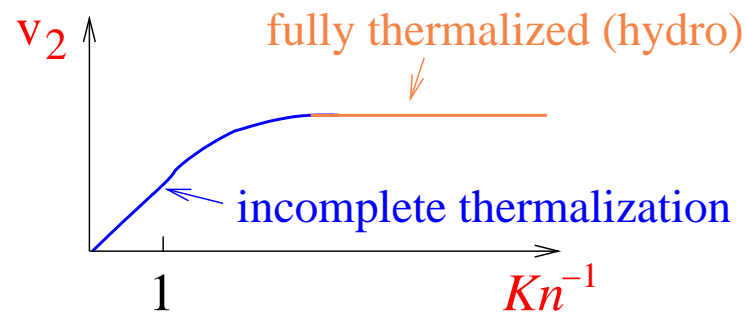
Anisotropic flow: out-of-equilibrium scenario

v_n proportional to the number of collisions Kn^{-1}



Anisotropic flow: out-of-equilibrium scenario

v_n proportional to the number of collisions $Kn^{-1} \Rightarrow \frac{v_4}{(v_2)^2} \propto \frac{1}{Kn^{-1}}$



👉 in the out-of-equilibrium scenario $\frac{v_4}{(v_2)^2} > \frac{1}{2}$

STAR (PRC 72 (2005) 014904) & PHENIX (QM'05) find $\frac{v_4}{(v_2)^2} \approx 1-1.5$

Out-of-equilibrium scenario: a control parameter

The natural time (resp. length) scale for v_2 is \bar{R}/c_s (resp. \bar{R})
⇒ mean number of collisions per particle to build up v_2 :

$$Kn^{-1} \simeq \frac{\bar{R}}{\lambda}$$

In the out-of-equilibrium scenario, v_2 depends on Kn^{-1} , hence on

- the system size \bar{R}

☞ breakdown of the scale-invariance of hydrodynamics

R.S. Bhalerao, J.-P. Blaizot, N.B., J.-Y. Ollitrault, PLB 627 (2005) 49

Out-of-equilibrium scenario: a control parameter

The natural time (resp. length) scale for v_2 is \bar{R}/c_s (resp. \bar{R})
 \Rightarrow **mean number of collisions** per particle to build up v_2 :

$$Kn^{-1} \simeq \frac{\bar{R}}{\lambda} = \bar{R} \sigma n \left(\frac{\bar{R}}{c_s} \right)$$

σ interaction cross section, $n(\tau)$ particle density

In the out-of-equilibrium scenario, v_2 depends on Kn^{-1} , hence on

- the **system size** \bar{R}

👉 breakdown of the **scale-invariance** of **hydrodynamics**

R.S. Bhalerao, J.-P. Blaizot, N.B., J.-Y. Ollitrault, PLB **627** (2005) 49

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⇒ mean number of collisions per particle to build up v_2 :

$$Kn^{-1} \simeq \frac{\bar{R}}{\lambda} = \bar{R} \sigma n \left(\frac{\bar{R}}{c_s} \right) \simeq \frac{c_s}{c} \frac{\sigma}{S} \frac{dN}{dy}$$

