Hints of incomplete thermalization in RHIC data

Nicolas BORGHINI

in collaboration with

R.S. BHALERAO J.-P. BLAIZOT J.-Y. OLLITRAULT

CERN

RHIC Au–Au results: the fashionable view

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RHIC Scientists Serve Up “Perfect” Liquid

New state of matter more remarkable than predicted -- raising many new questions

April 18, 2005

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RHIC Sci

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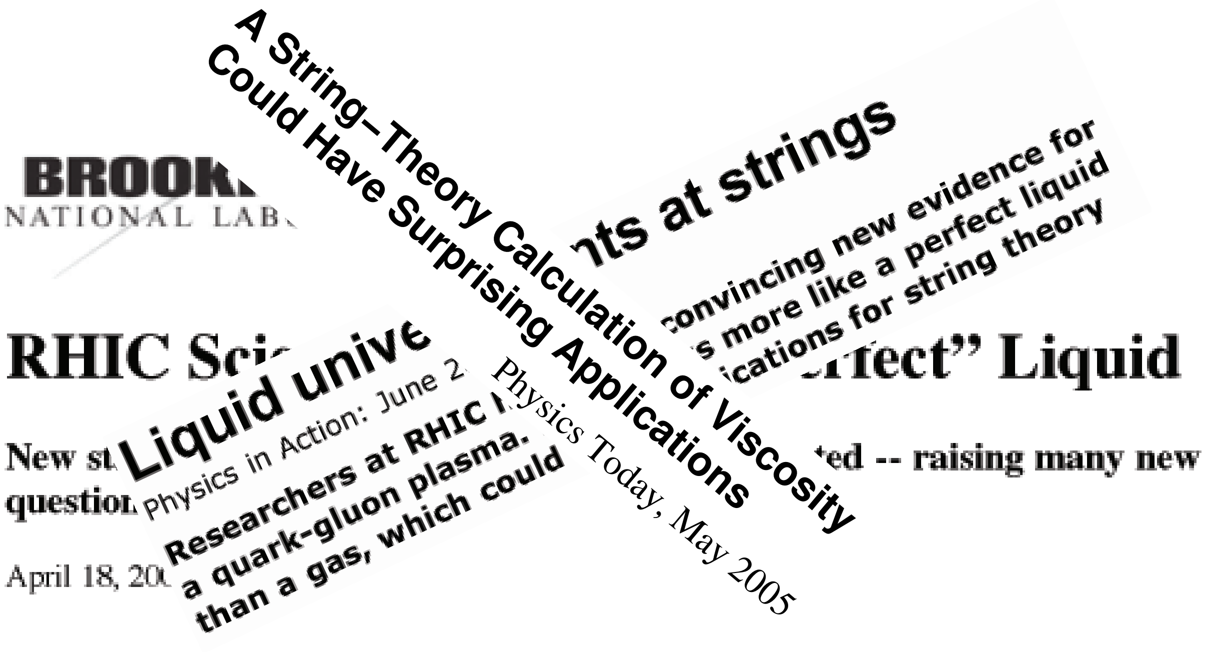
Liquid universe hints at strings

Physics in Action: June 2005

Researchers at RHIC have seen convincing new evidence for a quark-gluon plasma. But it looks more like a perfect liquid than a gas, which could have implications for string theory

RHIC Au–Au results: the fashionable view



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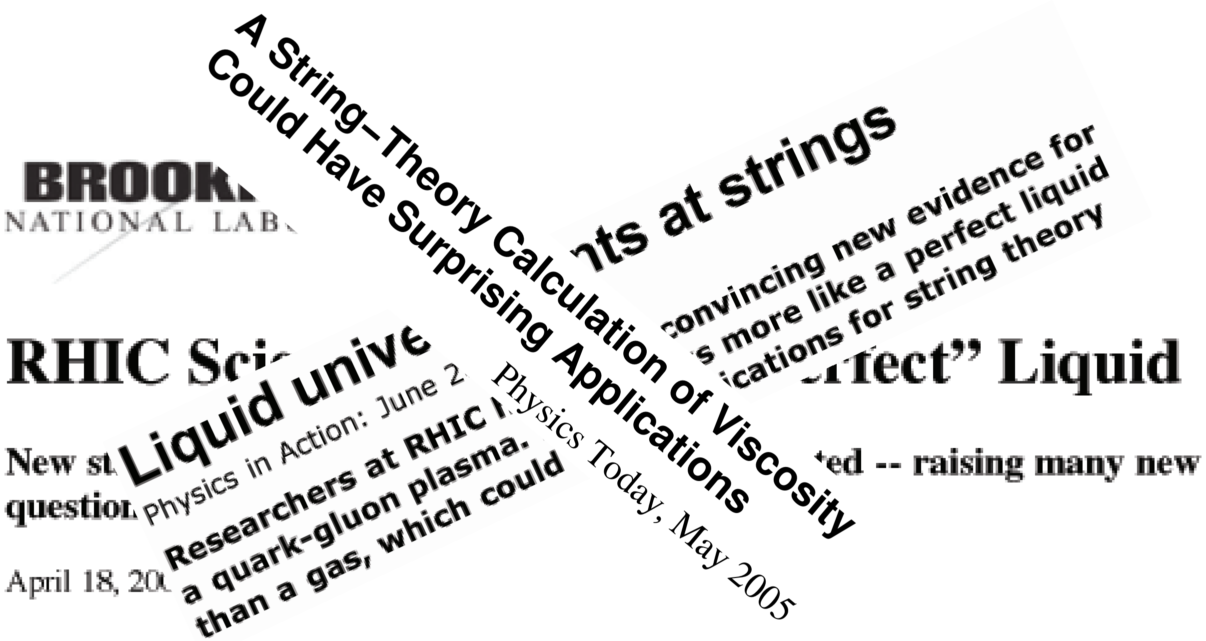

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Ideal fluid dynamics reproduce both p_t spectra and $v_2(p_t)$ of soft ($p_t \lesssim 2$ GeV/c) identified particles for minimum bias collisions, near central rapidity.

This agreement necessitates a soft equation of state, and very short thermalization times: $\tau_{\rm thermalization} < 0.6$ fm/c.

$\Rightarrow$ **strongly interacting Quark-Gluon Plasma**

Ideal fluid dynamics in heavy-ion collisions

- A few reminders on fluid dynamics
- Fluid dynamics and heavy ion collisions: theory
  - Overall scenario
  - General predictions of ideal fluid dynamics
    - Momentum spectra
    - Anisotropic flow
- Fluid dynamics and heavy ion collisions: theory vs. data
- Reconciling data and theory

  (including predictions for Cu–Cu@RHIC and Pb–Pb@LHC)

Fluid dynamics: physical quantities

- Microscopic parameters
  - λ = mean free path between two collisions
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- Microscopic parameters
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- Macroscopic parameters
  - L = system size
  - v_{fluid} = fluid velocity
- Micro and macro are connected: kinetic theory
  - c_s = sound velocity $\sim v_{\text{thermal}}$
  - η = viscosity $\sim \lambda\, v_{\text{thermal}}$

Fluid dynamics: various types of flow

- **Thermodynamic equilibrium?** ☞ Knudsen number $Kn = \frac{\lambda}{L}$
  - $Kn \gg 1$: Free-streaming limit
  - $Kn \ll 1$: Thermalization : Fluid (hydro) limit

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- **Compressible or Incompressible?** ☞ Mach number $Ma = \frac{v_{\text{fluid}}}{c_s}$
  - $Ma \ll 1$: Incompressible flow
  - $Ma > 1$: Compressible (supersonic) flow

Fluid dynamics:
various types of flow

Three numbers:

$$Kn = \frac{\lambda}{L}, \qquad Re = \frac{L v_{\text{fluid}}}{\eta}, \qquad Ma = \frac{v_{\text{fluid}}}{c_s}$$

$\Rightarrow$ **an important relation:**

$$Kn \times Re = \frac{\lambda\, v_{\text{fluid}}}{\eta} \sim \frac{v_{\text{fluid}}}{c_s} = Ma$$

Compressible fluid: Thermalized means Ideal

Viscosity $\equiv$ departure from equilibrium

General scenario of a heavy-ion collision

⓪ Creation of a dense gas of particles

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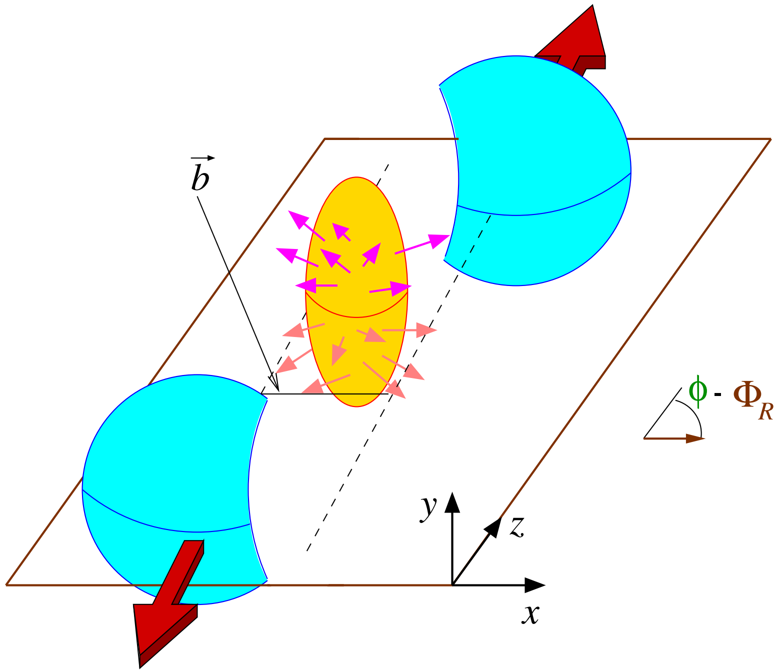
"(kinetic) freeze-out"

Freeze-out usually parameterized in terms of a temperature $T_{\mathrm{f.o.}}$

If the mean free path varies smoothly with temperature, consistency requires $T_{\mathrm{f.o.}} \ll T_0$

Heavy-ion observable: Anisotropic flow

Non-central collision:



Initial anisotropy of the source (in the transverse plane)

$\Rightarrow$ anisotropic pressure gradients, larger along the impact parameter $\vec{b}$

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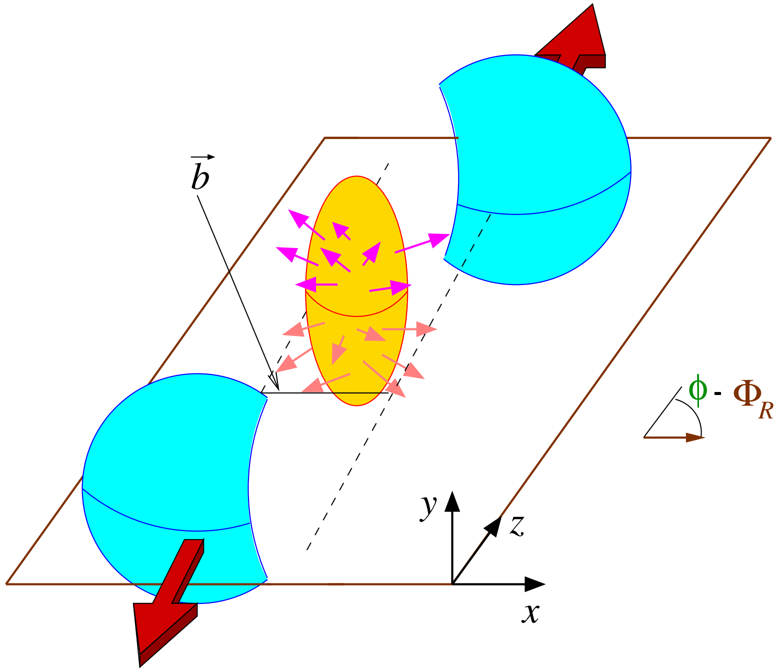

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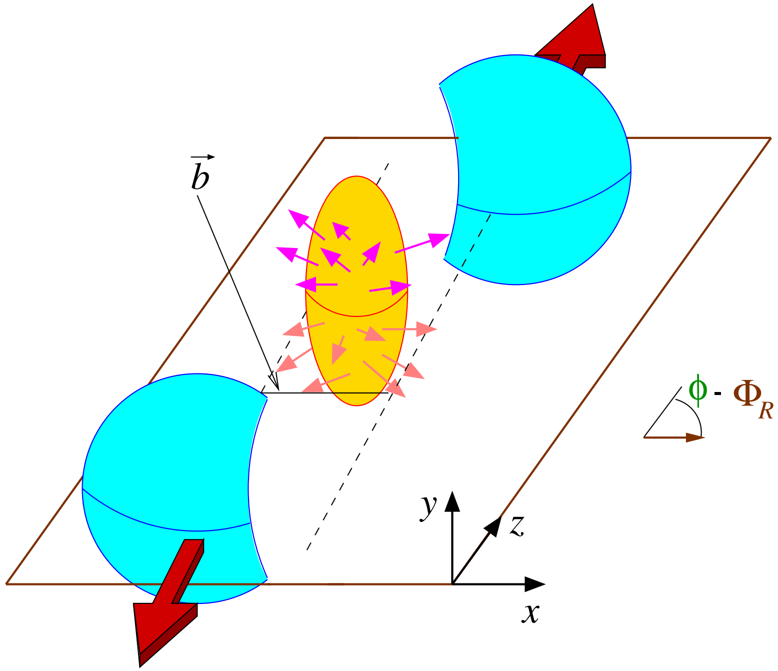

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$$E\frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{p}} \propto \frac{\mathrm{d}N}{p_t\,\mathrm{d}p_t\,\mathrm{d}y}\left[1 + 2v_1\cos(\phi - \Phi_R) + 2v_2\cos 2(\phi - \Phi_R) + \ldots\right]$$

“directed” “elliptic”

“Flow”: misleading terminology; does NOT imply fluid dynamics!

