A NEW METHOD OF FLOW ANALYSIS

N. Borghini (Brussels),
P.M. Dinh (Saclay),
J.-Y. Ollitrault (Saclay)

- Standard analysis methods

  → two-particle correlations

  – Limited sensitivity

  Phys. Lett. B**477** (2000) 51
  Phys. Rev. C**62** (2000) 034902

- New method(s)

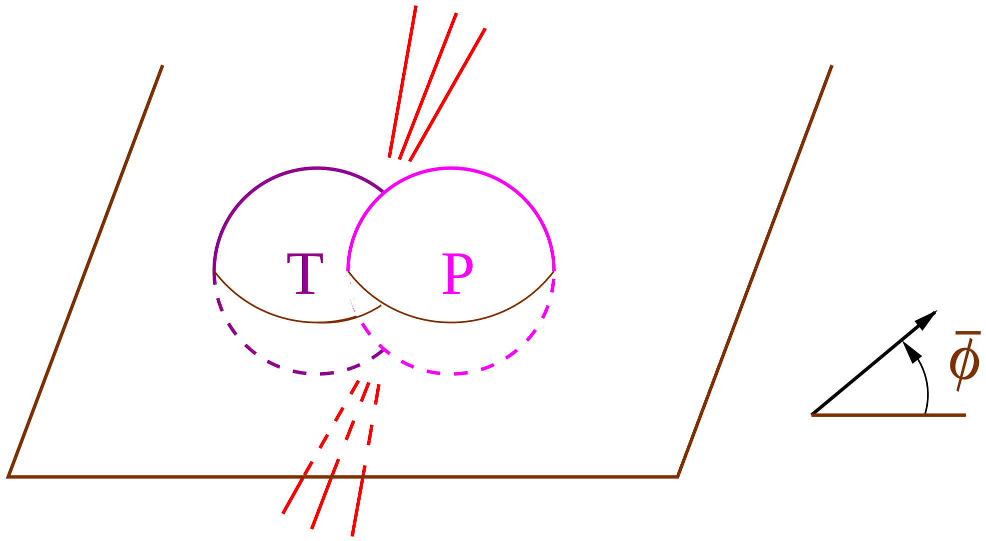
  → multiparticle correlations

  – Integrated flow
  – Differential flow
  – Increased sensitivity
  – Acceptance corrections

  Phys. Rev. C**63** (2001) 054906
  `nucl-th/0105040` (Phys. Rev. C, in press)

FLOW

Flow $\equiv$ azimuthal correlation with the reaction plane:



Fourier expansion of the azimuthal distributions of outgoing particles with respect to the unknown reaction plane:

$$\frac{dN}{d\bar{\phi}} = A\left(1 + 2v_1 \cos\bar{\phi} + 2v_2 \cos 2\bar{\phi} + \cdots\right)$$

where:

$$v_n = \left\langle e^{in\bar{\phi}} \right\rangle.$$

v_1 "directed" flow, v_2 "elliptic" flow.

