Aspects of the phenomenology of nucleus–nucleus collisions

Nicolas BORGHINI

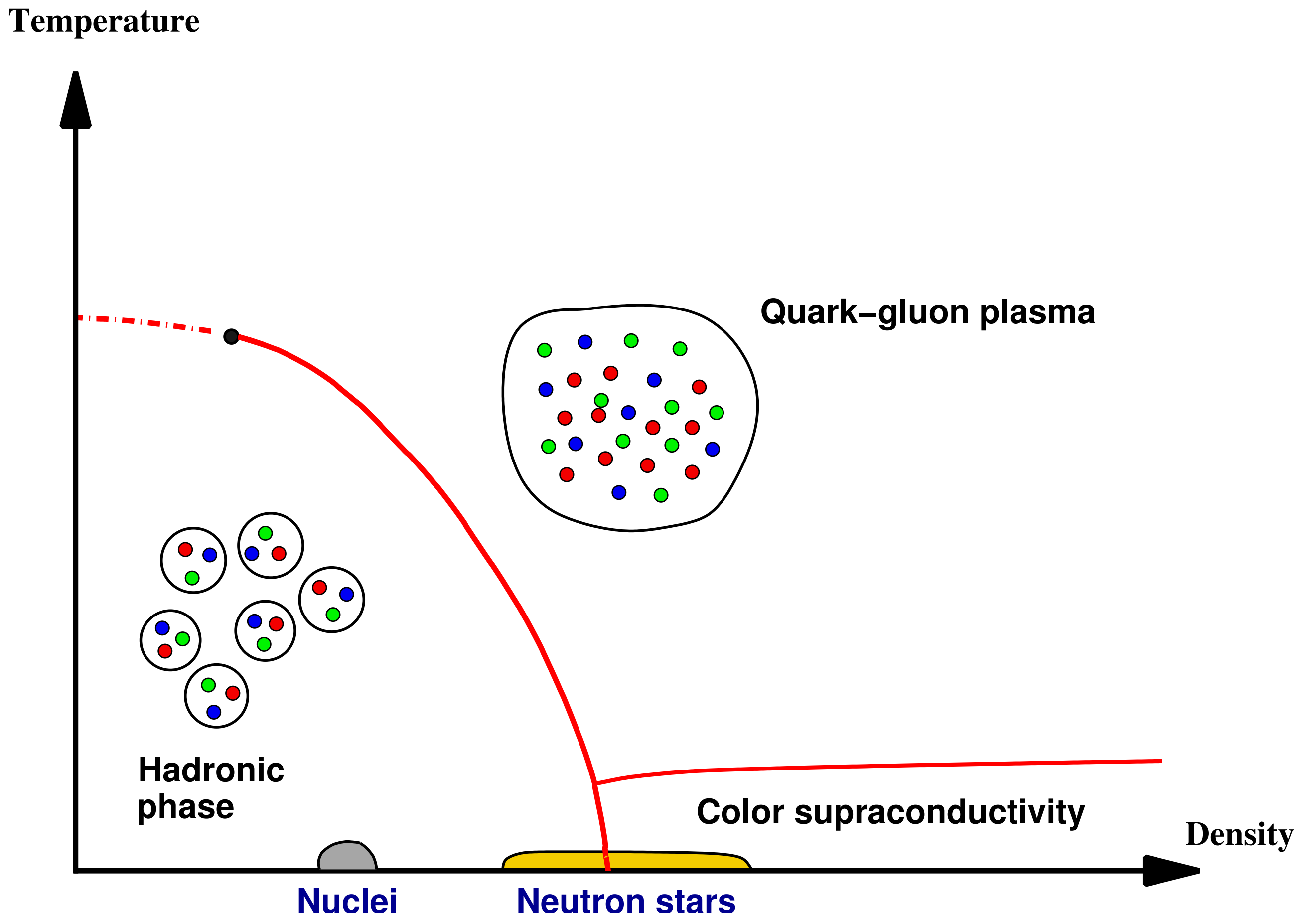
CERN

Phenomenology of nucleus–nucleus collisions

- Heavy-ion collisions: general issues
- A global observable: anisotropic collective flow
  - underlying physics: thermalization of the medium?
  - a not-so-trivial problem: measuring anisotropic flow
- A hard probe: jets propagating through the medium
  - modification of the jet shape