Ideal fluid dynamics: general predictions

Consistent ideal fluid dynamics picture requires $T_{\rm f.o.} \ll T_0$

$$\Leftrightarrow$$

Ideal-fluid limit = $T_{\rm f.o.} \to 0$ limit

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☞ one can compute in a model-independent way

- the spectrum $E\dfrac{\mathrm{d}N}{\mathrm{d}^3\mathbf{p}} = C\displaystyle\int_\Sigma \exp\left(-\frac{p^\mu u_\mu(x)}{T_{\mathrm{f.o.}}}\right) p^\mu\, \mathrm{d}\sigma_\mu$
- the anisotropic flow $v_n = \dfrac{\displaystyle\int_0^{2\pi} \frac{\mathrm{d}\phi}{2\pi}\, E\frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{p}} \cos n\phi}{\displaystyle\int_0^{2\pi} \frac{\mathrm{d}\phi}{2\pi}\, E\frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{p}}}$

using saddle-point approximations

N.B. & J.-Y. Ollitrault, `nucl-th/0506045`

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fluid velocity

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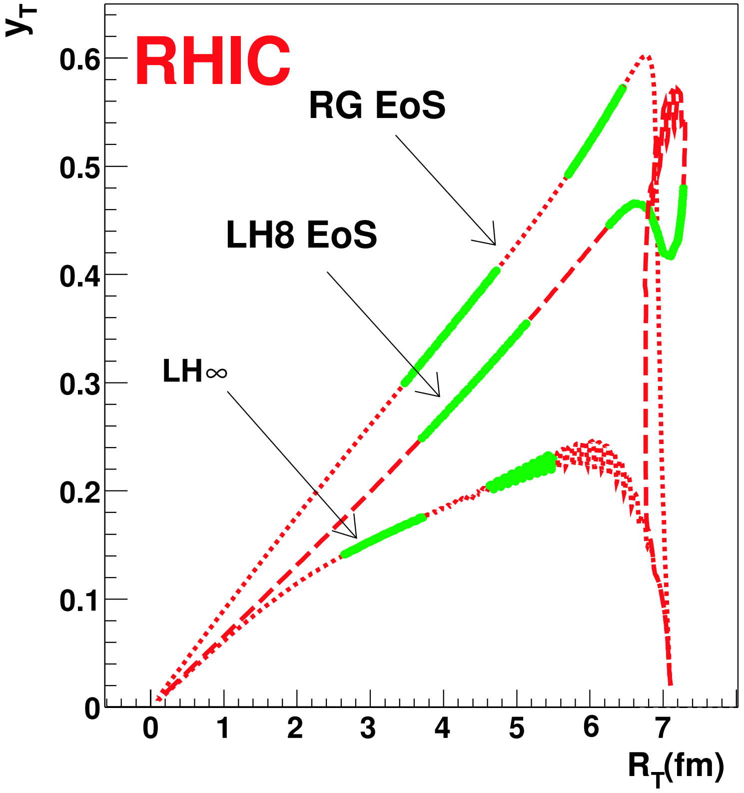
using saddle-point approximations around the minimum of

N.B. & J.-Y. Ollitrault, `nucl-th/0506045`

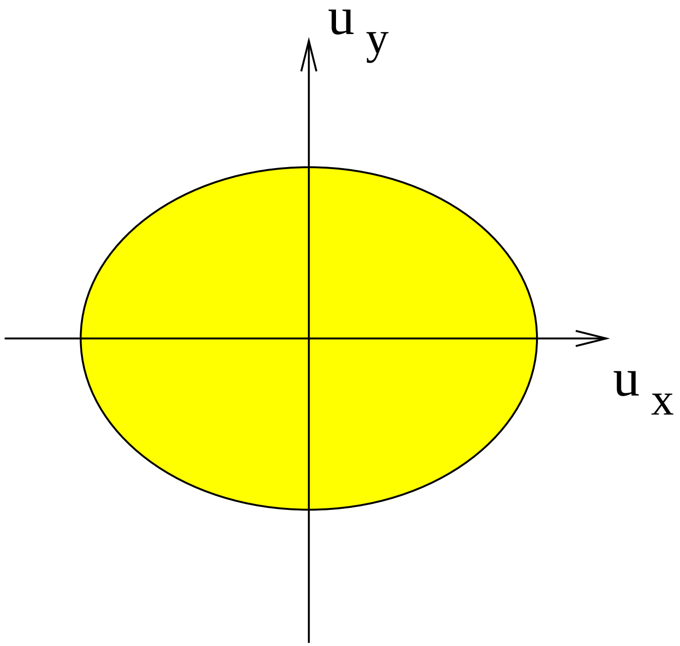
Ideal fluid dynamics: general predictions

Fluid velocity profiles: $u^\mu = \frac{1}{\sqrt{1-\vec{v}^2}}\begin{pmatrix}1\\ \vec{v}\end{pmatrix}$



Kolb & Heinz, `nucl-th/0305084`

Profile in non-central collisions

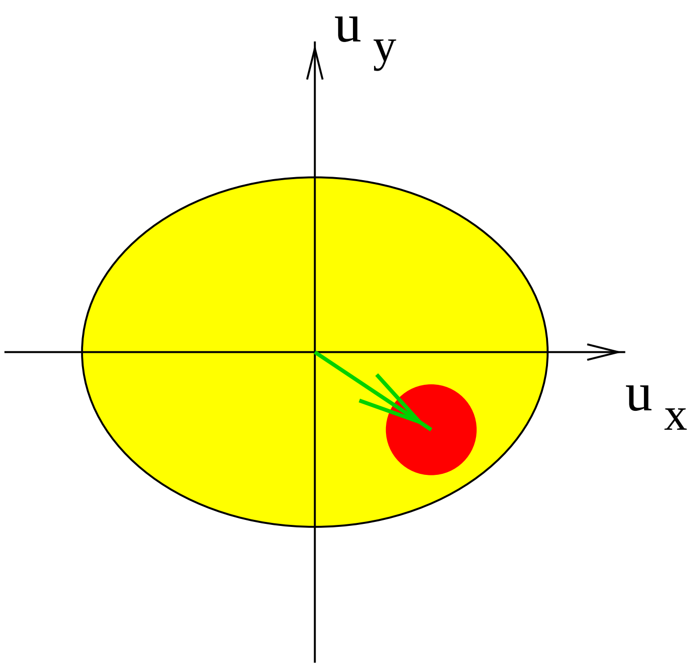


(velocity larger along the direction of impact parameter)

Ideal fluid dynamics: general predictions

Slow particles ($p_t/m < u_{\rm max}(\frac{\pi}{2})$) move together with the fluid

There is a point where the fluid velocity equals the particle velocity



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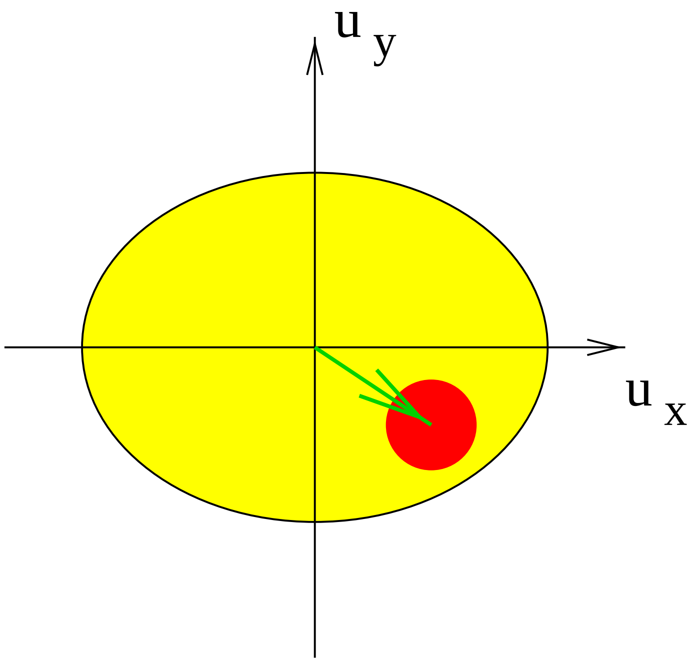

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- Similar spectra for different hadrons, up to normalization constants:

$$E\frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{p}} = c^h(m)\,f\left(\frac{p_t}{m}, y, \phi\right)$$

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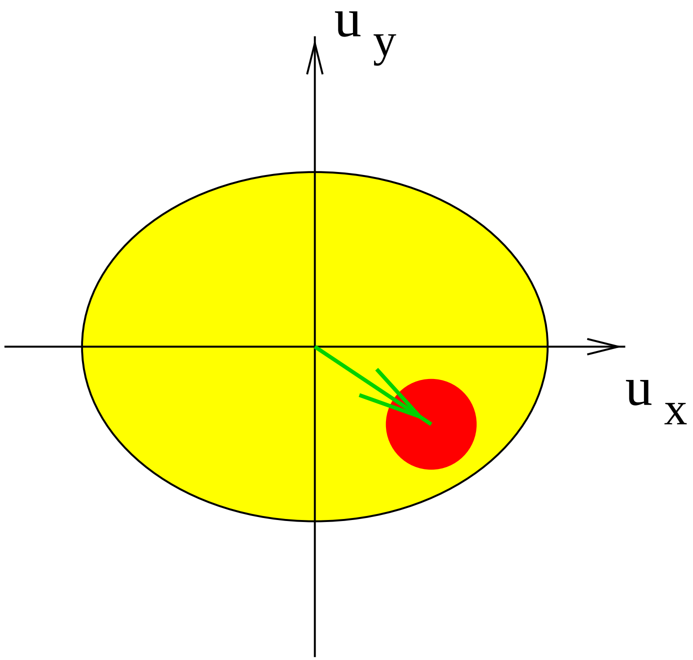

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- $v_n\left(\frac{p_t}{m}, y\right)$ universal!

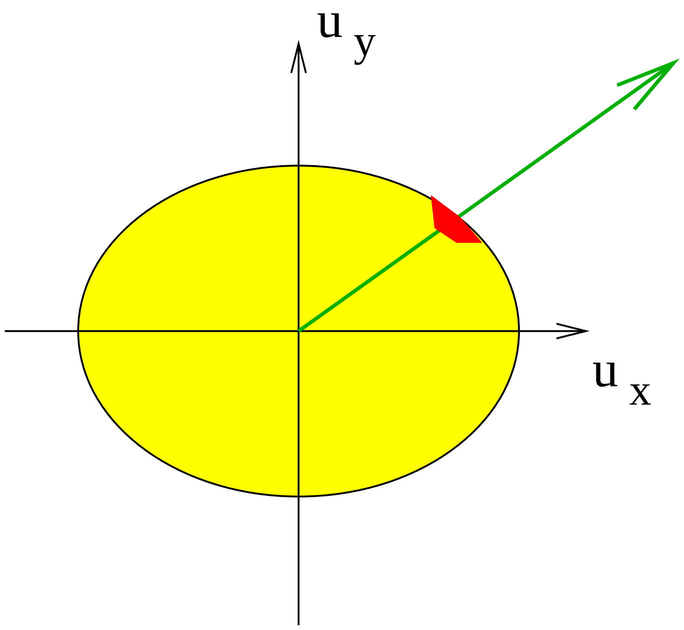
  $\Rightarrow$ mass-ordering of $v_2(p_t, y)$

Calculations valid if $T_{\rm f.o.} \ll m v_{\rm max}^2$ ($\Rightarrow$ not for pions)

Ideal fluid dynamics: general predictions

Fast particles ($p_t/m > u_{\max}(0)$) move faster than the fluid

Particle comes from where the fluid is fastest along the direction of its velocity:



Saddle-point method even more predictive:

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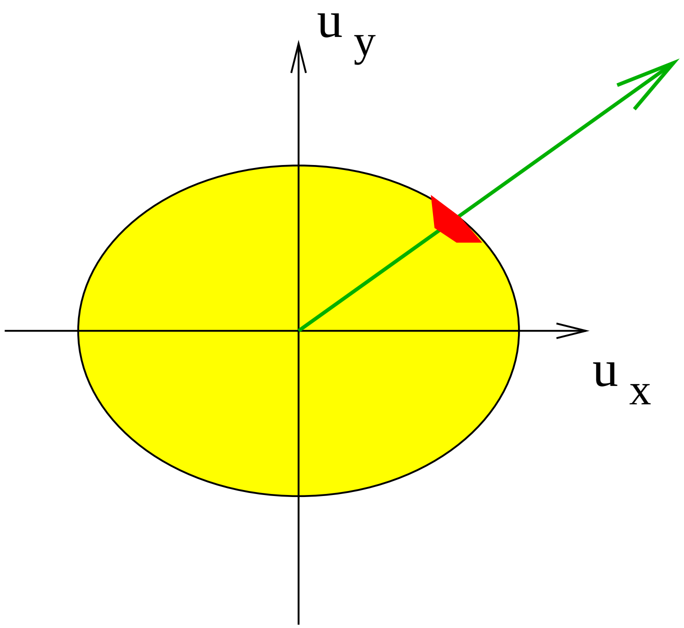

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- $$E\frac{\mathrm{d}N}{\mathrm{d}^2\mathbf{p}_t\,\mathrm{d}y} \propto \frac{1}{\sqrt{p_t - m_t v_{\max}}} \exp\left(\frac{p_t u_{\max} - m_t u^0_{\max}}{T_{\mathrm{f.o.}}}\right)$$

p_t-dependent slopes of m_t spectra

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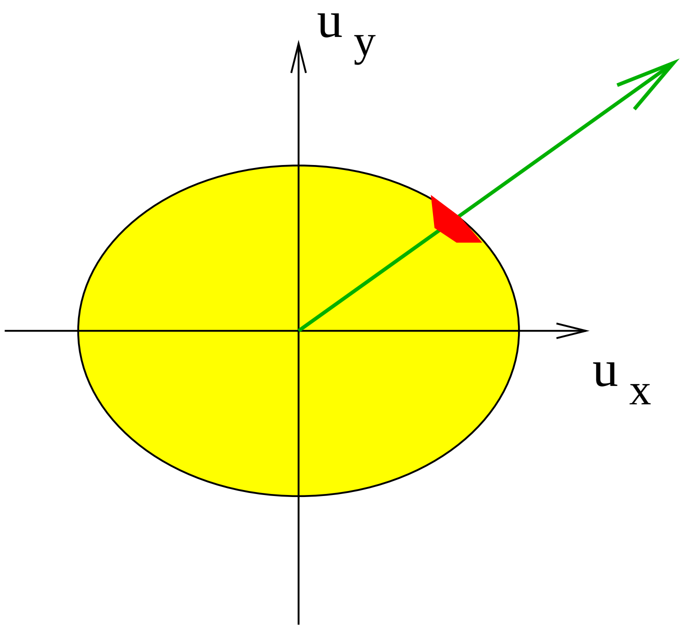

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  p_t-dependent slopes of m_t spectra