σ interaction cross section, $n(\tau)$ particle density, S transverse surface

In the out-of-equilibrium scenario, v_2 depends on Kn^{-1} , hence on

- the system size \bar{R}

☞ breakdown of the scale-invariance of hydrodynamics

- the “control parameter” $\frac{1}{S} \frac{dN}{dy}$

R.S. Bhalerao, J.-P. Blaizot, N.B., J.-Y. Ollitrault, PLB 627 (2005) 49

Out-of-equilibrium scenario: a control parameter

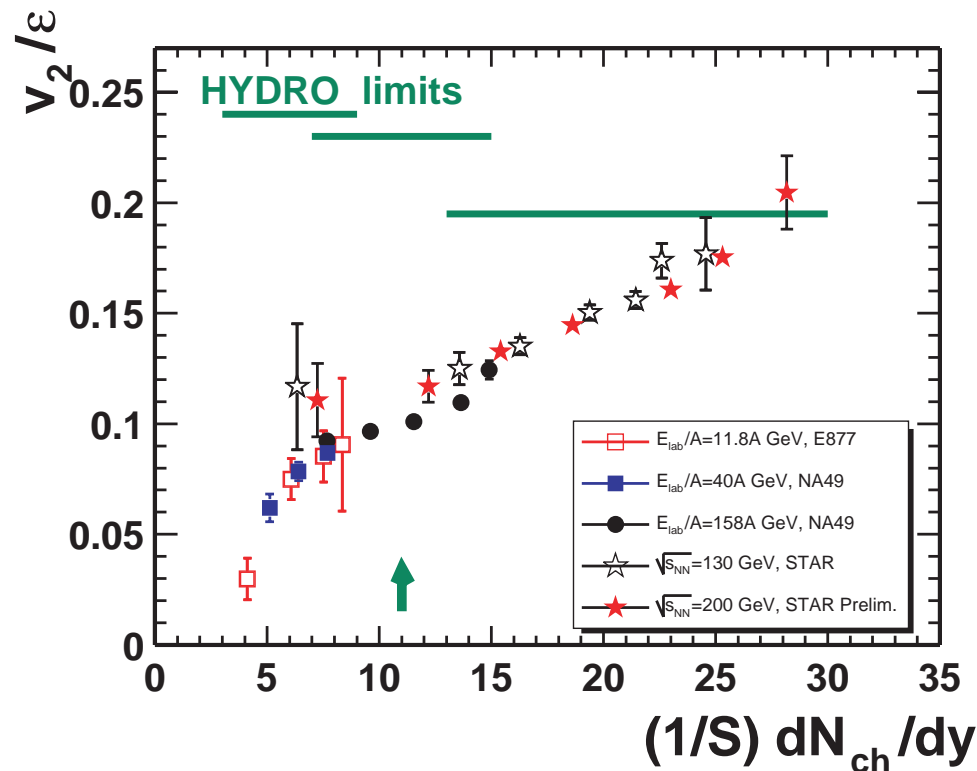
Number of collisions per particle to build up v_2 : $Kn^{-1} \simeq \frac{c_s}{c} \frac{\sigma}{S} \frac{dN}{dy}$

👉 the variation (out-of-equilibrium scenario) or independence (ideal liquid paradigm) of v_2 with Kn^{-1} can be checked using its

- centrality dependence (using the universality of v_2/ϵ)
- beam-energy dependence
- system-size dependence → importance of lighter systems
- rapidity dependence
- transverse momentum dependence

RHIC anisotropic flow data and incomplete equilibration

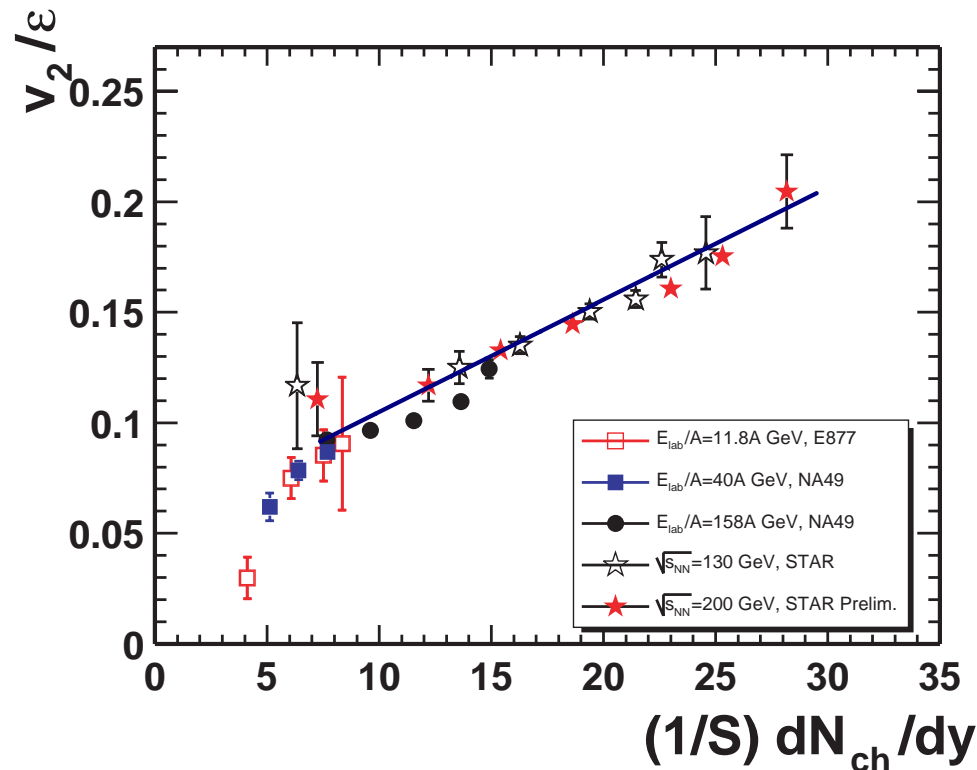
Centrality and beam-energy dependence:



NA49 Collaboration, PRC 68 (2003) 034903

RHIC anisotropic flow data and incomplete equilibration

Centrality and beam-energy dependence:



NA49 Collaboration, PRC 68 (2003) 034903

Scaling law seems to work for RHIC data (+ matching with SPS)

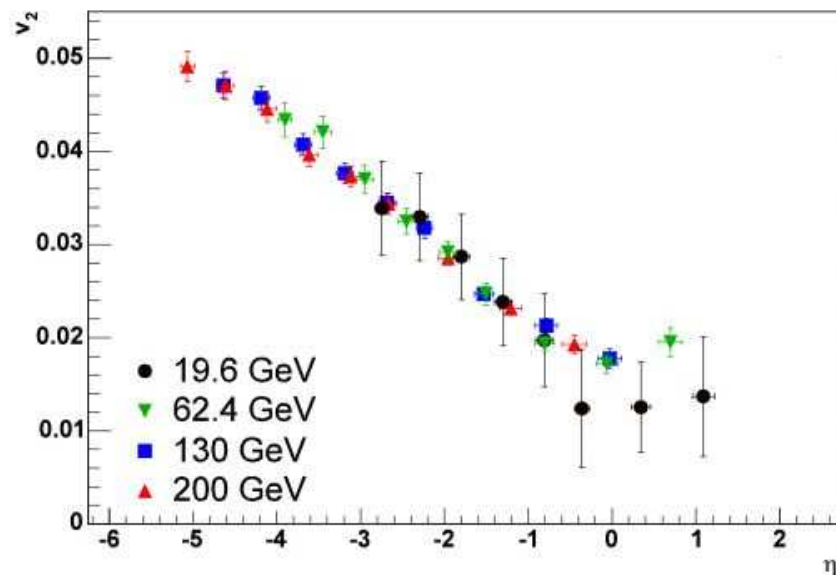
$v_2(Kn^{-1})$ increases steadily (no hint at hydro saturation in the data)

RHIC anisotropic flow data and incomplete equilibration

(Pseudo)rapidity dependence of v_2

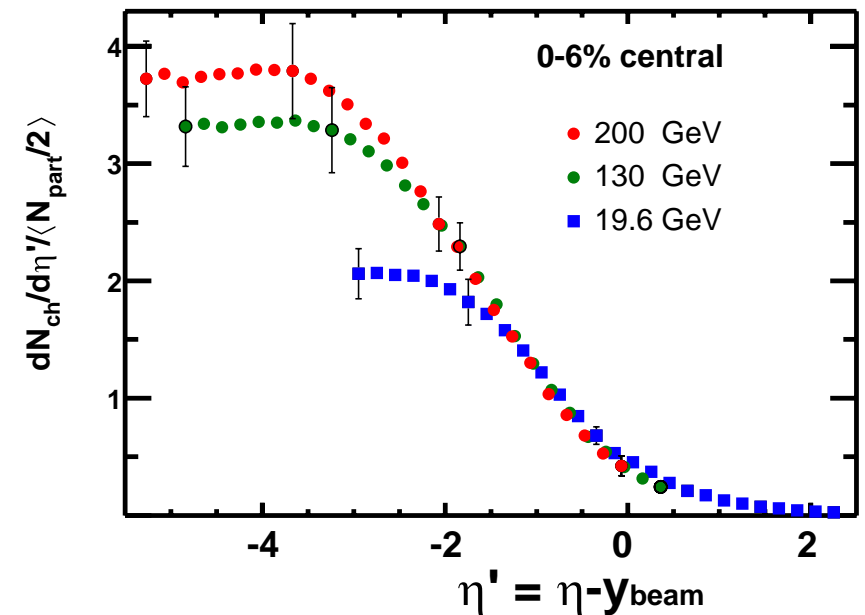
Steve Manly (PHOBOS Coll.)

QM'05



PHOBOS Collaboration

PRL 91 (2003) 052303



👉 $v_2(\eta)$ and $\frac{dN}{dy}$ approximately proportional $\Leftrightarrow v_2 \propto Kn^{-1}$

Hirano, PRC 65 (2002) 011901



Ideal liquid dynamics
vs.
out-of-equilibrium scenario

Anisotropic flow and thermalization at RHIC

- On the theoretical side...
 - bottom-up approaches cannot accommodate short **thermalization** times (?)
 - new mechanisms are emerging, but their outcome is unclear
- What do the **data** tell us?
Conflicting interpretations of Au–Au **measurements**!
 - early **thermalization**, which allows one to use **hydrodynamics**
👉 $v_2(p_t)$, spectra
 - **incomplete thermalization**, which manifests itself by the breakdown of several scaling laws of **hydro**
👉 $v_2(y)$, $v_2(E_{\text{beam}})$, $v_4/(v_2)^2$

Anisotropic flow and thermalization at RHIC

- On the theoretical side...