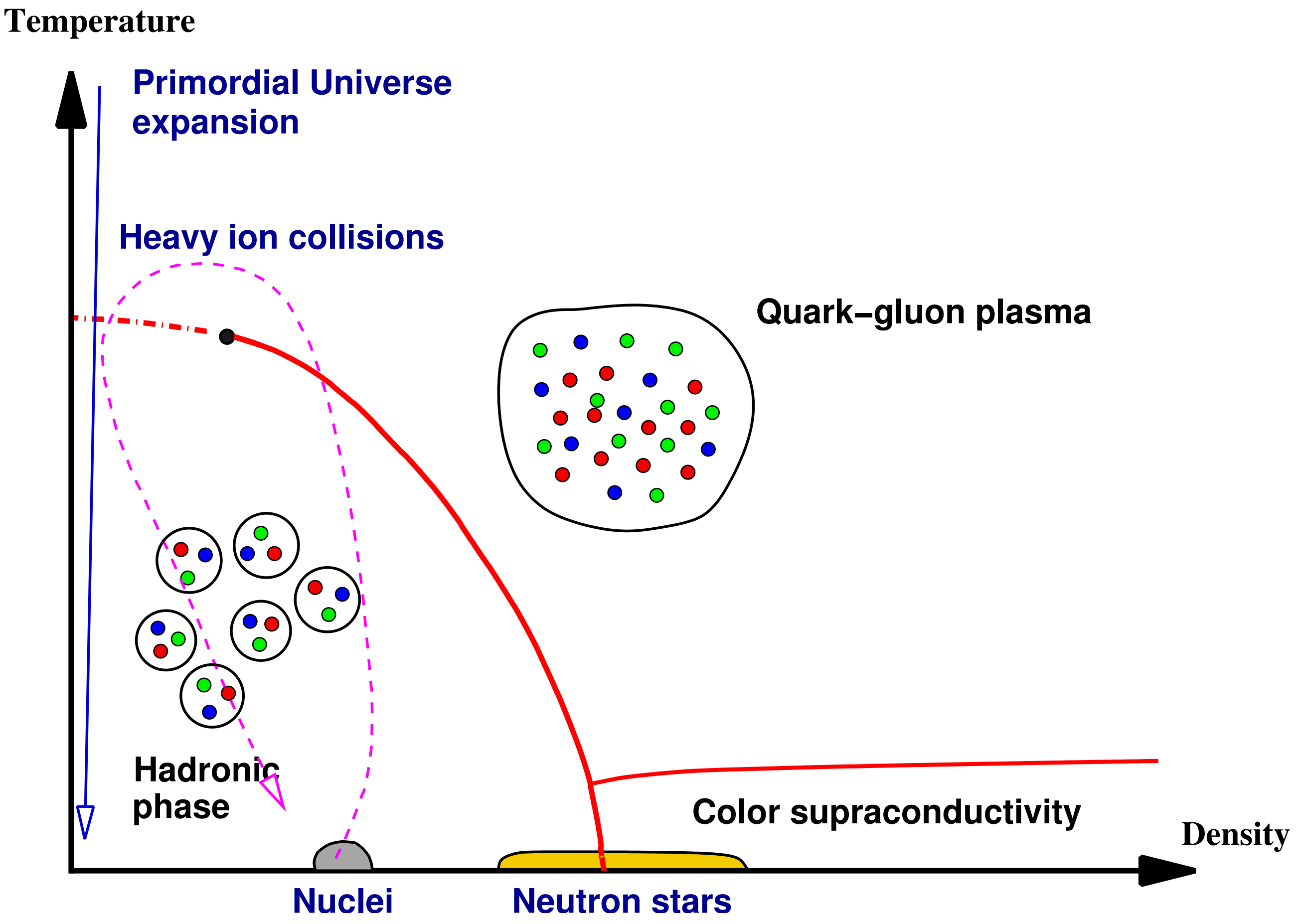
Why heavy ion collisions?