- $$v_2(p_t) \propto \frac{u_{\max}}{T_{\text{f.o.}}}(p_t - m_t v_{\max})$$

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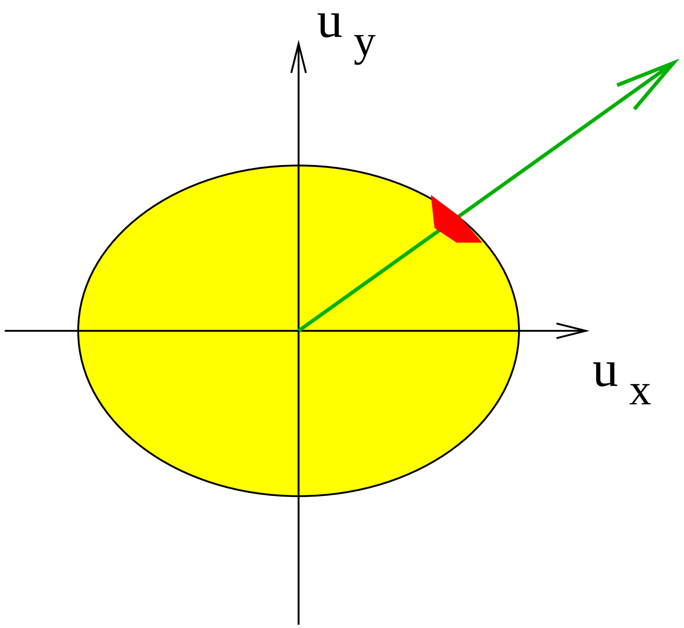

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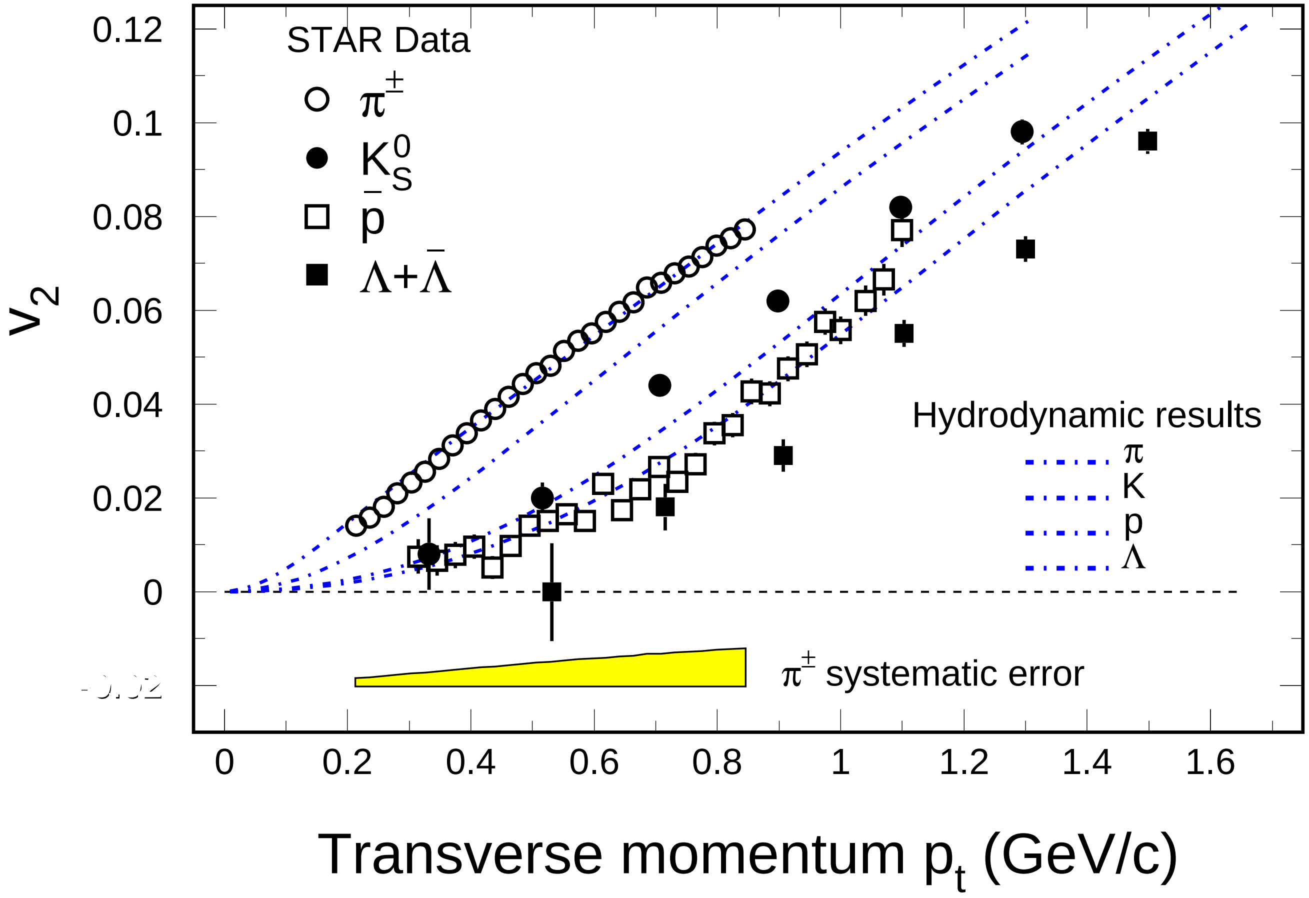
- $$v_2(p_t) \propto \frac{u_{\rm max}}{T_{\rm f.o.}}(p_t - m_t v_{\rm max})$$
  $\Rightarrow$ mass-ordering of $v_2(p_t)$
- $$v_4(p_t) = \frac{v_2(p_t)^2}{2}$$

RHIC data: a personal choice [1/5]

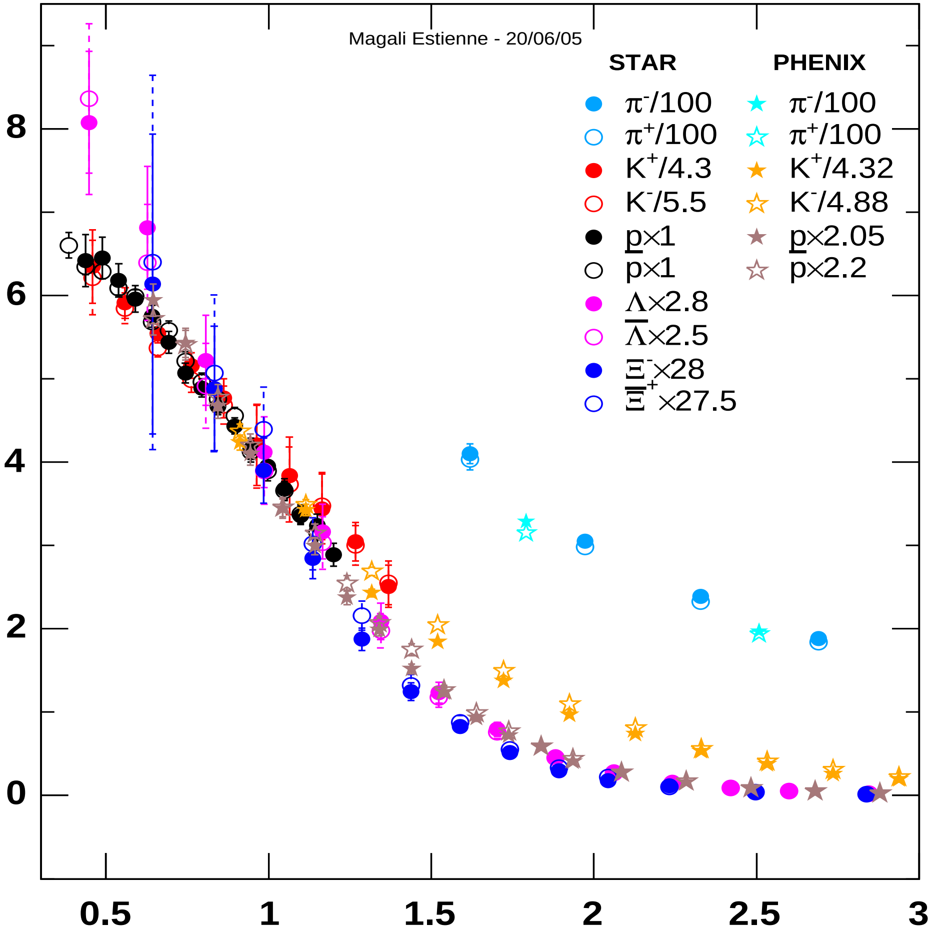
$v_2(p_t)$ at midrapidity, minimum bias collisions:

STAR Collaboration, `nucl-ex/0409033`



RHIC data: a personal choice [2/5]

p_t spectra
at midrapidity
vs. p_t/m



Magali Estienne
private communication

All particles (except pions) with $\frac{p_t}{m} \lesssim \underline{1.2}$ flow with the same velocity!

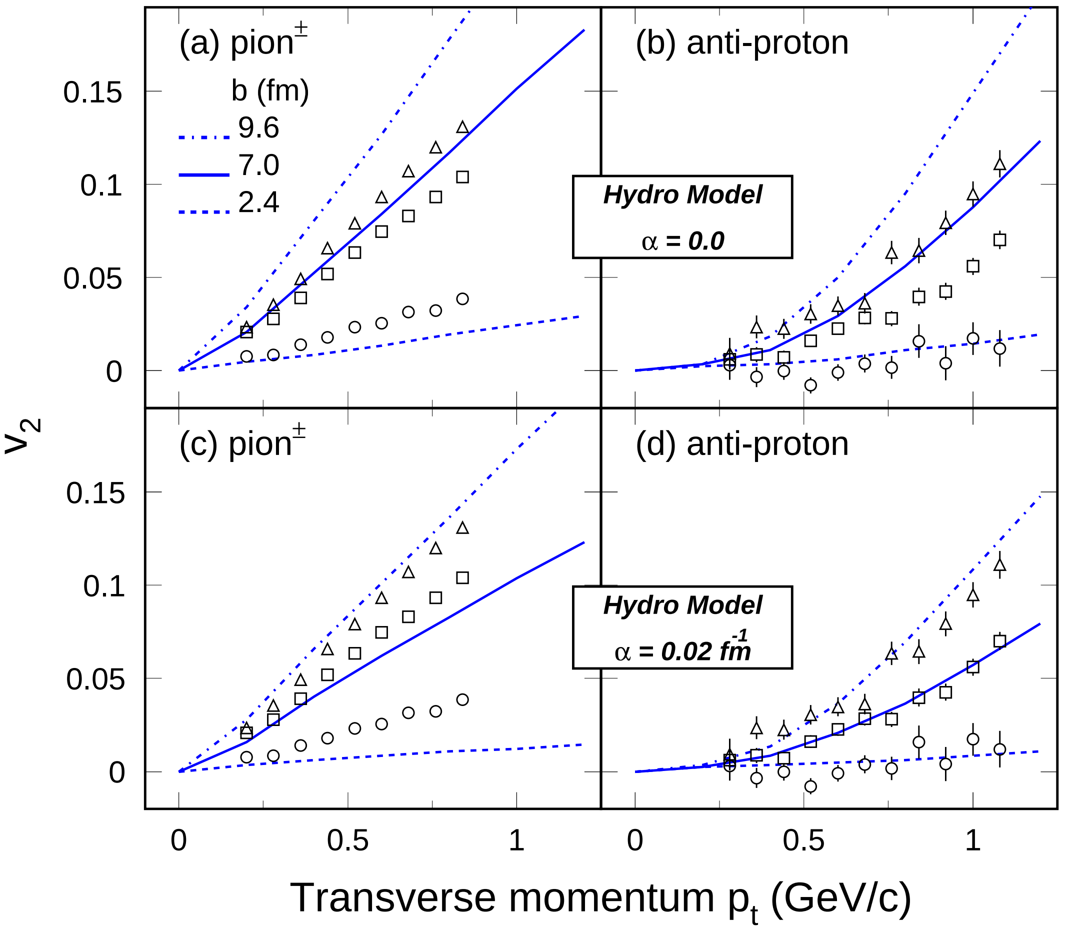
slow

$\simeq u_{\max}$

RHIC data:
a personal choice [3/5]

$v_2(p_t)$ for various centralities (impact parameters):

STAR Collaboration, `nucl-ex/0409033`



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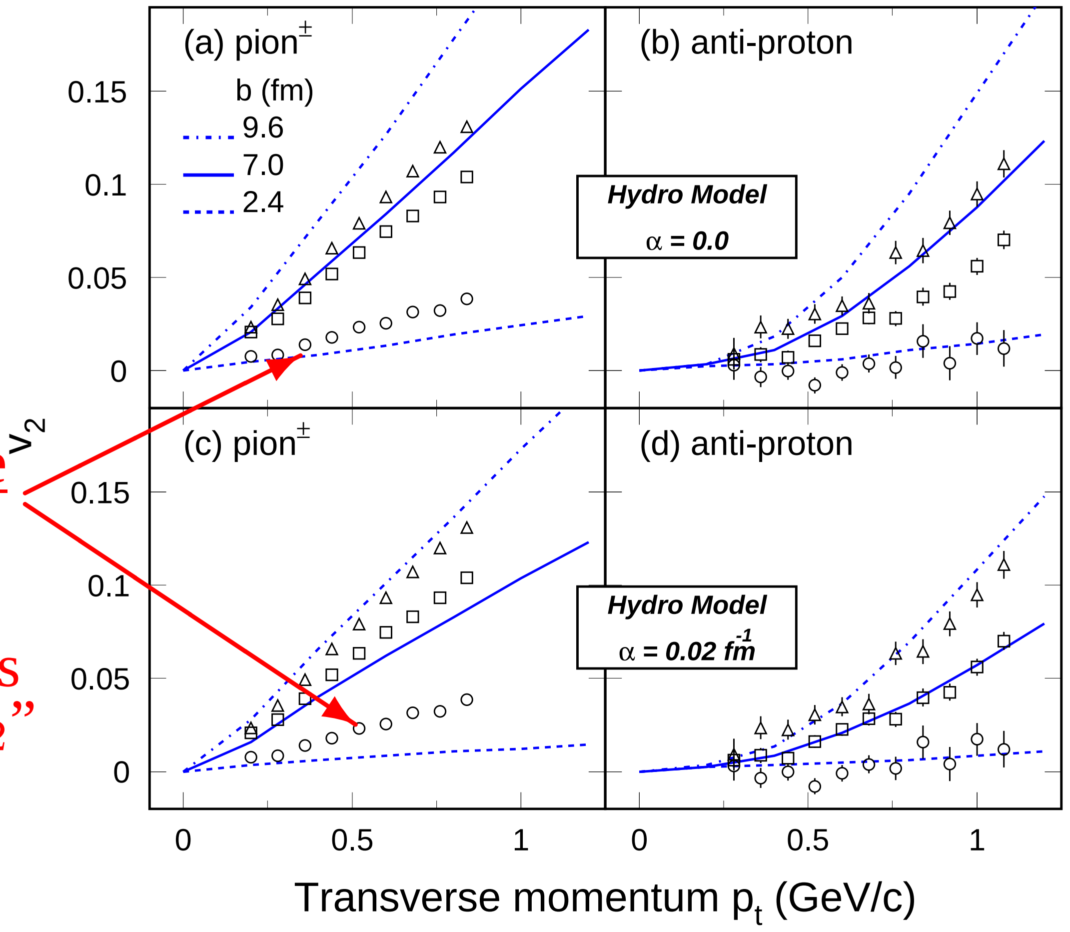