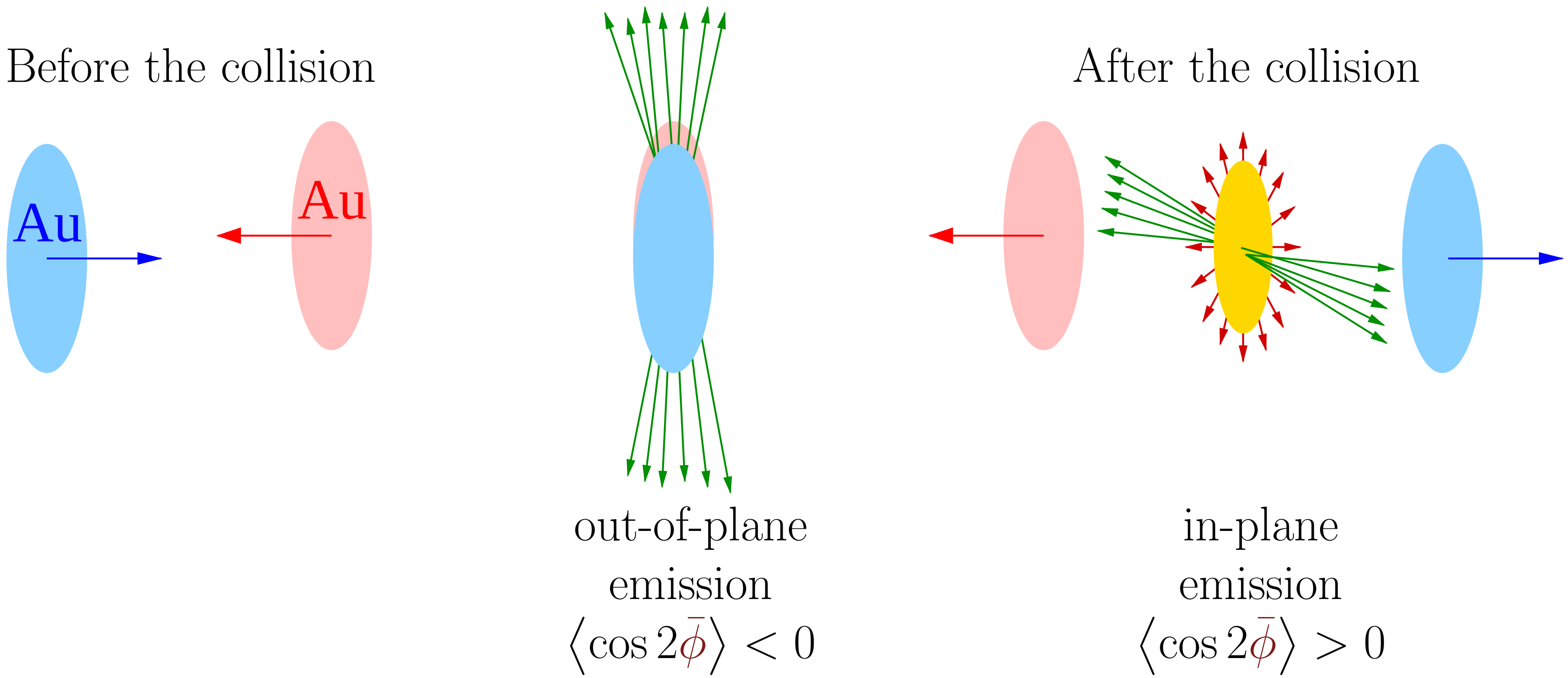
At CERN SPS, v_1 and $v_2 \simeq 3\%$ for pions and protons.

At RHIC (PHENIX, PHOBOS, STAR): $v_2 \simeq 5 - 6\%$

$\longrightarrow$ see R. Lacey, A. Poskanzer.

WHY FLOW?

- Flow determination ⇒ equation of state:



- Signature of collective behavior at ultrarelativistic energies.
- Influence of flow on two-particle correlations (HBT, Coulomb...).
- Observation of possible parity violation requires accurate flow determination.

FLOW ANALYSIS METHODS (simplified)

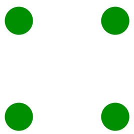
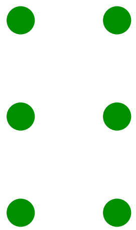
♣ Two-particle methods ( $= \left\langle e^{in(\phi_1-\phi_2)} \right\rangle$):

♣ "subevent" method (P. Danielewicz and G. Odyniec):

$\longrightarrow$ correlation between **2** subevents;

♣ **two**-particle correlation function analysis (R. Lacey): $C(\Delta\phi)$;

♡ Multiparticle methods

 $= \left\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)} \right\rangle$,  $= \left\langle e^{in(\phi_1+\phi_2+\phi_3-\phi_4-\phi_5-\phi_6)} \right\rangle$ $\cdots$

(♡) cumulants of the event flow vector;

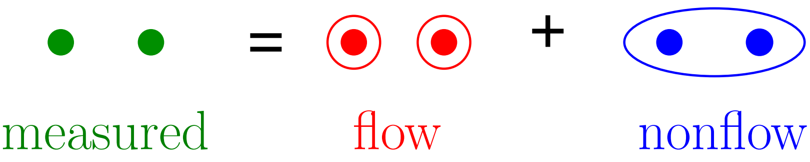
♡♡ cumulants of multiparticle azimuthal correlations.