Prediction of (lattice) QCD/ effective models:

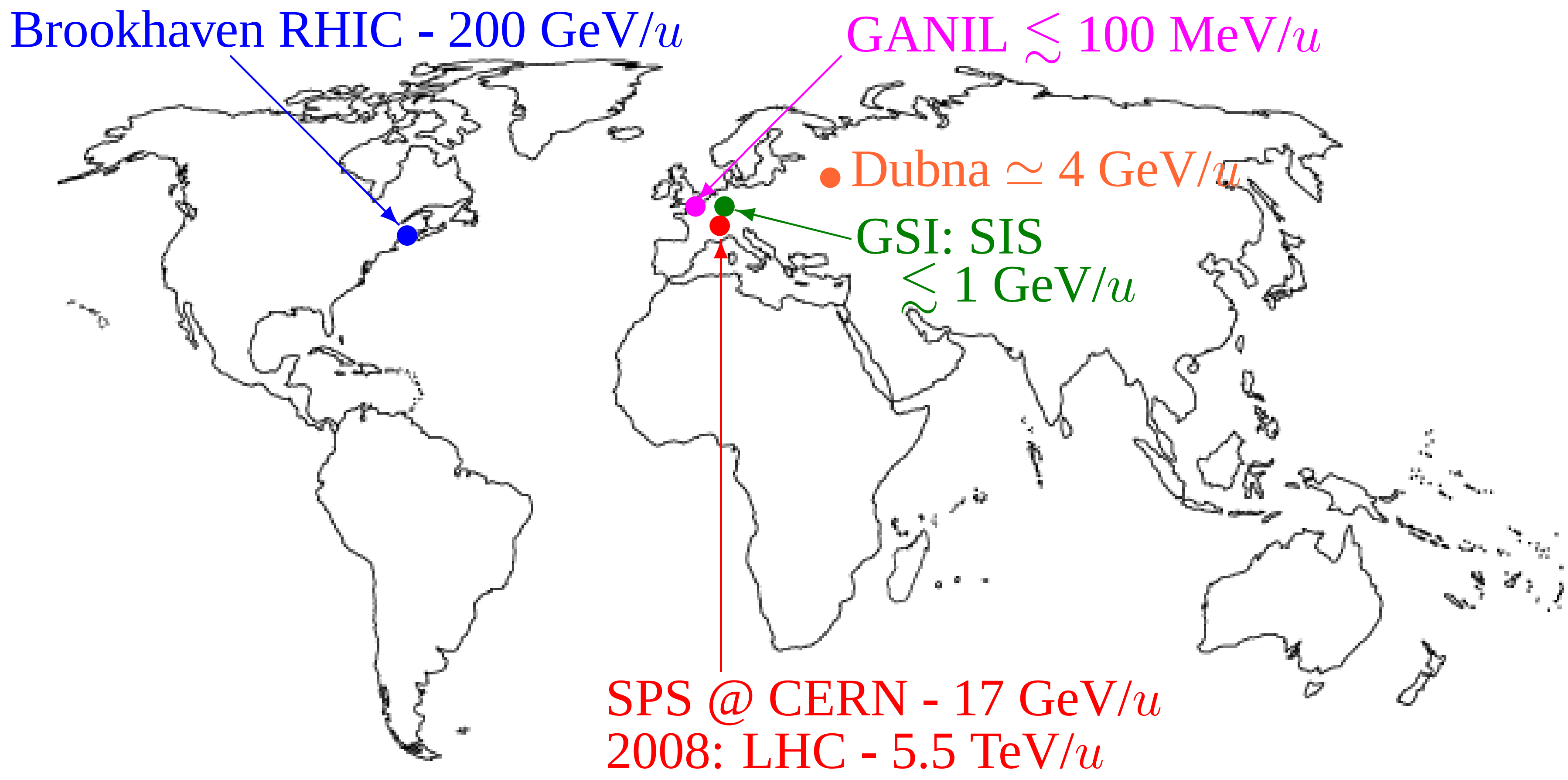


Why heavy ion collisions?

Prediction of (lattice) QCD/ effective models:



Experimental efforts



Brookhaven RHIC - 200 GeV/u

GANIL $\lesssim$ 100 MeV/u

● Dubna $\simeq$ 4 GeV/u

GSI: SIS $\lesssim$ 1 GeV/u

SPS @ CERN - 17 GeV/u
2008: LHC - 5.5 TeV/u

Heavy-ion collisions

In order to characterize the medium created in heavy-ion collisions, plenty of observables have been proposed.