data <u>above</u>
ideal fluid

$\neq$

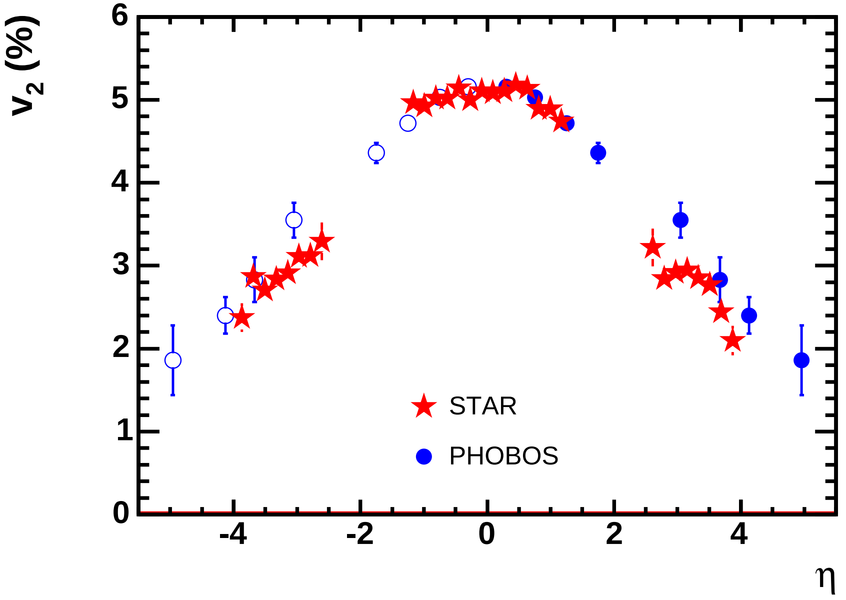
“hydro yields
maximum v_2”

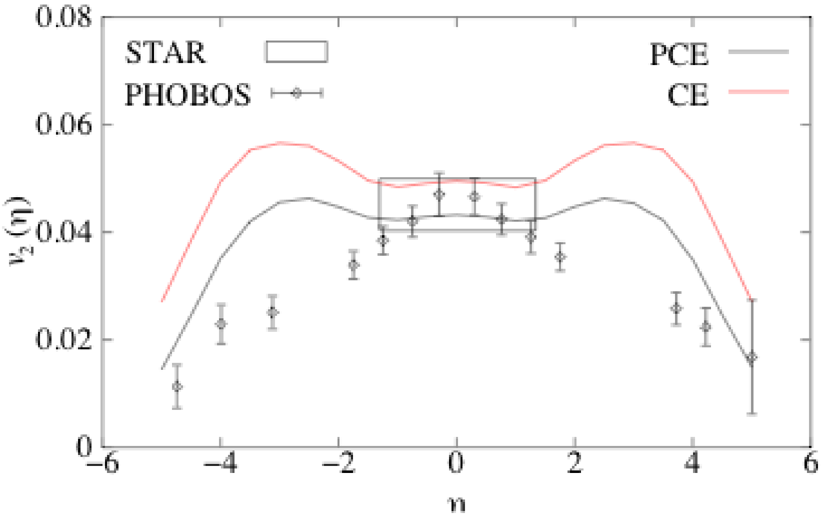
RHIC data: a personal choice [4/5]

(Pseudo)rapidity dependence of v_2

STAR Collaboration,
`nucl-ex/0409033`

Hirano & Tsuda,
Phys. Rev. C **66** (2002) 054905



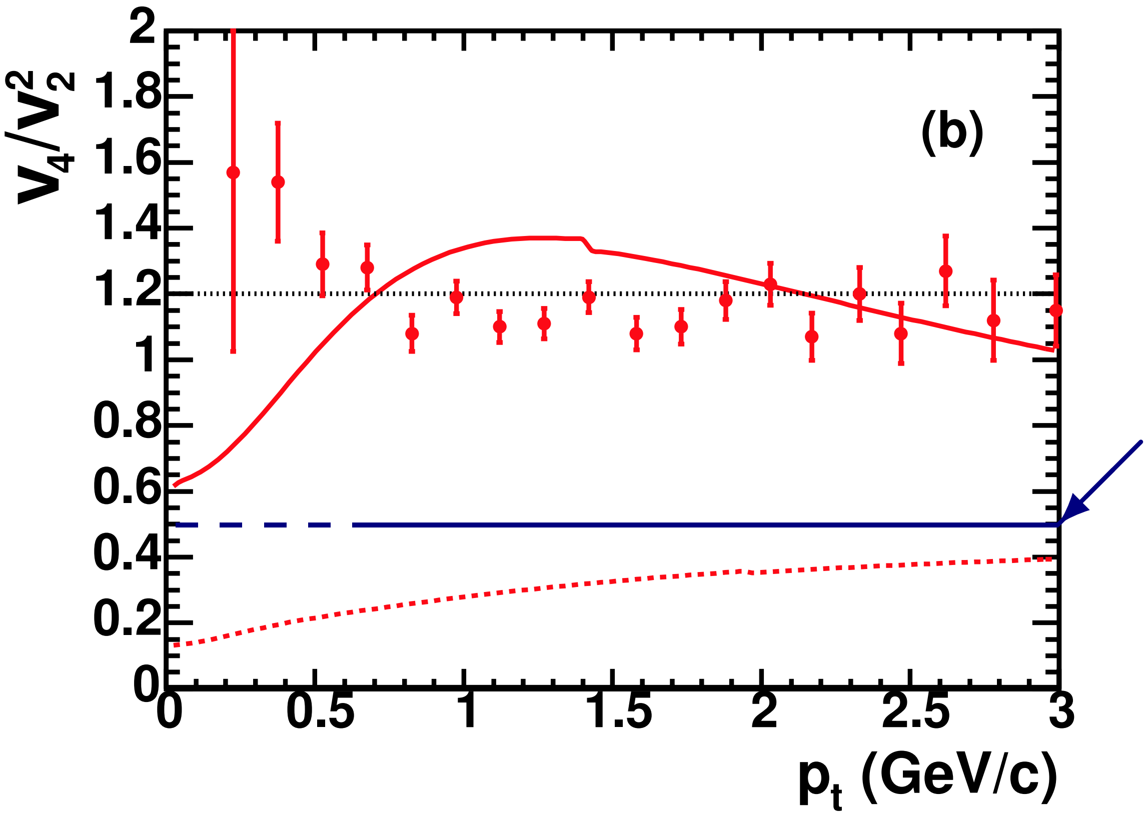


v_2(hydro) flatter than data

RHIC data: a personal choice [5/5]

Transverse momentum dependence of $\frac{v_4}{(v_2)^2}$

STAR Collaboration, `nucl-ex/0409033`



ideal fluid prediction

Ideal fluid dynamics vs. RHIC data

$$\left.\begin{array}{l} \spadesuit\ v_2(p_t)\ \text{hydro} \ < \ \text{data} \\ \spadesuit\ v_2(y)\ \text{hydro} \ \neq \ \text{data} \\ \spadesuit\ \dfrac{v_4}{(v_2)^2}\ \text{hydro} \ < \ \text{data} \end{array}\right\} \text{is the ideal fluid assumption valid?}$$

Ideal fluid dynamics vs. RHIC data

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Is this really true?

What are the length scales in the system at time τ_0?

Heavy ion collisions: length scales

At time τ_0, two possible choices for the system size L which enters Kn

- $L = c\tau_0$ longitudinal size (strong Lorentz contraction!)

Heavy ion collisions: length scales

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- $L = c\tau_0$ longitudinal size (strong Lorentz contraction!)
- $L = \bar{R}$ transverse size ($\bar{R}$ "reduced" radius, $\frac{1}{\bar{R}} = \sqrt{\frac{1}{\sigma_x^2} + \frac{1}{\sigma_y^2}}$)

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At short times, $\tau_0 \lesssim 1$ fm/c, there are several possibilities:

1. $\lambda \ll c\tau_0$: early thermalization (preferred by most?)
2. $\lambda \sim c\tau_0$
3. $c\tau_0 \ll \lambda \ll \bar{R}$: only "transverse" thermalization
4. $\lambda \sim \bar{R}$
5. $\lambda \gg \bar{R}$: "initial state" dominates

☞ $\left\{ \begin{array}{l} \text{Anisotropic flow cannot resolve 1–3} \\ \text{RHIC data favor 4} \end{array} \right.$

Dependence of v_2 on centrality

The natural time scale for v_2 is $\bar{R}/c_s$:

massless particles
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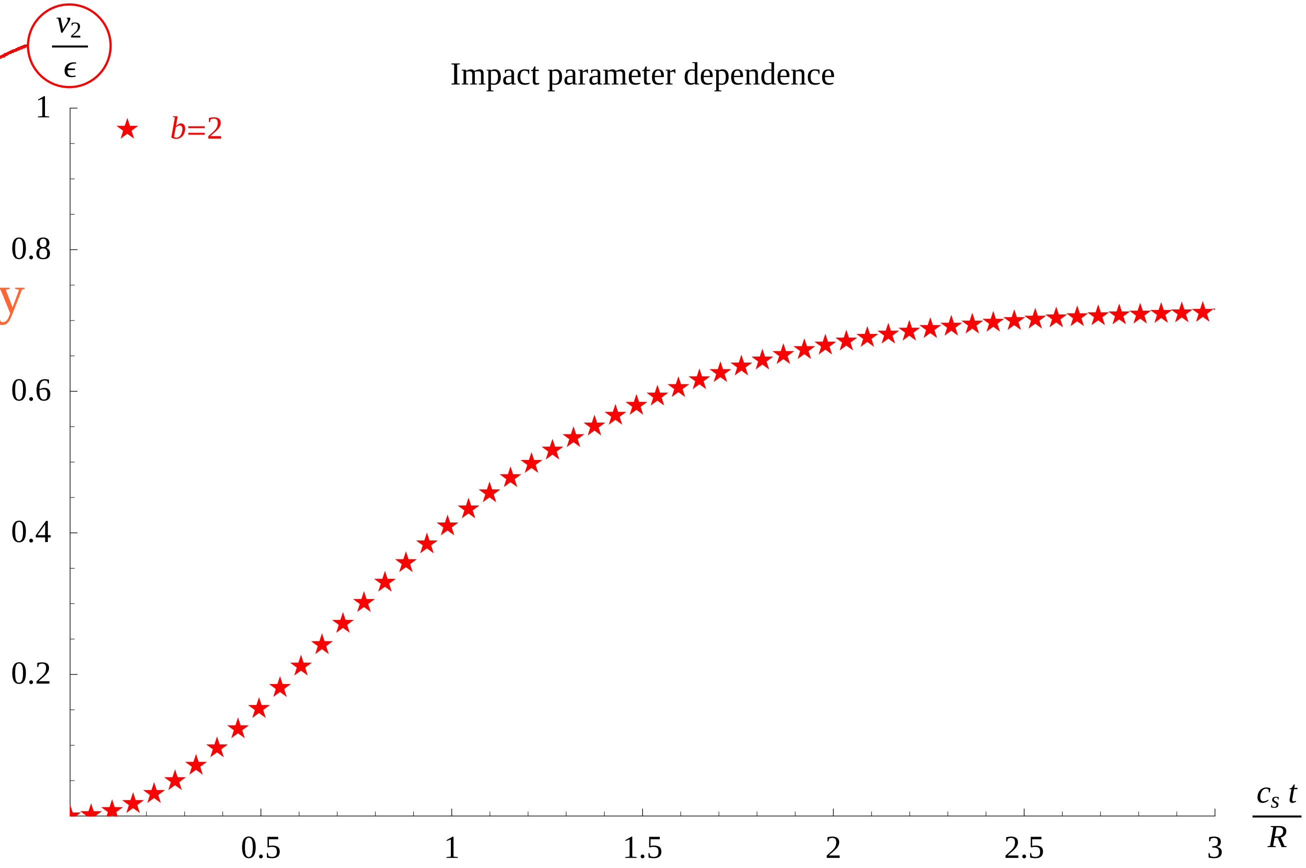