- bottom-up approaches cannot accommodate short

Mass-ordering is not unique to hydro!

also found transport approaches:

- QGSM (Phys. Lett. B **631** (2005) 109)
- RQMD/UrQMD (nucl-th/0602009)

their outcome is unclear

measurements!

- early thermalization, which allows one to use hydrodynamics

👉 $v_2(p_t)$, spectra

- incomplete thermalization, which manifests itself by the breakdown of several scaling laws of hydro

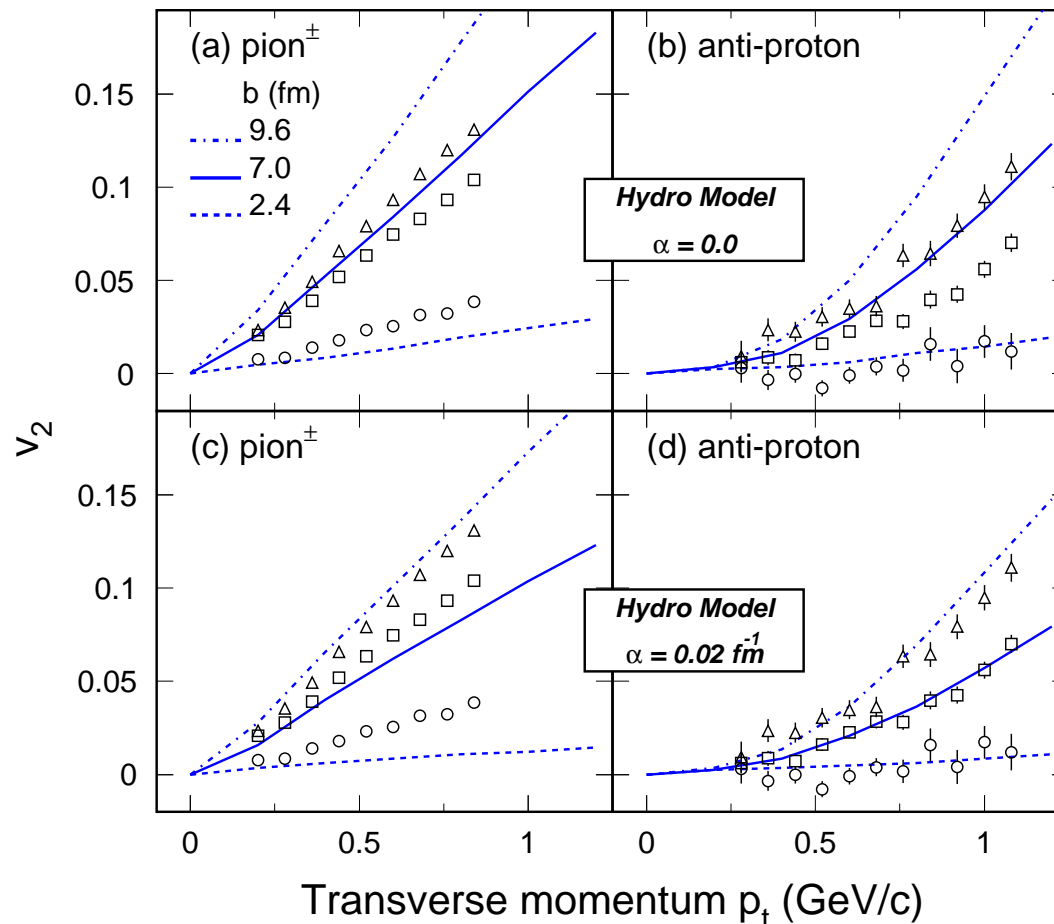
👉 $v_2(y)$, $v_2(E_{\text{beam}})$, $v_4/(v_2)^2$

Rescuing ideal fluid dynamics



The use of **hydrodynamics** at RHIC may have been too simplistic...