- "Global" observables quantify bulk features in the collisions

  Particle multiplicity, abundance ratios, momentum distributions, flow phenomena…

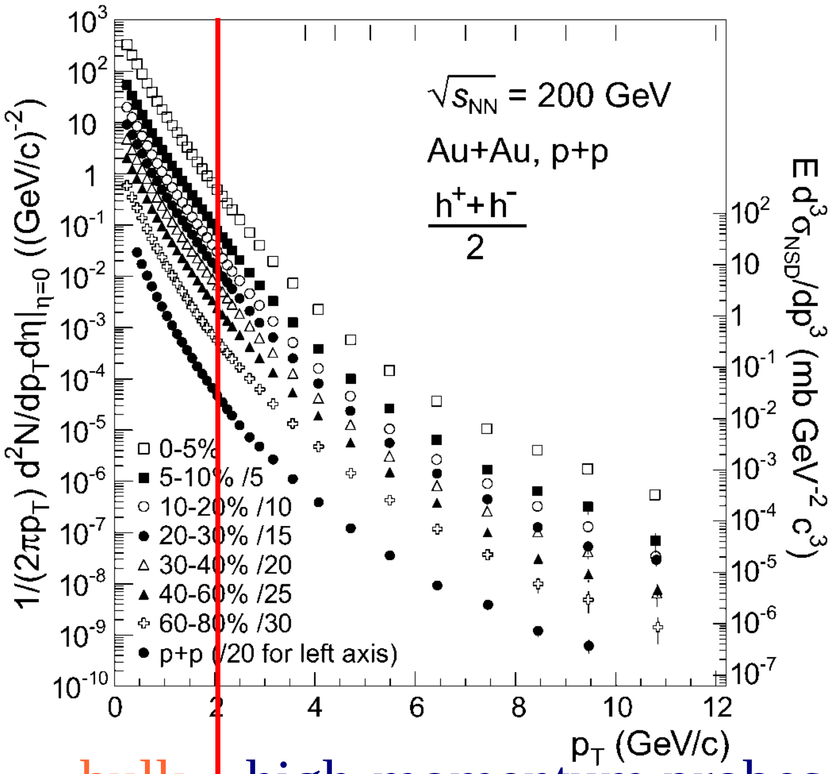
  ☞ naturally call for *macroscopic* concepts: statistical physics, hydrodynamics, …

- "Hard" probes address the medium-induced modification of processes known in elementary-particle collisions