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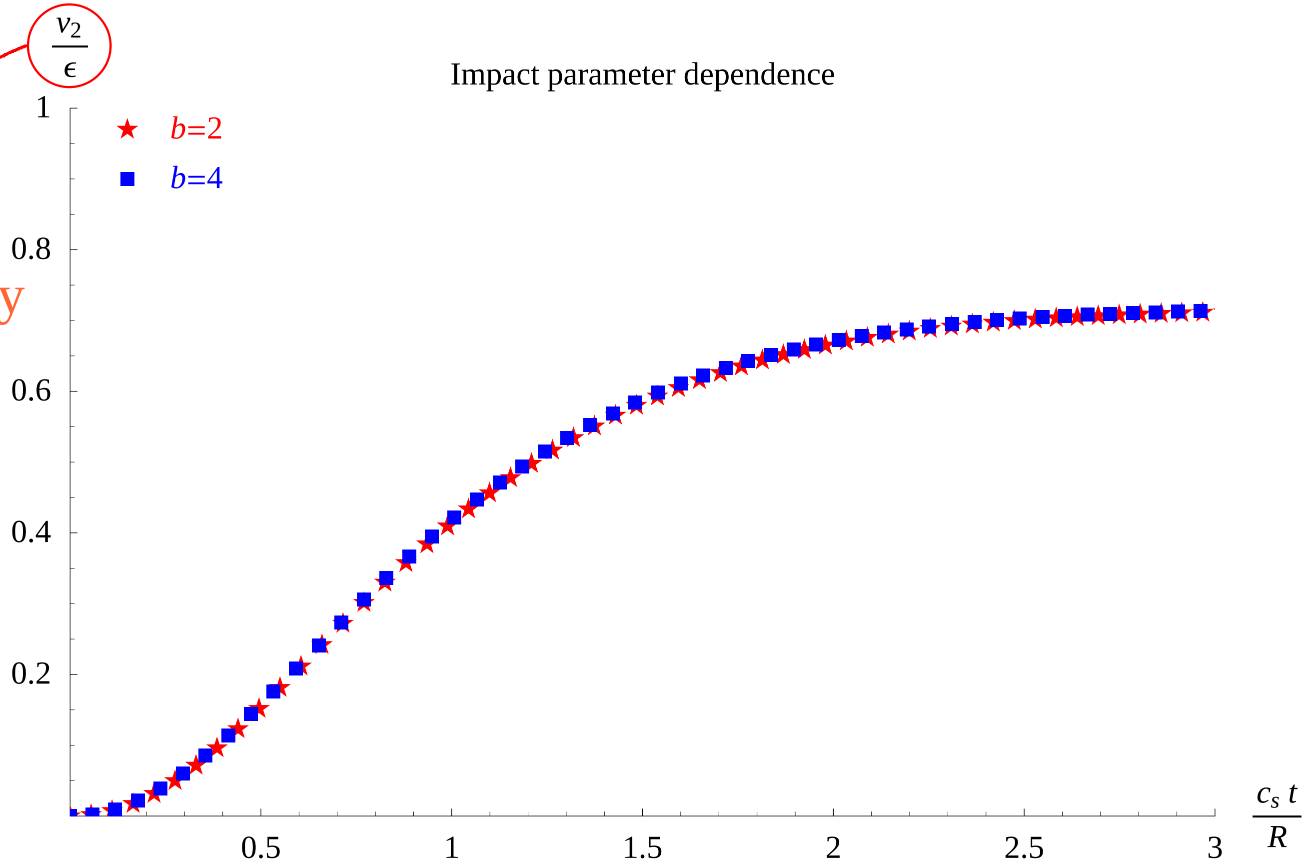

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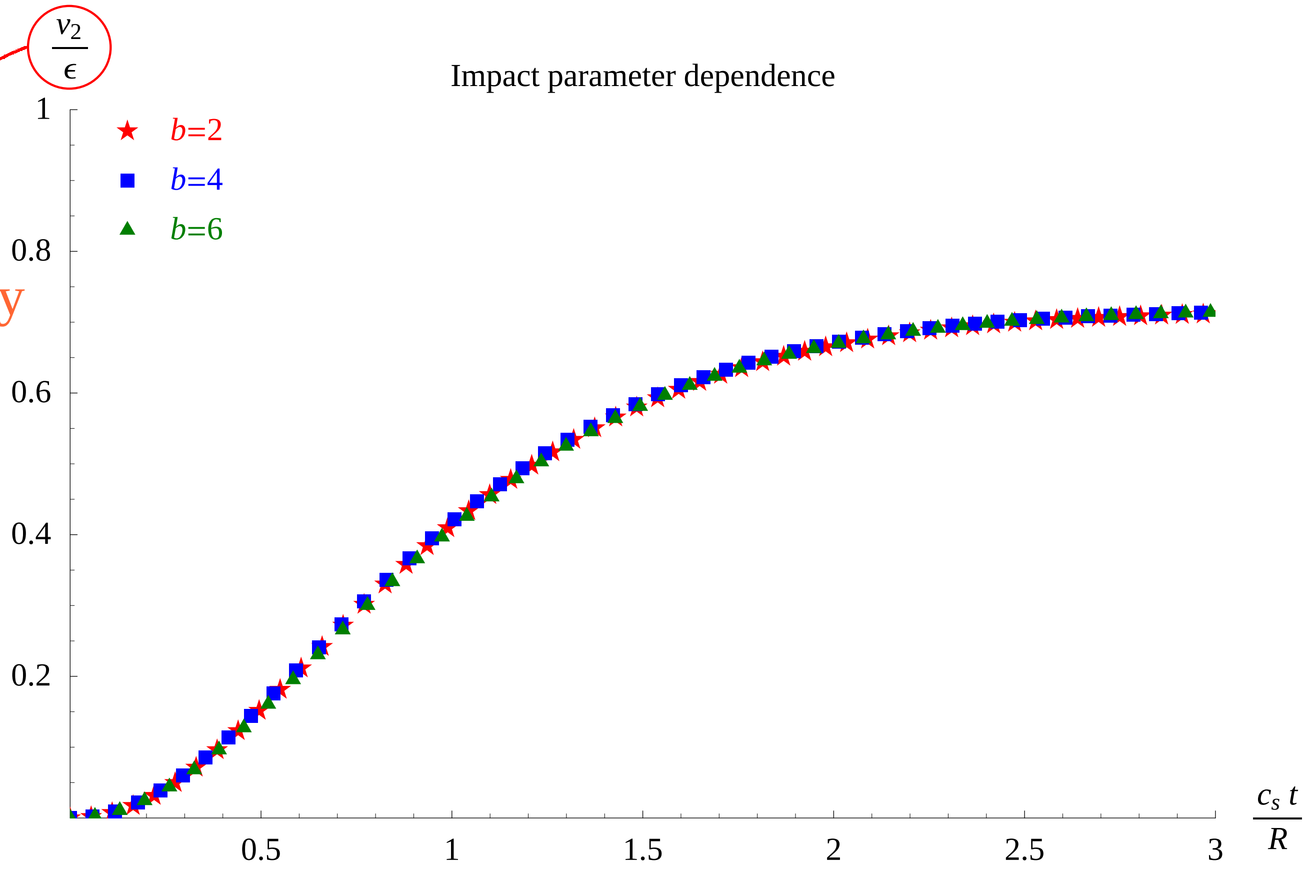

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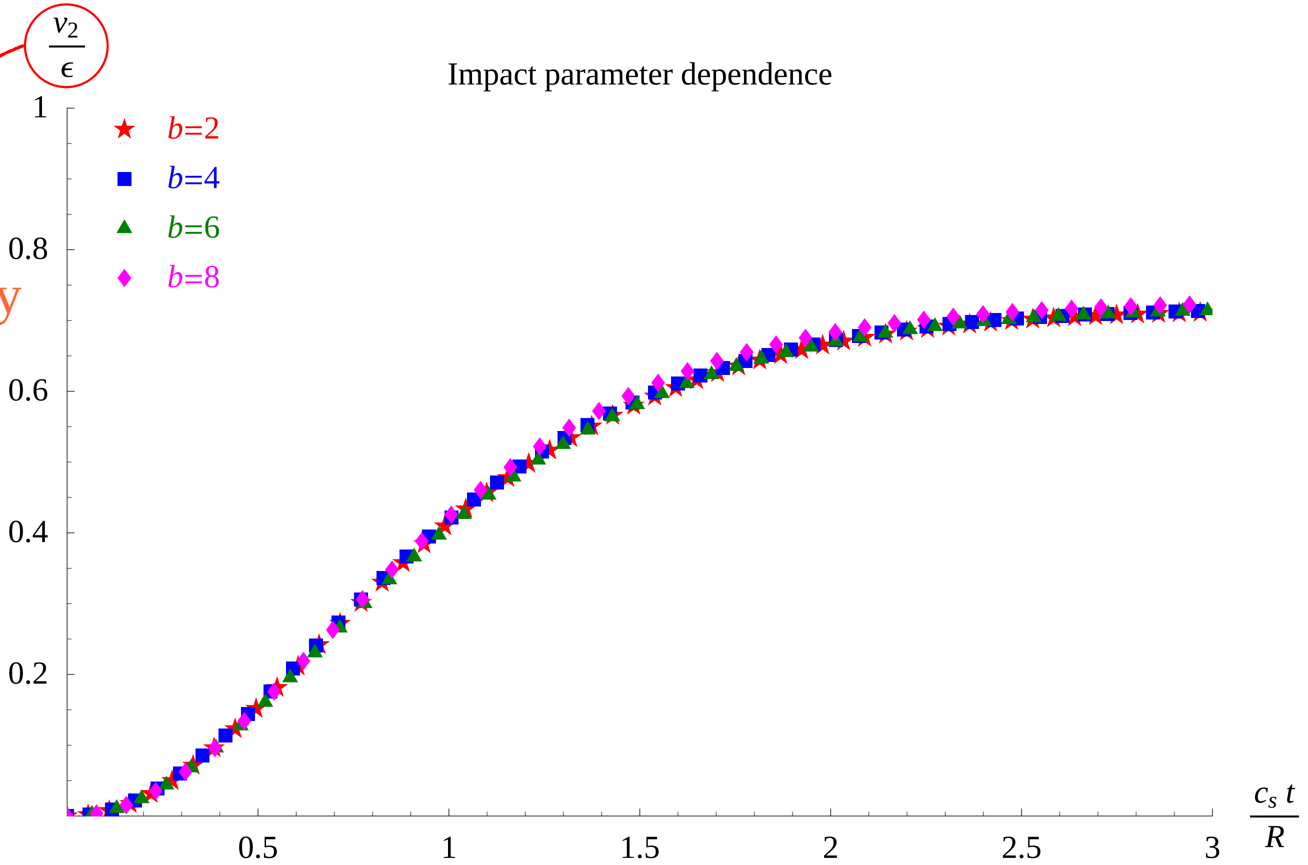

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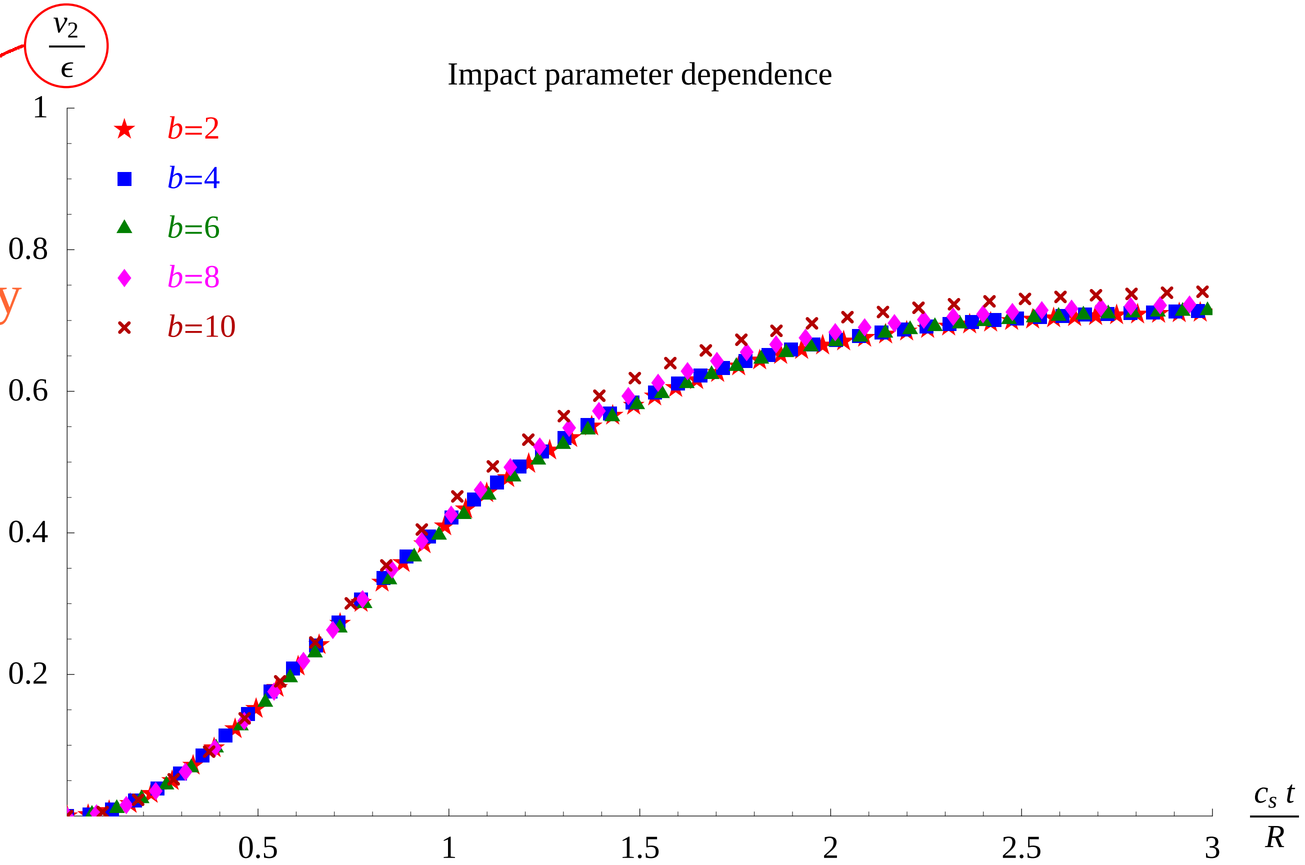