$v_2(p_t)$ for various centralities STAR Collaboration, PRC 72 (2005) 014904

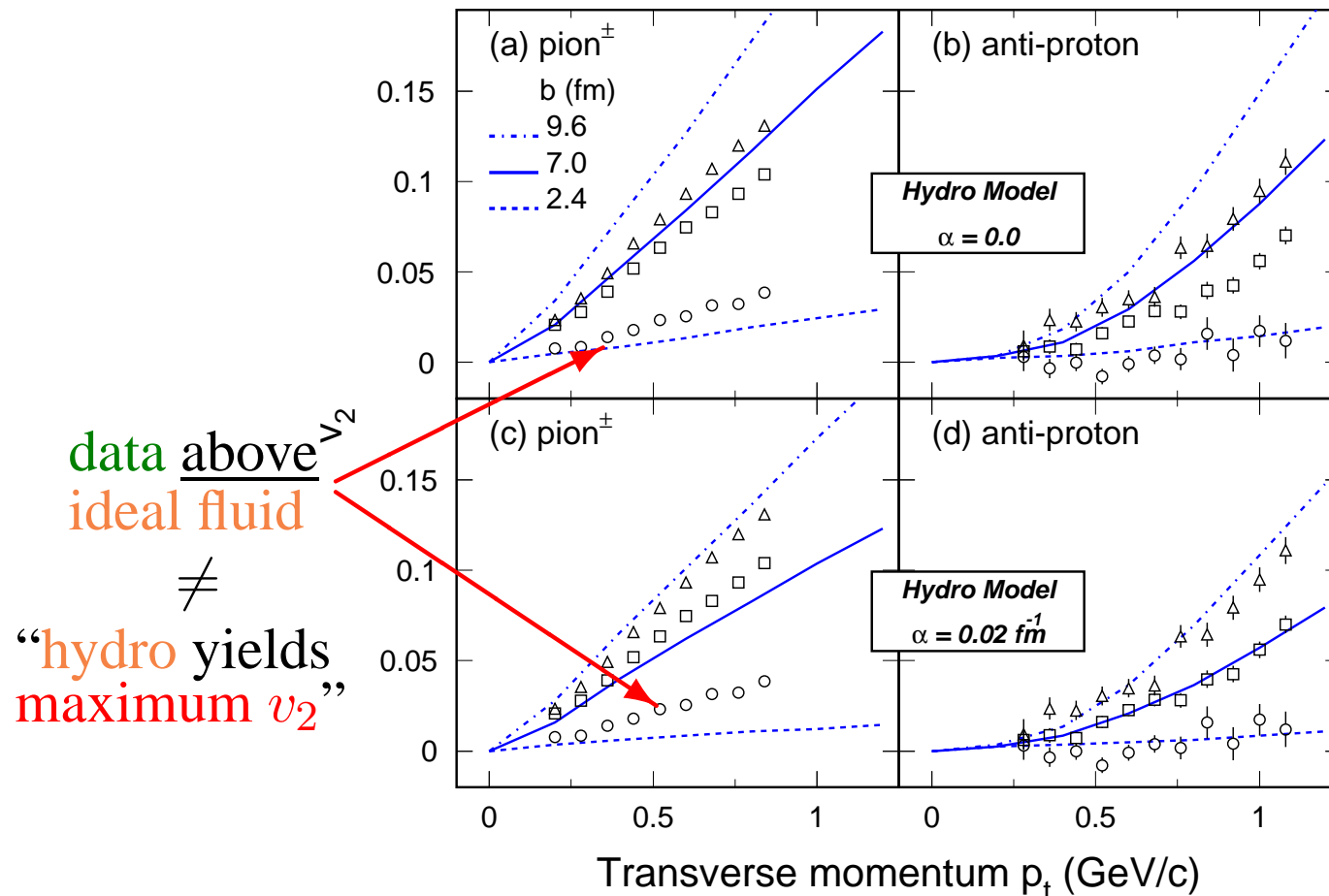


Rescuing ideal fluid dynamics



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Rescuing ideal fluid dynamics



In hydrodynamical fits, the speed of sound is constrained by p_t spectra, which require a soft equation of state

→ with a hard equation of state, the energy per particle is too high

All relies on the assumption that the energy per particle is related to the density, i.e., that chemical equilibrium is maintained

- chemical equilibrium is more fragile than kinetic equilibrium
- the only experimental indication of chemical equilibrium is in the particle-abundance ratios (cf. however $e^+e^- \dots$)

If there is no chemical equilibrium, energy per particle and density are independent variables, as in ordinary thermodynamics