STANDARD FLOW ANALYSIS

Coefficient v_n extracted from the measured two-particle azimuthal correlations:

$$\left\langle e^{in(\phi_1-\phi_2)}\right\rangle = \left\langle e^{in\bar{\phi}_1}\right\rangle\left\langle e^{-in\bar{\phi}_2}\right\rangle + \left\langle e^{in(\phi_1-\phi_2)}\right\rangle_c$$
$$\equiv v_n^2 + \left\langle e^{in(\phi_1-\phi_2)}\right\rangle_c .$$

Expansion of two-particle correlations:



“STANDARD” ASSUMPTION: nonflow sources of two-particle azimuthal correlations are negligible:

$$v_n^2 \gg \left\langle e^{in(\phi_1-\phi_2)}\right\rangle_c .$$

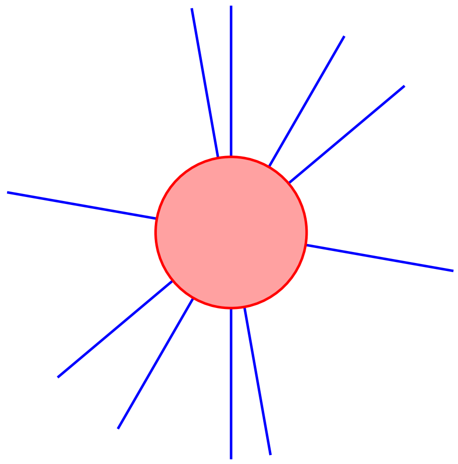
The measured two-particle azimuthal correlations are only due to flow:

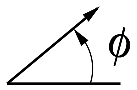
$$v_n = \pm\sqrt{\left\langle e^{in(\phi_1-\phi_2)}\right\rangle}.$$

TWO-PARTICLE NONFLOW CORRELATIONS
a simple example

Central collision → NO flow, $v_n = 0$.

Strong direct back-to-back correlations:



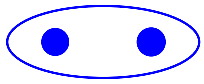


$\Rightarrow \langle \cos 2(\phi_1 - \phi_2) \rangle > 0.$

The standard analysis assumes $v_2 = \sqrt{\langle \cos 2(\phi_1 - \phi_2) \rangle}$...

$$\Rightarrow v_2 \neq 0$$

TWO-PARTICLE NONFLOW ("DIRECT") CORRELATIONS

Many sources for $\left\langle e^{in(\phi_1-\phi_2)}\right\rangle_c =$  :

- ◇ total momentum conservation;
- ◇ quantum "HBT" correlations;
- ◇ final state (strong/Coulomb) interactions;
- ◇ resonance decays;
- ◇ other sources? (minijets...)

} order $\frac{1}{N}$

⇒ the assumption $v_n^2 \gg \left\langle e^{in(\phi_1-\phi_2)}\right\rangle_c$ underlying the standard analysis holds only if

$$\boxed{v_n \gg \frac{1}{N^{1/2}}}.$$

Possibility: compute and subtract nonflow correlations.

OK, but nonflow correlations may not be under control. . .

Important: two-particle nonflow correlations scale as $\frac{1}{N}$
⇒ dominant for peripheral collisions

see A. Poskanzer's talk!!