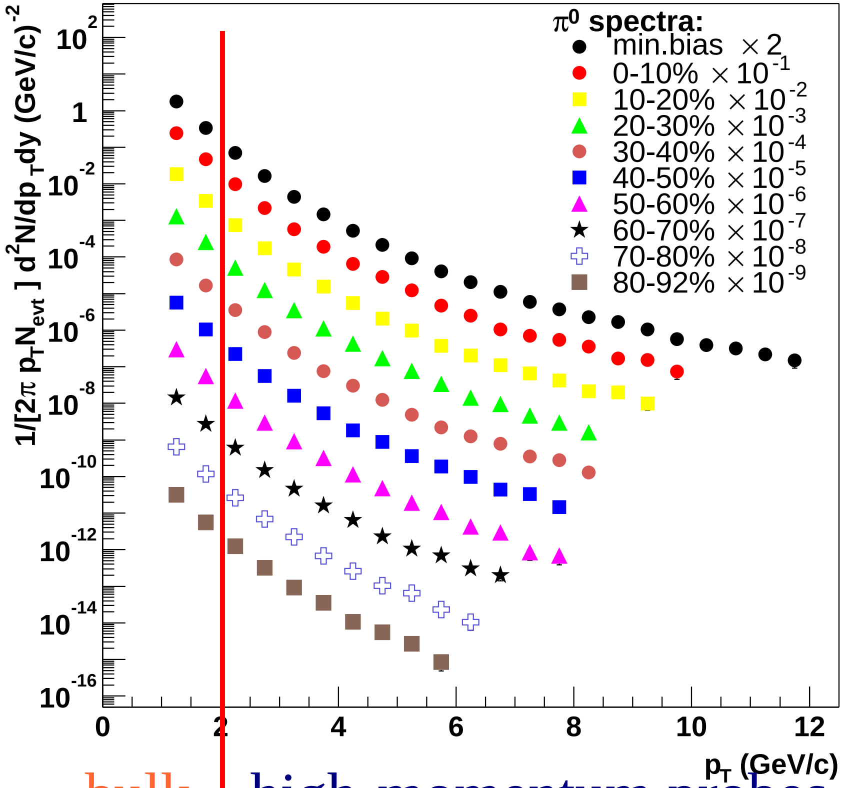
  J/ψ suppression, jets…

  ☞ rely on more *microscopic* approaches

Heavy-ion collisions: bulk vs. hard probes



bulk | high-momentum probes



bulk | high-momentum probes

Particles with high momenta are rare, but their production mechanism is *a priori* better understood (perturbative QCD): can probe the bulk

Heavy ion collision: hydrodynamic description

⓪ Creation of a dense "gas" of particles

① At some time τ_0, the mean free path λ is much smaller than *all* dimensions in the system

$\Rightarrow$ thermalization (T_0), ideal fluid dynamics applies

② The fluid expands: density decreases, λ increases (system size also)

③ At some time, the mean free path is of the same order as the system size: ideal fluid dynamics is no longer valid

"(kinetic) freeze-out"

Freeze-out usually parameterized in terms of a temperature $T_{\rm f.o.}$

If the mean free path varies smoothly with temperature, consistency requires $T_{\rm f.o.} \ll T_0$

Heavy ion collision: hydrodynamic description

At freeze-out, particles are emitted according to thermal distributions (Bose–Einstein, Fermi–Dirac) boosted with the fluid velocity:

$$E\frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{p}} = C\int_\Sigma \exp\left(-\frac{p^\mu u_\mu(x)}{T_{\mathrm{f.o.}}}\right) p^\mu\, \mathrm{d}\sigma_\mu$$

particle momentum

freeze-out hypersurface

Consistent ideal fluid dynamics picture requires $T_{\mathrm{f.o.}} \ll T_0$

$$\Leftrightarrow$$

Ideal-fluid limit = small-$T_{\mathrm{f.o.}}$ limit

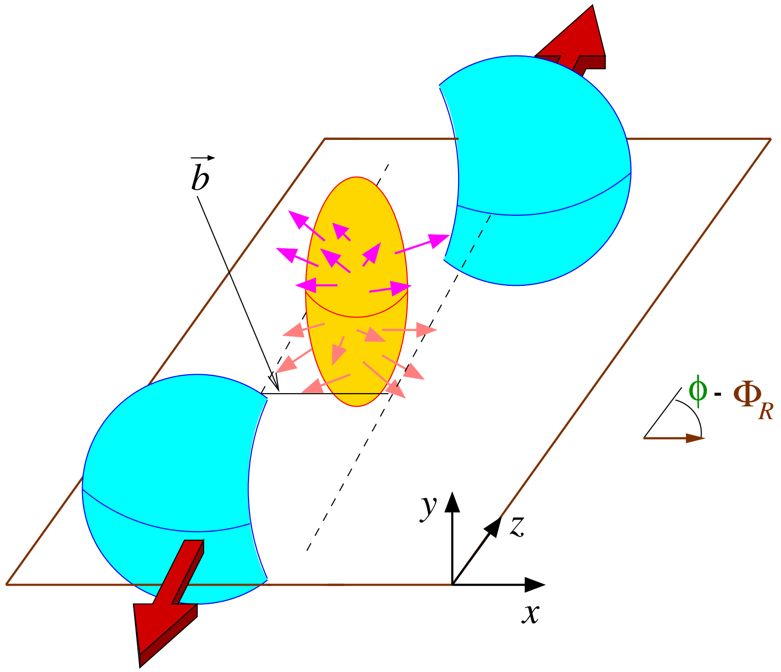
☞ one can compute the spectrum in a model-independent way using saddle-point approximations (or the steepest-descent method)

N.B. & J.-Y. Ollitrault, **nucl-th/0506045**

Similarly, one can obtain analytical results for anisotropic flow…

Heavy-ion observable: Anisotropic flow

Non-central collision:



Initial anisotropy of the source
(in the transverse plane)

⇒ anisotropic pressure gradients,
larger along the impact parameter $\vec{b}$

⇒ anisotropic emission of particles:

anisotropic (collective) flow

$$E\frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{p}} \propto \frac{\mathrm{d}N}{p_t\,\mathrm{d}p_t\,\mathrm{d}y}\left[1 + 2v_1\cos(\phi - \Phi_R) + 2v_2\cos 2(\phi - \Phi_R) + \ldots\right]$$

“directed” “elliptic”

“Flow”: misleading terminology; does NOT imply fluid dynamics!