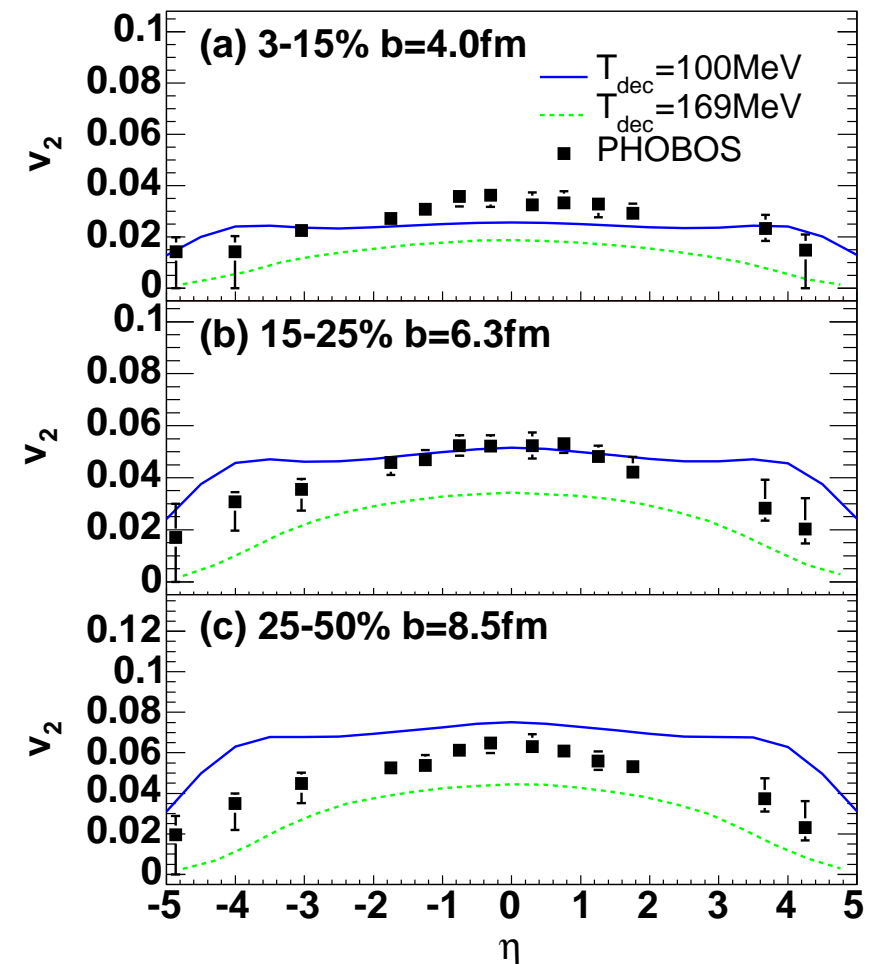
☞ there is no constraint on the equation of state from p_t spectra:
one can consider a larger c_s to increase v_2 in central collisions

Rescuing ideal fluid dynamics



T. Hirano, U. Heinz, D. Kharzeev, R. Lacey, Y. Nara, [nucl-th/0511046](#)

- Assume a quickly equilibrated, perfect liquid
- Vary the initial conditions of its hydro evolution
 - quasi-Gaussian in rapidity
 - CGC
- $\Rightarrow v_2(\eta)$ is not reproduced