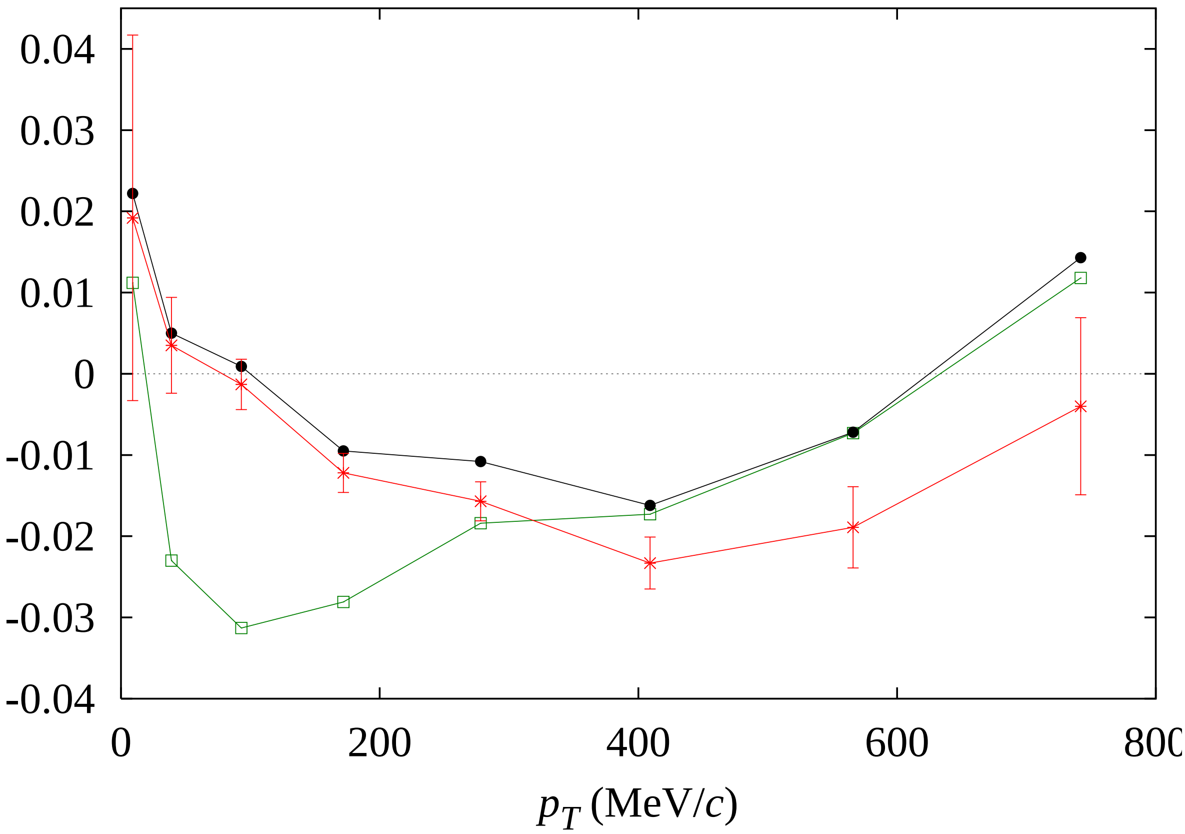
STANDARD FLOW ANALYSIS AT SPS

"Standard" assumption: $v_n^2 \gg \left\langle e^{in(\phi_1-\phi_2)} \right\rangle_c \sim \frac{1}{N}$.

- v_1 and $v_2 \simeq 3\%$ for pions and protons;
- total multiplicity in the collision $N \simeq 2500$.

$\Rightarrow$ the assumption is not valid.

Pion directed flow at SPS (1996 data)



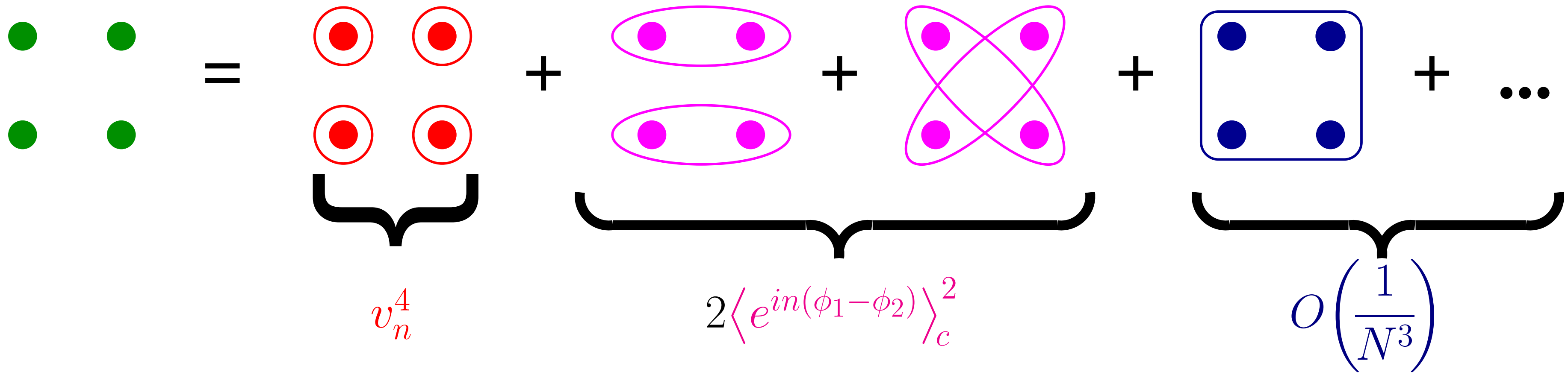
□: "data" [NA49, Phys. Rev. Lett. **80** (1998) 4136]

•: data − HBT [Phys. Lett. B**477** (2000) 51]

×: data − (HBT & p_T conservation) [PRC**62**, 034902]

NEW METHOD

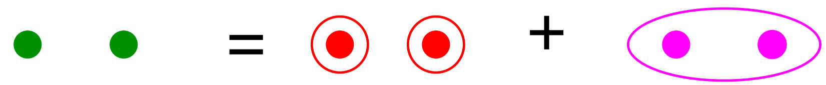
Idea: extract flow from multiparticle azimuthal correlations.