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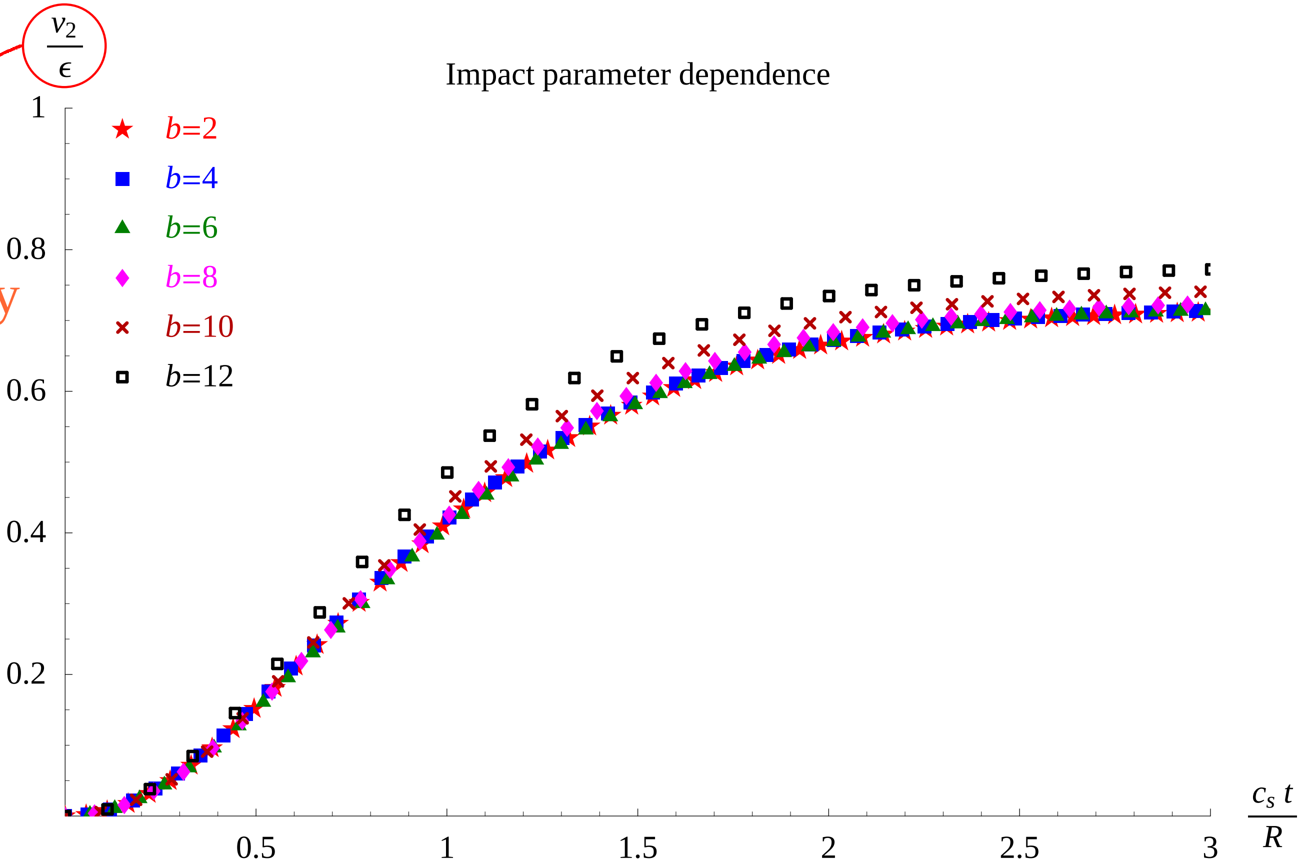


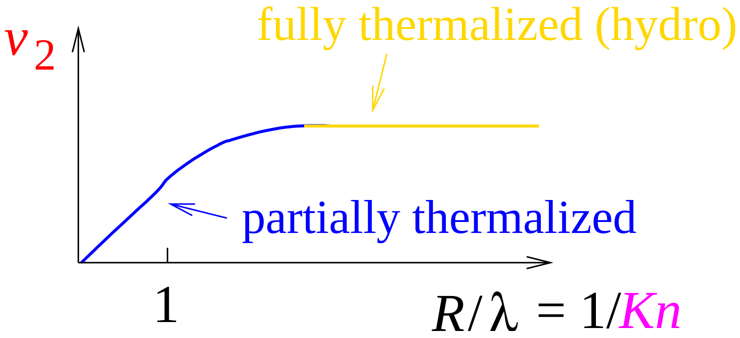
v_2 knows nothing about early times!

Anisotropic flow: a control parameter

The natural time scale for v_2 is $\bar{R}/c_s$

$\Rightarrow$ number of collisions to build up v_2:

$$\frac{1}{Kn} \simeq \frac{\bar{R}}{\lambda} = \bar{R}\sigma\, n\left(\frac{\bar{R}}{c_s}\right) \simeq \frac{c_s}{c}\,\frac{\sigma}{S}\,\frac{\mathrm{d}N}{\mathrm{d}y}$$



σ interaction cross section, $n(\tau)$ particle density, S transverse surface

System NOT thermalized $\Leftrightarrow$ $v_2 \propto 1/Kn$

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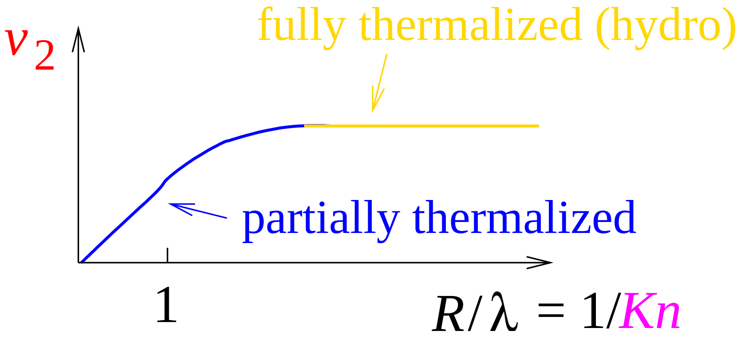

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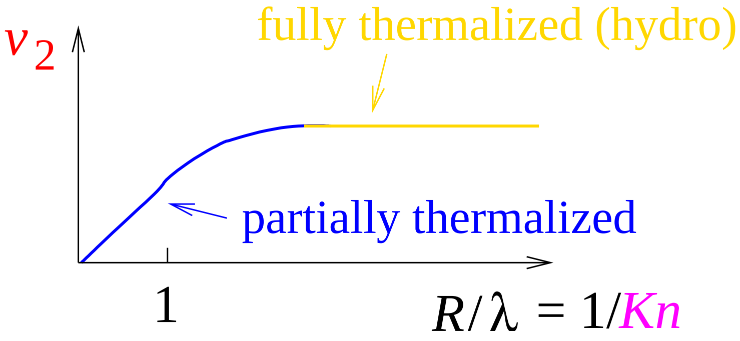

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☞ $\frac{1}{S}\frac{\mathrm{d}N}{\mathrm{d}y}$ control parameter for v_2: to vary Kn, one can study

- centrality dependence (using the universality of v_2/ϵ)
- beam-energy dependence
- system-size dependence → importance of lighter systems!
- rapidity dependence

Control parameter: centrality dependence

The number of collisions to build up v_2 is $\dfrac{1}{Kn} \simeq \bar{R}\sigma\, n\left(\dfrac{\bar{R}}{c_s}\right) \propto \dfrac{\sigma}{S}\dfrac{\mathrm{d}N}{\mathrm{d}y}$

In Au–Au collisions at RHIC:

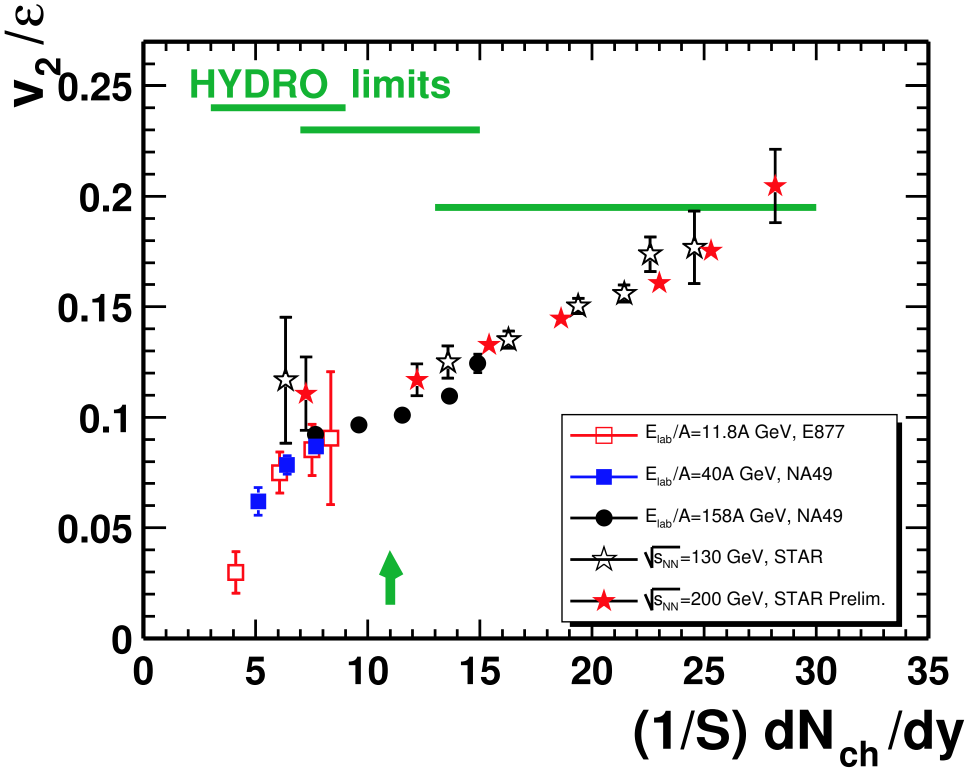
b	$\bar{R}$ (fm)	$\dfrac{\mathrm{d}N}{\mathrm{d}y}$	$n\left(\dfrac{\bar{R}}{c_s}\right)$ (fm^{-3})
0	2.07	1050	5.4
2	2.02	975	5.4
4	1.89	790	5.5
6	1.68	562	5.3
8	1.45	344	4.9
10	1.22	167	3.8

$n\left(\dfrac{\bar{R}}{c_s}\right)$, hence λ, varies little for $b = 0$–8 fm, while $\bar{R}$ varies by 30%

☞ centrality-dependence of $\dfrac{v_2}{\epsilon} \Leftrightarrow \dfrac{1}{S}\dfrac{\mathrm{d}N}{\mathrm{d}y}$-dependence

Anisotropic flow: incomplete thermalization

Centrality and beam-energy dependence:



NA49 Collaboration, Phys. Rev. C **68** (2003) 034903

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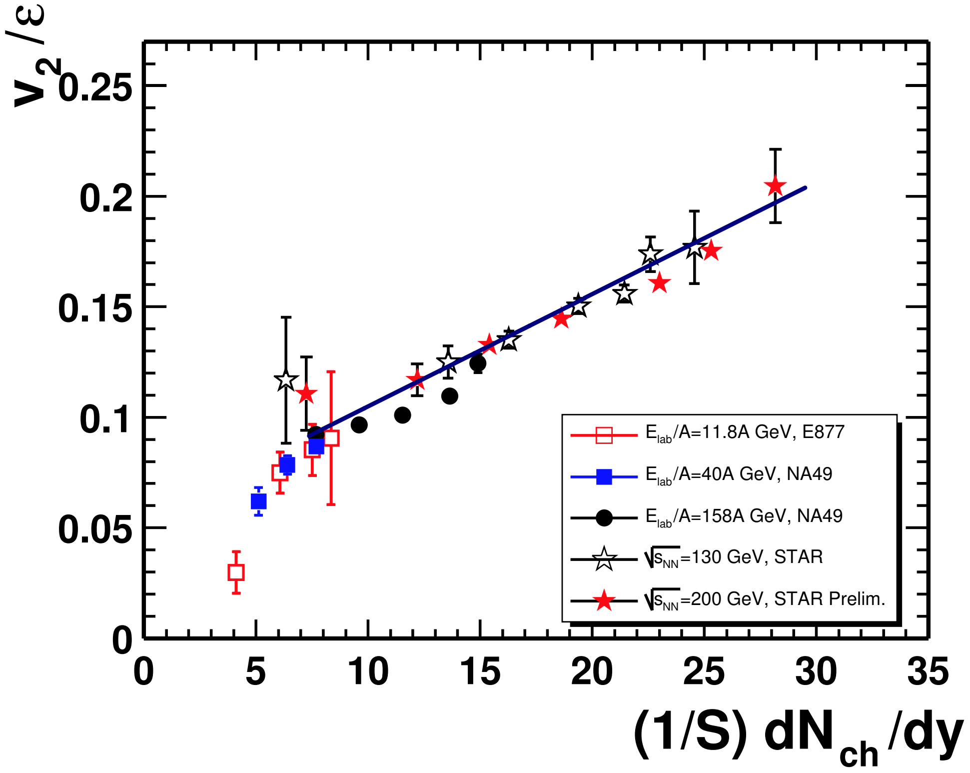


NA49 Collaboration, Phys. Rev. C **68** (2003) 034903

Scaling law seems to work for RHIC data (+ matching with SPS)

Data alone do not point to a saturation of v_2

Anisotropic flow: predictions for Cu–Cu

The matching between central SPS and peripheral RHIC suggests that we can even compare systems with different densities, i.e., different σ

☞ we can compare Au–Au at $b = 8$ fm with Cu–Cu at $b = 5.5$ fm (similar centrality)

- If hydro holds, v_2 should scale like ϵ:

$$v_2(\mathrm{Cu}) = 0.69\, v_2(\mathrm{Au})$$

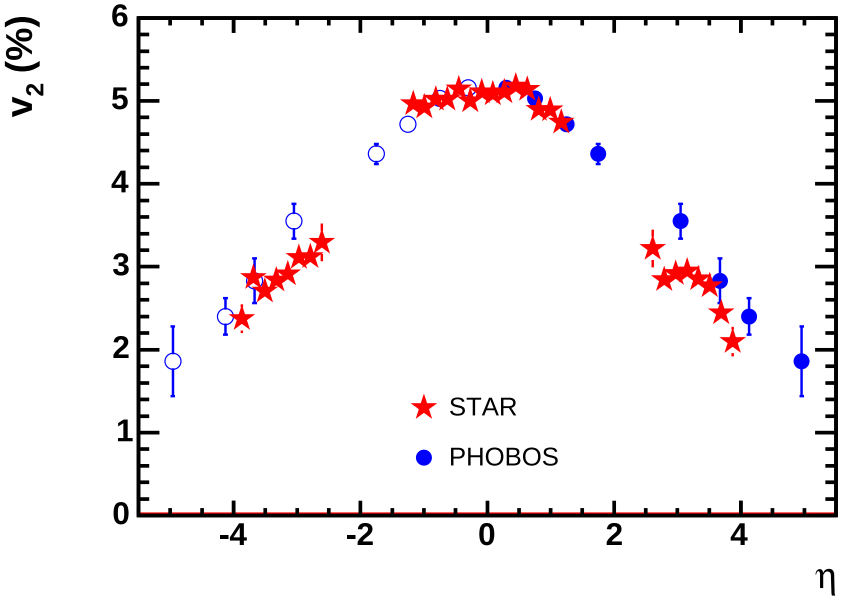
- If thermalization is incomplete, $\dfrac{v_2}{\epsilon}$ should scale like $\dfrac{1}{S}\dfrac{\mathrm{d}N}{\mathrm{d}y}$, i.e.