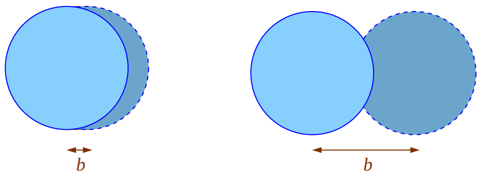
Anisotropic flow: predictions of hydro

- Characteristic build-up time of v_2 is $\bar{R}/c_s$

  typical system size ↗ ↖ speed of sound

- v_2/ϵ constant across different centralities

  ↖ system eccentricity



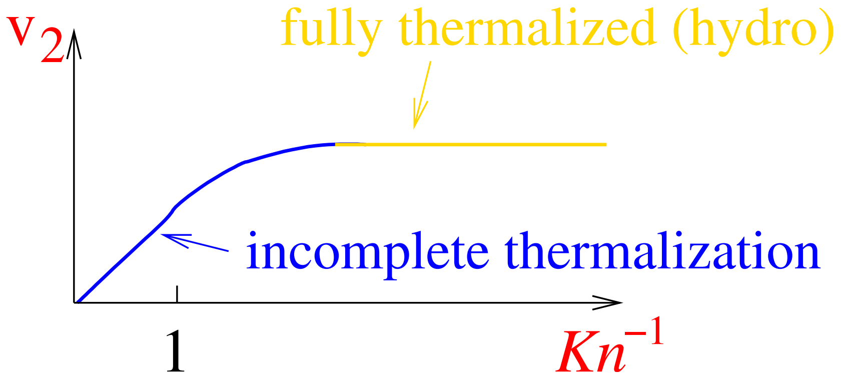
- v_2 roughly independent of the system size (Au–Au vs. Cu–Cu)
- v_2 increases with increasing speed of sound c_s
- Mass-ordering of the $v_2(p_T)$ of different particles

  (the heavier the particle, the smaller its v_2 at a given momentum)
- Relationship between different harmonics: $\dfrac{v_4}{(v_2)^2} = \dfrac{1}{2}$

…can be tested experimentally!

Anisotropic flow:
out-of-equilibrium scenario

The flow grows with the number of collisions per particle $\frac{1}{Kn} = \frac{\bar{R}}{\lambda}$:



- v_2 varies with the number of collisions undergone by particles

  ☞ v_2 depends on the system size $\bar{R}$:

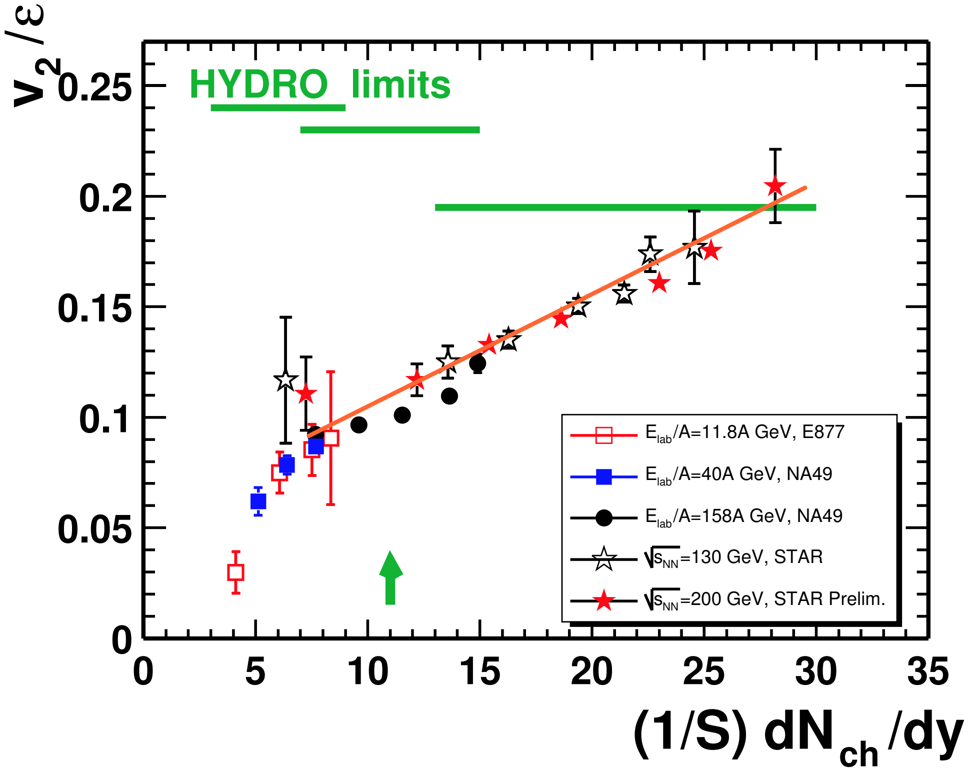
  breakdown of the scale-invariance of hydrodynamics

- $$\frac{v_4}{(v_2)^2} > \frac{1}{2}$$

R.S. Bhalerao, J.-P. Blaizot, N.B., J.-Y. Ollitrault, PLB **627** (2005) 49

Incomplete equilibration & RHIC data

Experimental results seem to favor the out-of-equilibrium scenario:



NA49 Collaboration, Phys. Rev. C **68** (2003) 034903

Scaling law seems to work for RHIC data (+ matching with SPS)

$v_2(Kn^{-1})$ increases steadily (no hint at hydro saturation in the data)

Measuring collective flow

Complicated issue $v_n = \langle \cos n(\phi - \Phi_R) \rangle$
...but the impact parameter (and its direction Φ_R) is not measured

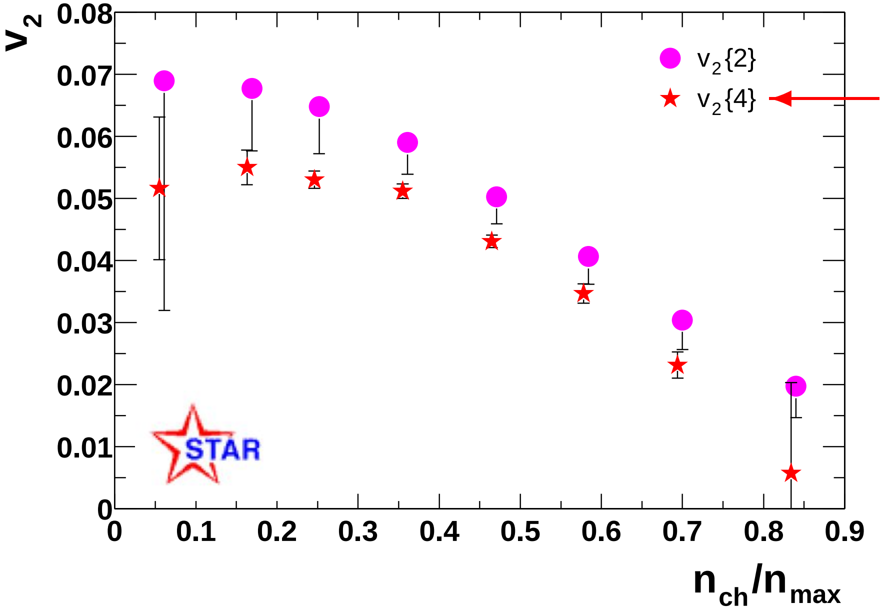
- We showed that “standard” methods used to determine flow are unreliable
- We developed new methods, which allow the measurement of unambiguous v_n values

  Original application of several tools of statistical physics:
  generating functions, cumulants, Lee–Yang zeroes
- These new methods have been adopted by experimentalists!
  STAR, PHENIX, PHOBOS, NA49, NA45, WA98, E895, FOPI...