Method: compare flow with direct **4**-particle correlations

$\Rightarrow$ eliminate (non-negligible) extra terms:

cumulant of the multiparticle correlations.

Remember that 

NEW METHOD: INTEGRATED FLOW $v(\mathcal{D})$

Cumulant of the four-particle azimuthal correlation:

$$\left\langle\!\left\langle e^{in(\phi_1-\phi_2+\phi_3-\phi_4)} \right\rangle\!\right\rangle \equiv \left\langle e^{in(\phi_1-\phi_2+\phi_3-\phi_4)} \right\rangle - 2\left\langle e^{in(\phi_1-\phi_2)} \right\rangle^2$$

$$= -v_n^4 + O\left(\frac{1}{N^3}\right)$$

Increased sensitivity: analysis valid if $\boxed{v_n \gg \frac{1}{N^{3/4}}}$, better than $v_n \gg \frac{1}{N^{1/2}}$.

$$\text{systematic error } \delta(v_n^4) \simeq \frac{1}{N^3}$$

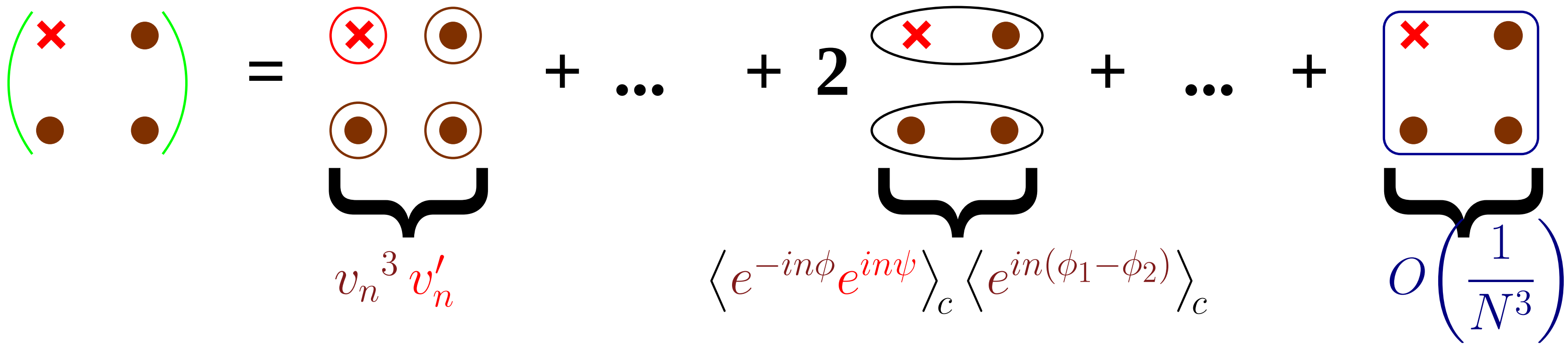
Statistics: $N_{\rm evts}$ events, M particles per event $\longrightarrow N_{\rm evts}M^4$ quadruplets

$$\text{statistical error } \delta(v_n^4) \simeq \frac{1}{M^2\sqrt{N_{\rm evts}}}.$$

DIFFERENTIAL FLOW $v'(p_T, y)$

1. Measure the integrated flow $\langle e^{in\phi} \rangle = v_n$ using many particles (●): reaction plane determination.

2. Study the correlation between the azimuth ψ of a given particle (✖) and the reaction plane: $\langle e^{-in\phi} e^{in\psi} \rangle$.



Idea: compare the flow term with the direct multiparticle azimuthal correlation.