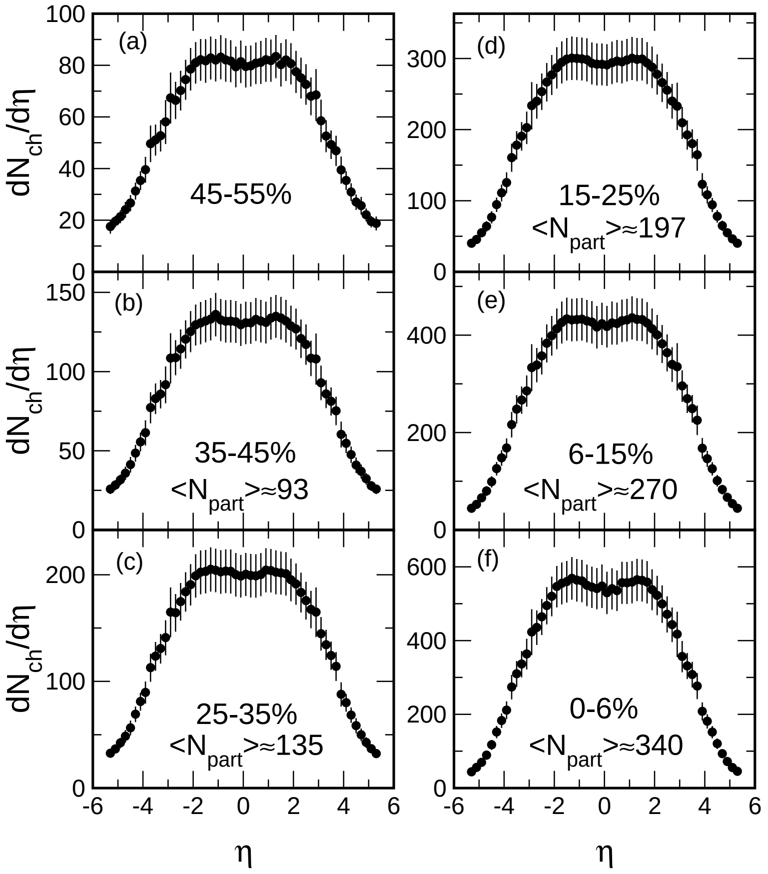
$$v_2(\mathrm{Cu}) = 0.34\, v_2(\mathrm{Au})$$

RHIC data: incomplete thermalization

(Pseudo)rapidity dependence of v_2

STAR Collaboration,
`nucl-ex/0409033`





☞ can be explained by incomplete thermalization

Hirano, Phys. Rev. C **65** (2002) 011901

RHIC data:
incomplete thermalization

Ideal fluid dynamics predicts $\dfrac{v_4}{(v_2)^2} = \dfrac{1}{2}$, RHIC data are above ($\sim 1.2$)

RHIC data: incomplete thermalization

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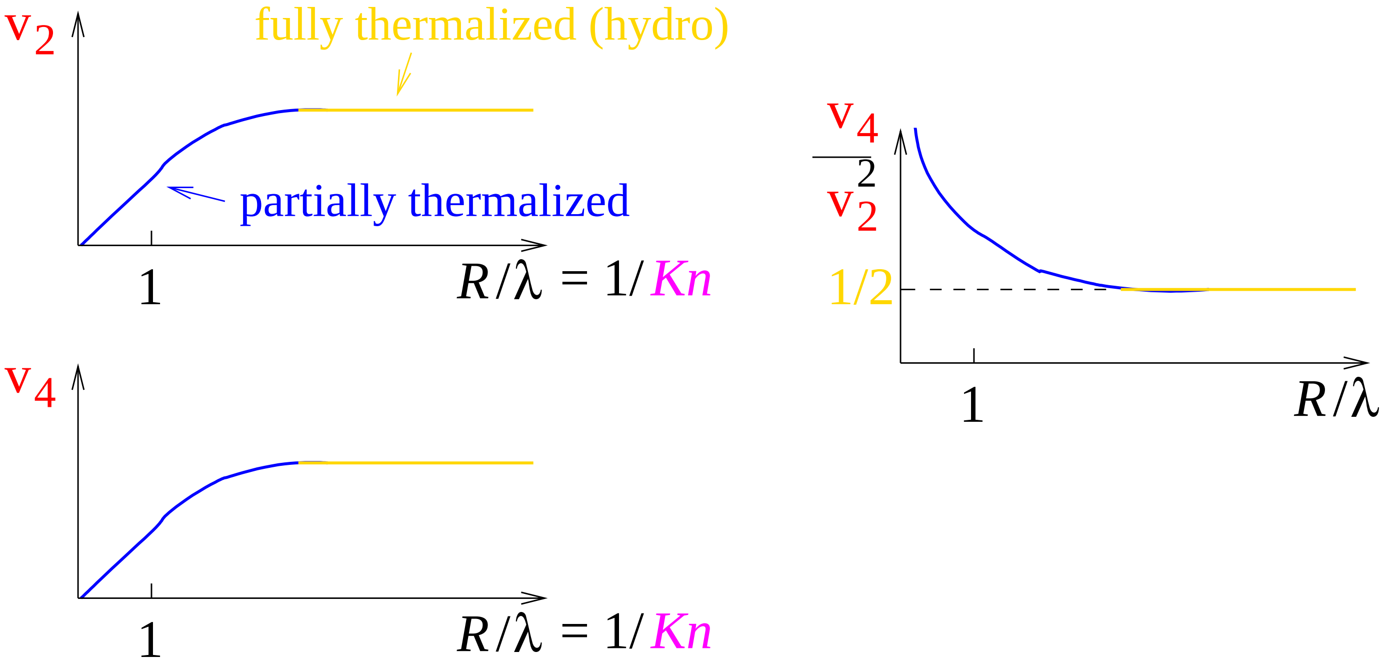
☞ increase can be explained by incomplete thermalization naturally:

v_n proportional to the number of collisions $\frac{1}{Kn}$ $\Rightarrow$ $\frac{v_4}{(v_2)^2} \propto Kn$



Dependence of v_2 on the speed of sound

How can data overshoot the “ideal fluid limit”?

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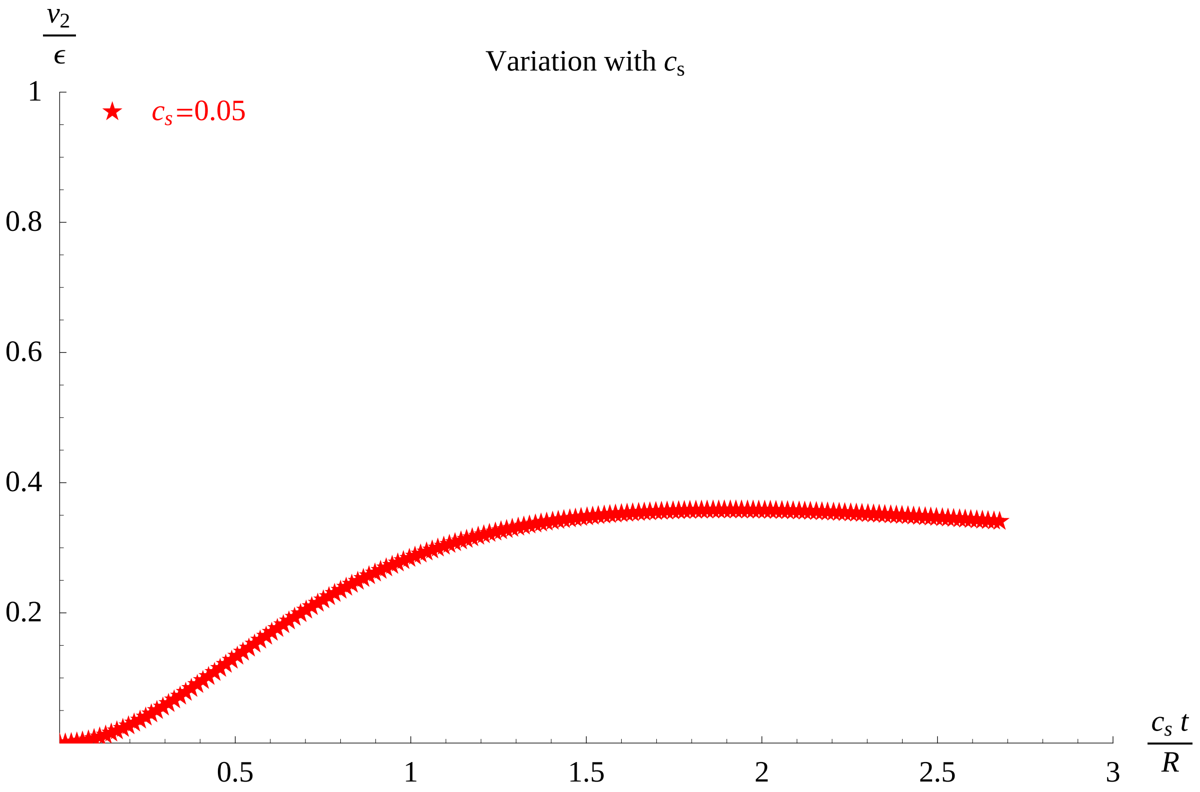

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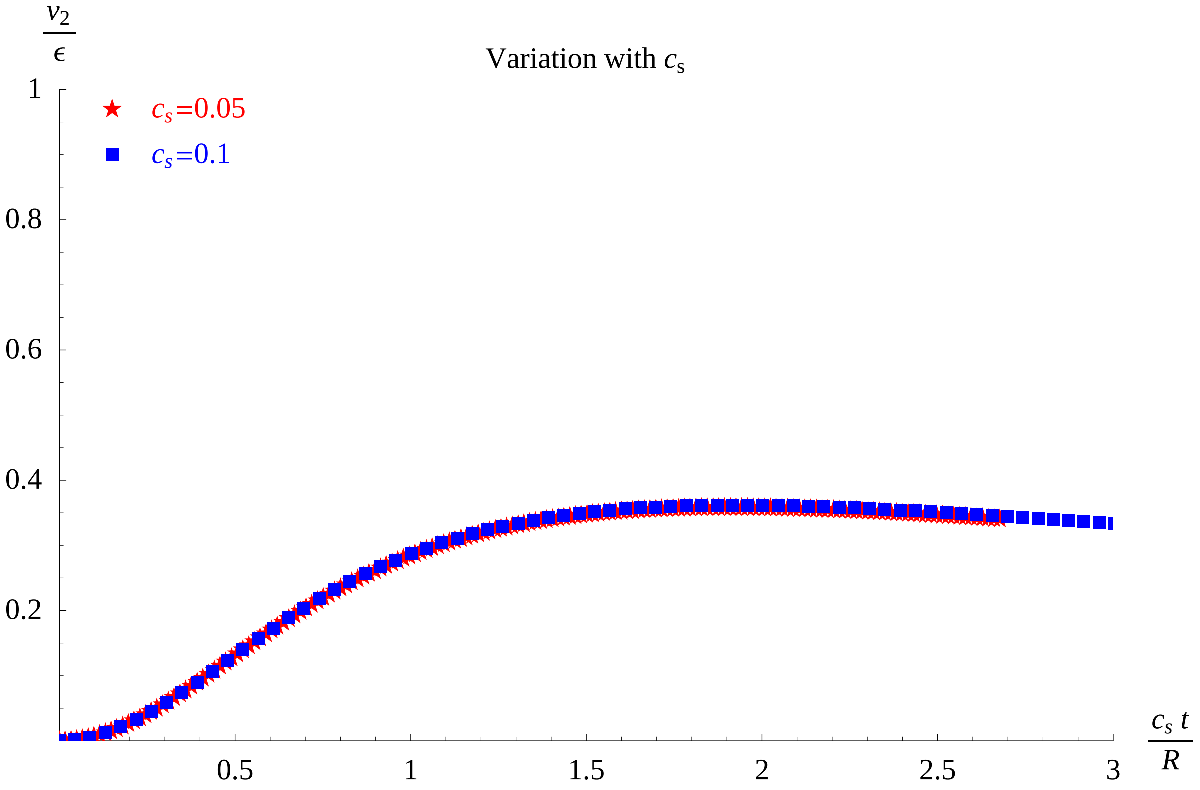

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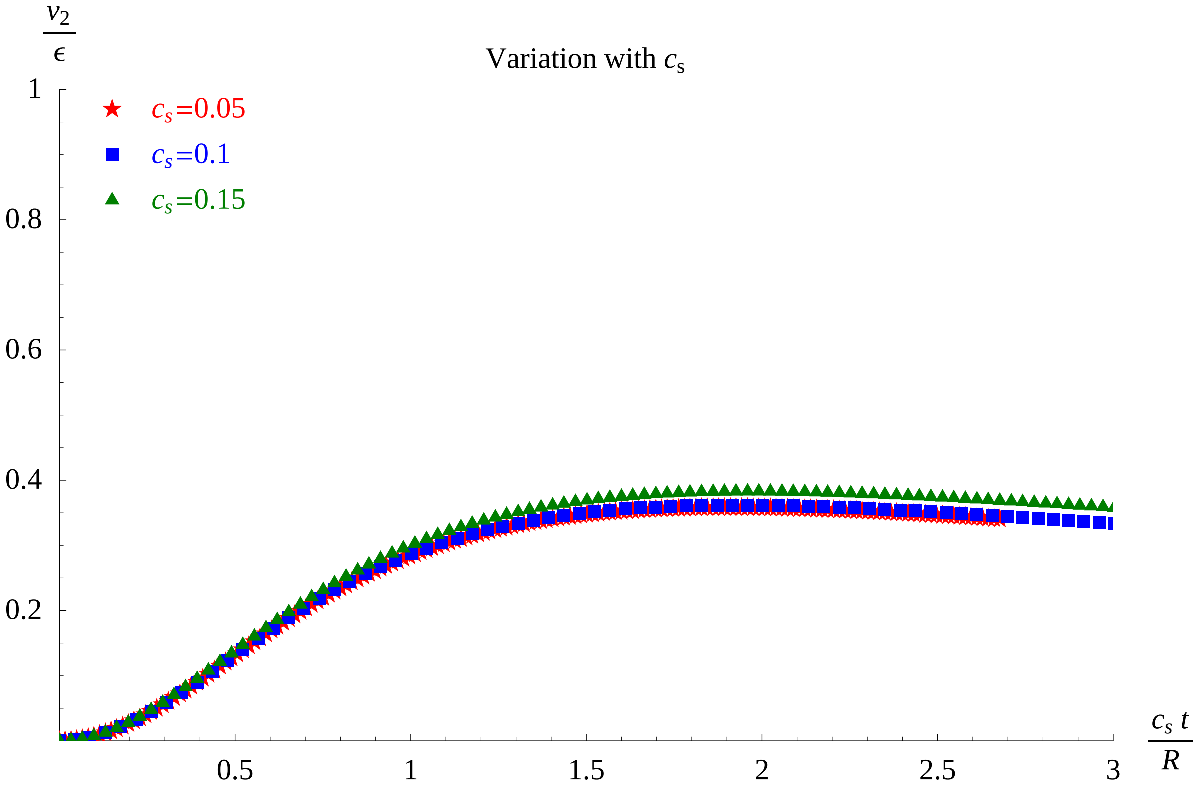