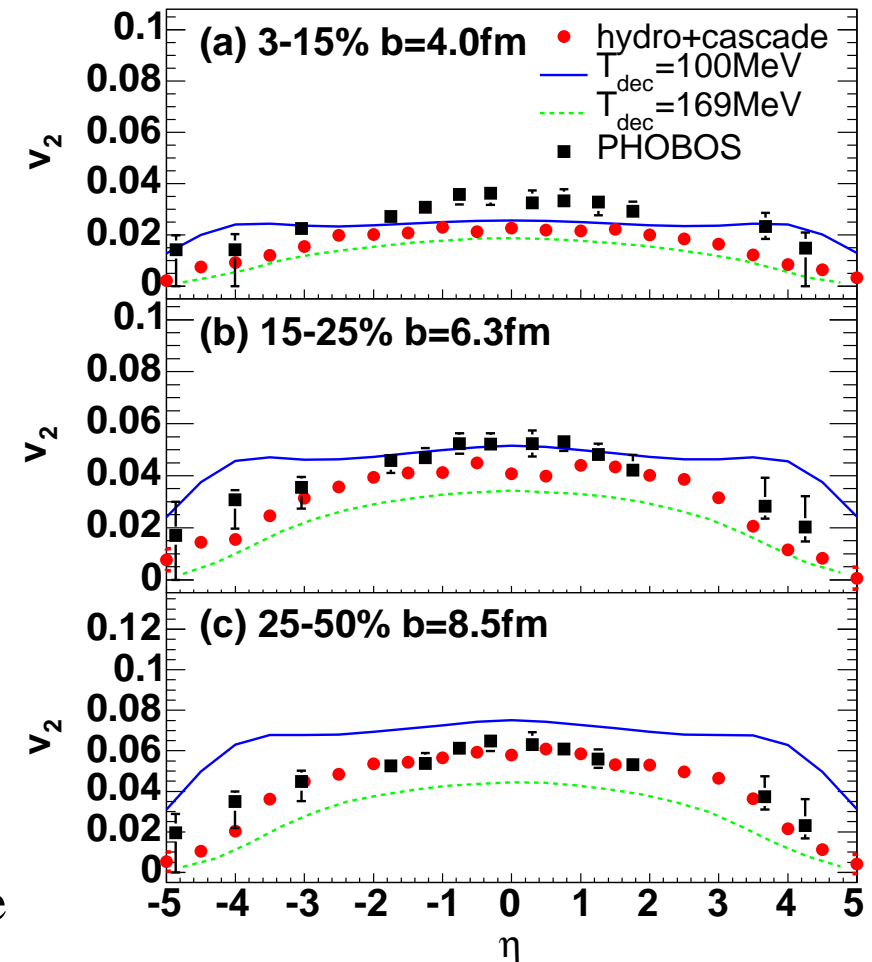


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 - modeled by hadronic cascade

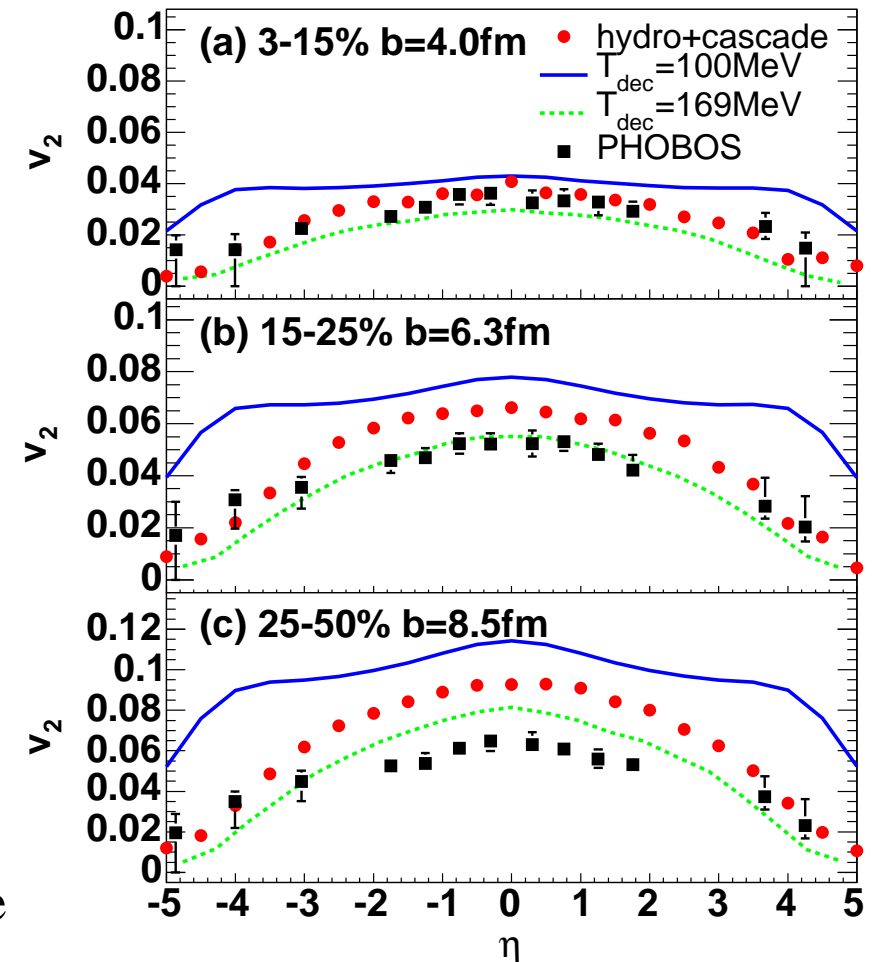


Rescuing ideal fluid dynamics



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Hints of incomplete thermalization in RHIC data

A wealth of experimental measurements on anisotropic flow is already available from RHIC, and much more will come.