⇒ Cumulant of the (1+3)-particle azimuthal correlation:

$$\left\langle\!\left\langle e^{in(\phi_1-\phi_2-\phi_3)} e^{in\psi} \right\rangle\!\right\rangle \equiv \left\langle e^{in(\phi_1-\phi_2-\phi_3)} e^{in\psi} \right\rangle - 2\left\langle e^{-in\phi} e^{in\psi} \right\rangle \left\langle e^{in(\phi_1-\phi_2)} \right\rangle$$

$$= -v_n{}^3 \left[v'_n + O\left(\frac{1}{(N v_n)^3} \right) \right].$$

CUMULANTS $\langle\langle|Q_n|^{2p}\rangle\rangle$: PRACTICAL FLOW ANALYSIS

"old version": Phys. Rev. C**63** (2001) 054906

(1.) Compute $Q_n = \frac{1}{\sqrt{M}}\sum_{k=1}^{M} e^{in\phi_k}$ for a given event.

(2.) Calculate the generating function $\mathcal{G}(z) = e^{z^* Q_n + z Q_n^*}$, then average over events.

Why? because $\langle\mathcal{G}(z)\rangle = 1 + \cdots + |z|^2\langle|Q_n|^2\rangle + \cdots + \frac{|z|^4}{4}\langle|Q_n|^4\rangle + \cdots$, and

the $|Q_n|^{2p}$ give the multiparticle azimuthal correlations: $|Q_n|^2 = \frac{1}{M}\sum_{j,k=1}^{M} e^{in(\phi_j-\phi_k)}$

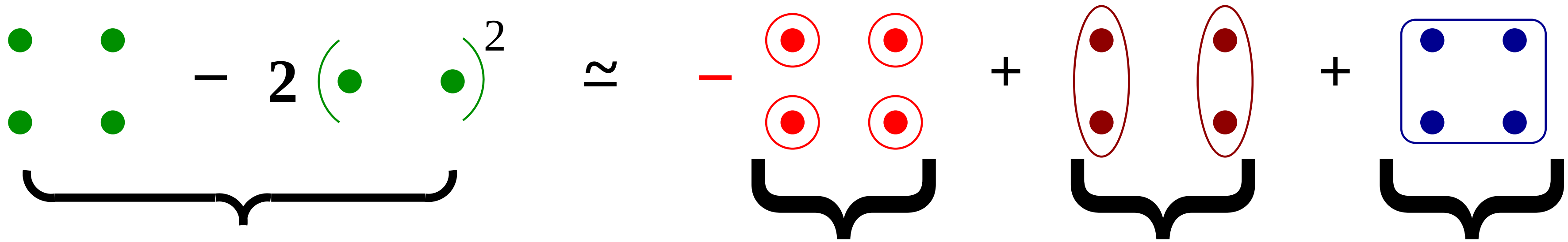
(3.) Deduce the cumulants, taking $\ln\langle\mathcal{G}(z)\rangle$:

$$\ln\langle\mathcal{G}(z)\rangle = 1 + \cdots + |z|^2\langle\langle|Q_n|^2\rangle\rangle + \cdots + \frac{|z|^4}{4}\langle\langle|Q_n|^4\rangle\rangle + \cdots$$

(4.) Extract the flow, using $\ln\langle\mathcal{G}(z)\rangle = \ln I_0(2|z|\langle\bar{Q}_n\rangle)$.

$\longrightarrow$ for instance, $\langle\langle|Q_n|^4\rangle\rangle \equiv \langle|Q_n|^4\rangle - 2\langle|Q_n|^2\rangle^2 = -\langle\bar{Q}_n\rangle^4 = -M^2 v_n^4$.

INTERFERENCE BETWEEN v_1 AND v_2



$\langle\!\langle e^{in(\phi_1-\phi_2+\phi_3-\phi_4)} \rangle\!\rangle \qquad v_n^4 \qquad O\left(\frac{v_{2n}^2}{N^2}\right) \qquad O\left(\frac{1}{N^3}\right)$

$\Rightarrow$ Measurements of v_n require $|v_{2n}| \ll N\, v_n^2$.

Problem for directed flow at RHIC, not for elliptic flow.