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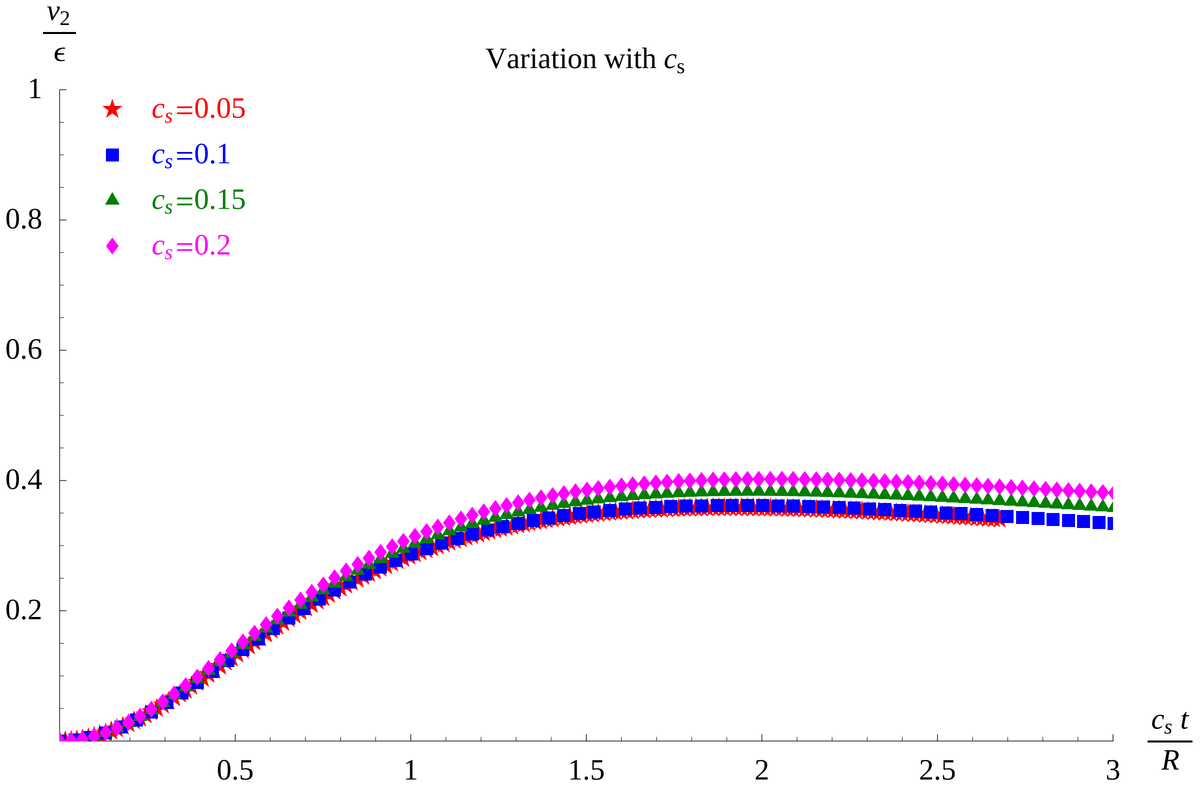

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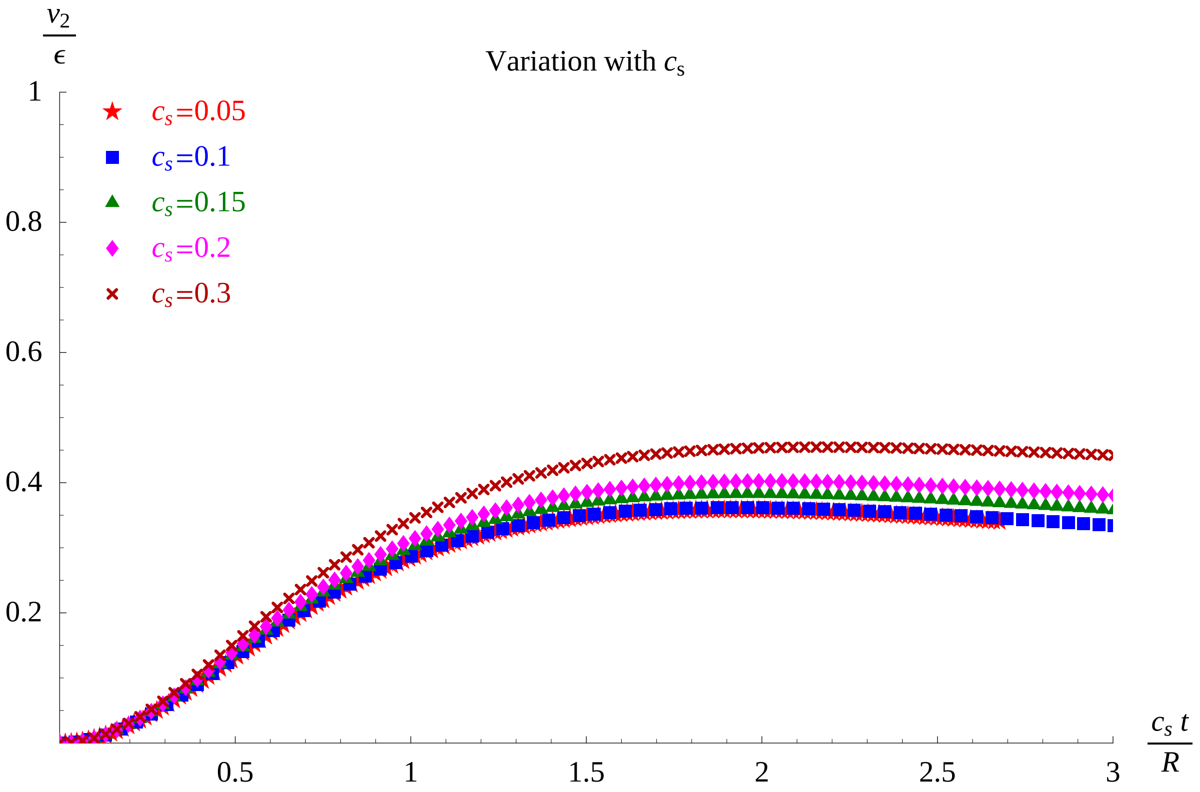

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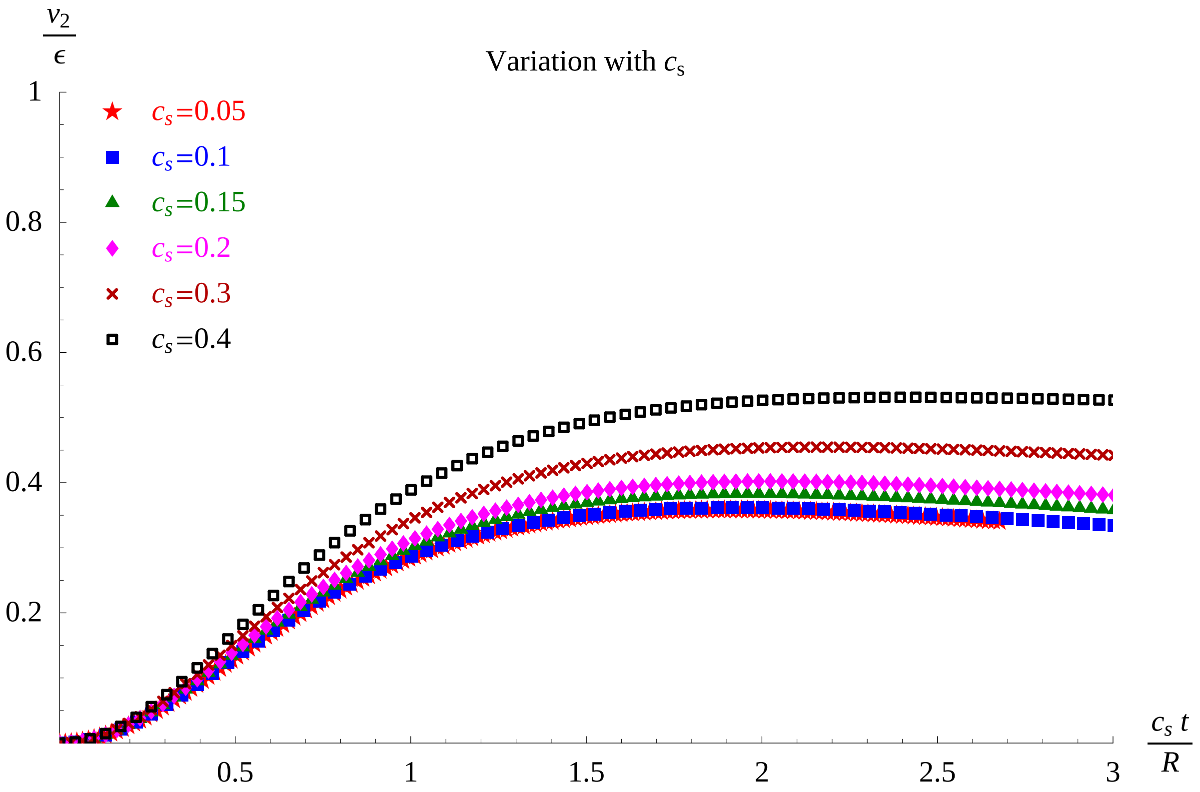

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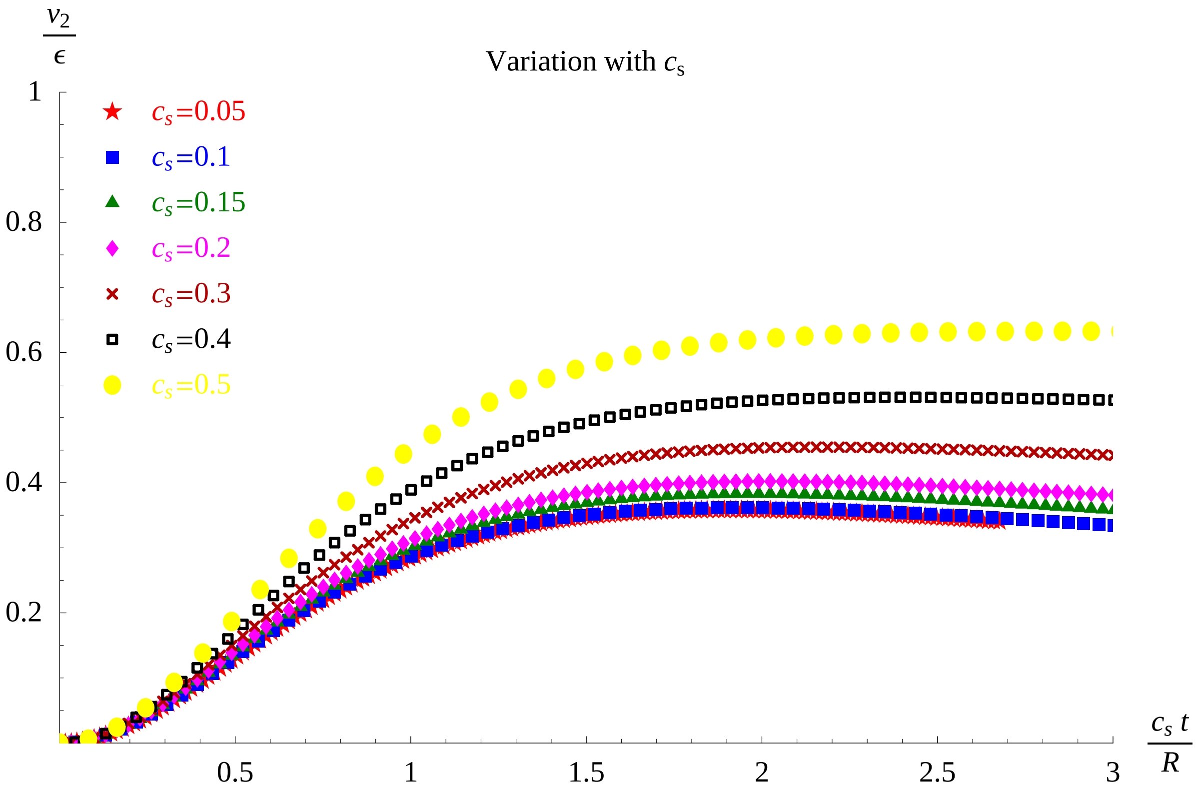

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☛ one can increase v_2 by increasing c_s!

Reconciling data and theory

In hydrodynamical fits, the speed of sound is constrained by p_t spectra, which require a soft equation of state

→ with a hard equation of state, the energy per particle is too high

All relies on the assumption that the energy per particle is related to the density, i.e., that chemical equilibrium is maintained

- chemical equilibrium is more fragile than kinetic equilibrium
- the only experimental indication of chemical equilibrium is in the particle ratios (cf. however $e^+e^- \ldots$)

If there is no chemical equilibrium, energy per particle and density are independent variables, as in ordinary thermodynamics

☞ there is no constraint on the equation of state from p_t spectra: one can consider a larger c_s

Incomplete thermalization at RHIC

- Ideal fluid dynamics: model-independent results
  $\Rightarrow$ slow vs. fast particles
- A reminder: the natural time scale for anisotropic flow is $\dfrac{\bar{R}}{c_s}$
  - no knowledge about early times
  - anisotropic flow cannot conclude on thermalization
- Size of v_2 controlled by $\dfrac{1}{S}\dfrac{\mathrm{d}N}{\mathrm{d}y}$, but no hint at saturation in the data

  incomplete transverse equilibration: $\lambda \sim \bar{R}$

  ☞ anisotropic flow is a tool to measure λ!
- v_2 overshoots the hydrodynamical prediction... because the latter is over-constrained by a non-existent chemical equilibrium
- Predictions for Cu–Cu collisions at RHIC

Predictions for LHC

Measuring anisotropic flow at LHC, you will find

- $\frac{v_2}{\epsilon}$ larger than at RHIC (getting closer to thermalization)

  larger signal, larger statistics ☞ easier measurement ☺

- $\frac{v_4}{(v_2)^2}$ smaller than at RHIC (closer to the ideal fluid value $\frac{1}{2}$)

  Well... that definitely means a smaller signal…

- Smaller systems yield complementary values of $\frac{1}{S}\frac{\mathrm{d}N}{\mathrm{d}y}$, allowing checks (thermalization or not?)