But these data still await a consistent quantitative interpretation: a model which aims at describing v_2 should simultaneously explain v_4 ! (not to mention fancier effects: Mach cone...)

Assuming that thermalization is incomplete at the time when flow develops gives a qualitative explanation of several features observed in the data (which do not come out naturally in a perfect-liquid picture).

Hopefully, this qualitative agreement may turn into a quantitative one, yielding information on the created medium: viscosity, other transport coefficients.



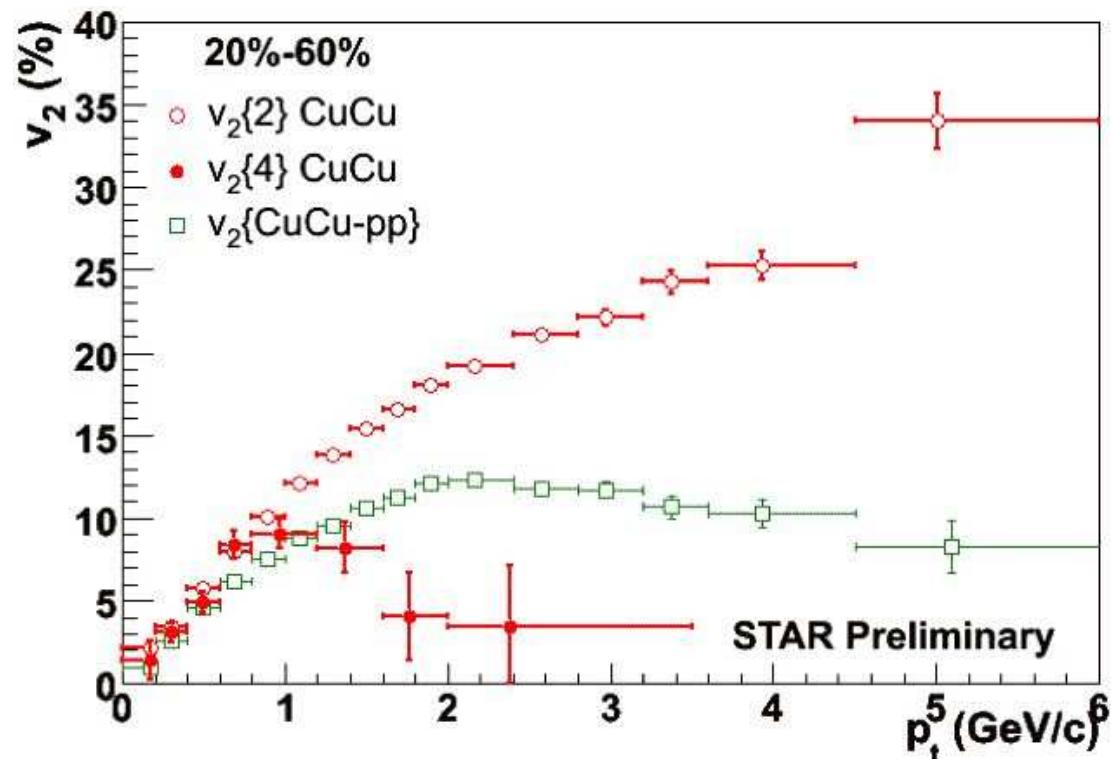
Extra slides

Incomplete equilibration: predictions for Cu–Cu flow

- The matching between central SPS and peripheral RHIC suggests that we can even compare **systems** with different densities, i.e., different σ (and c_s)
- ☞ compare **Au–Au** at $b = 8$ fm with **Cu–Cu** at $b = 5.5$ fm (similar **centrality**)
 - If **hydro** holds, v_2 should scale like ϵ : $v_2(\text{Cu}) = 0.69 v_2(\text{Au})$
 - If thermalization is incomplete, $\frac{v_2}{\epsilon} \propto \frac{1}{S} \frac{dN}{dy} \propto Kn^{-1}$, i.e.
$$v_2(\text{Cu}) = 0.34 v_2(\text{Au})$$
- **Cu–Cu** further from equilibrium than **Au–Au** $\Rightarrow \frac{v_4}{(v_2)^2} > 1.2$
- First results presented at QM'05 are too preliminary...

Cu–Cu collisions at RHIC: anisotropic flow

Gang Wang (STAR Collaboration) @ QM'05:



Measurements with different methods give very different v_2 values
(not a surprise...)

Wait and see!