BETTER CUMULANTS: ANY HARMONIC

"new version": `nucl-th/0105040`

(1.) Calculate the generating function $G_n(z) = \prod_{k=1}^{M} \left(1 + \frac{z^* e^{in\phi_k} + z e^{-in\phi_k}}{M}\right)$, then average over events.

$$\langle G_n(z) \rangle = 1 + \cdots + \frac{|z|^2}{M} \left\langle \sum_{j \neq k} e^{in(\phi_j - \phi_k)} \right\rangle + \cdots + \frac{|z|^4}{4M^4} \left\langle \sum_{j,k,l,m} e^{in(\phi_j + \phi_k - \phi_l - \phi_m)} \right\rangle + \cdots$$

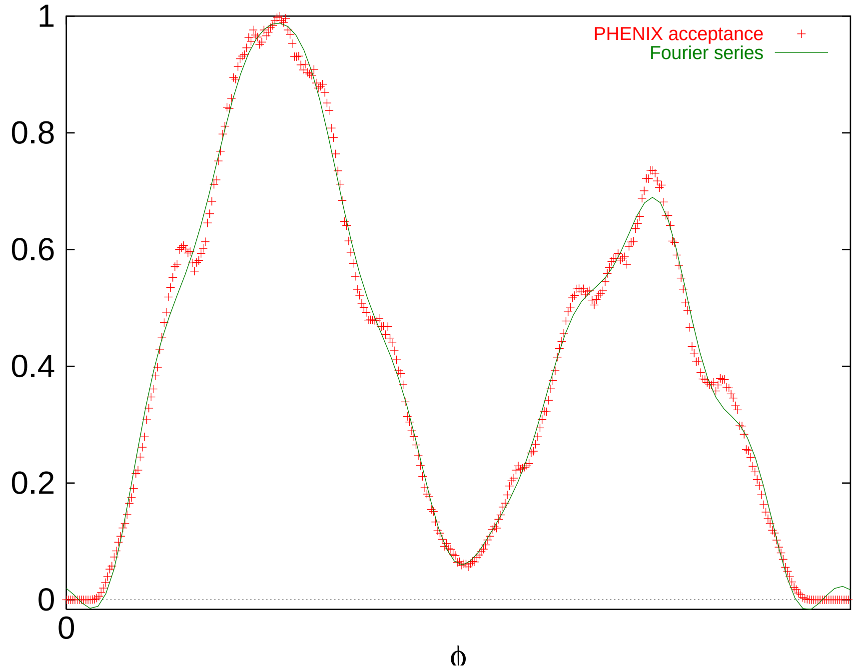
(2.) Deduce the cumulants, taking $M \left(\langle G_n(z) \rangle^{1/M} - 1 \right) = |z|^2 \left\langle\!\left\langle e^{in(\phi_j - \phi_k)} \right\rangle\!\right\rangle + \cdots$

(3.) Extract the flow, using (→ STAR ☺) $M \left(\langle G_n(z) \rangle^{1/M} - 1 \right) = \ln I_0(2 v_n |z|)$,
and/or performing the appropriate acceptance corrections (→ PHENIX ☹).

(4.) Post your paper on nucl-ex.

ACCEPTANCE CORRECTIONS

Detector acceptance/efficiency: $A(\phi) = \sum_{k=-\infty}^{+\infty} a_k e^{ik\phi}$.



Events with a fixed orientation of the reaction plane:

$$\left\langle e^{in\phi}\right\rangle = a_n + \sum_{k\neq 0}(a_{n+k} - a_n a_k)\, v_k\, e^{ik\Phi_R}$$

Imperfect acceptances mix different flow harmonics!

Insert $\left\langle e^{in\phi}\right\rangle$ in the generating function
$\longrightarrow$ new relations between cumulants and flow.

For instance:

$$\begin{aligned} c_2\{2\} &= 0.042\, v_1^2 + 0.659\, v_2^2 \\ c_2\{4\} &= -0.002\, v_1^4 - 0.487\, v_2^4 \end{aligned}$$

instead of $c_2\{2\} = v_2^2$, $c_2\{4\} = -v_2^4$ (perfect acceptance).

COLLECTIVE FLOW AND MULTIPARTICLE AZIMUTHAL CORRELATIONS

- At SPS energies, two-particle azimuthal correlations due either to collective flow or nonflow effects are of the same magnitude. $\Rightarrow$ the standard analysis is close to its validity limit $v_n \gg 1/N^{1/2}$.
- New method, using four-particle azimuthal correlations, allows measurements of smaller integrated flow values $v_n \gg 1/N^{3/4}$.

  Sensitivity (and accuracy) can still be improved, with $2p$-particle ($p > 2$) correlations ($\rightarrow$ higher statistics).
- Detector acceptance corrections.
- Differential flow.

Method currently tested/used by E895, NA49, PHENIX, STAR. First results available!

Two-particle and multiparticle methods may yield different values $v_n\{2\} \neq v_n\{4\}$...

"NEW" (unthought of) two-particle correlations!