Quantitative flow physics is now within reach

N.B., P.M. Dinh, J.-Y. Ollitrault, R.S. Bhalerao, 2000–2004

Measuring collective flow



with a new method

v_2 differs by about 20% according to the method…

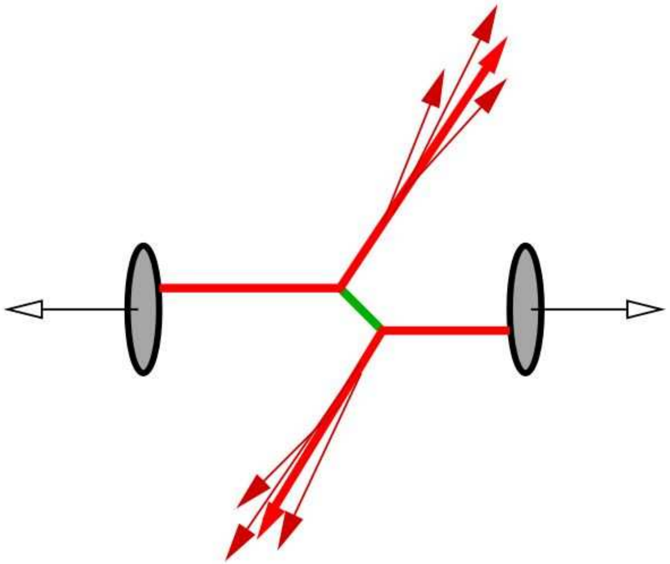
the new values are now compatible with well-established physical constraints (symmetry)

+ 1st measurement of v_1 at RHIC

+ 1st determination of the sign of v_2 (positive) at RHIC

Jet physics in elementary collisions

In proton–(anti)proton or e^+e^- interactions, one observes jets of collimated particles.

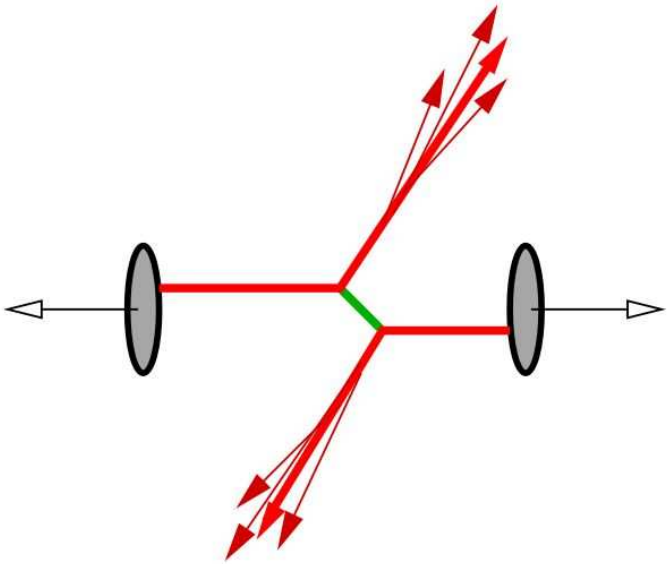


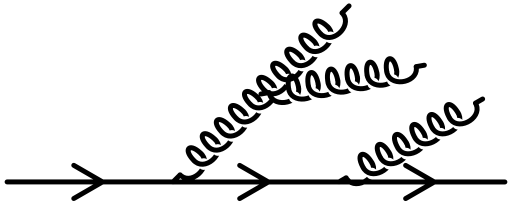
These jets are perfectly described by QCD:

A jet = the shower resulting from the successive emission of partons (mainly gluons) by a fast parton (quark or gluon) as it propagates in the vacuum.

Jet physics in elementary collisions

In proton–(anti)proton or e^+e^- interactions, one observes jets of collimated particles.





These jets are perfectly described by QCD:

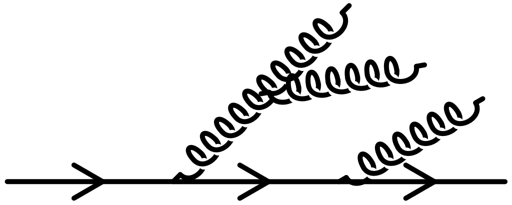
A jet = the shower resulting from the successive emission of partons (mainly gluons) by a fast parton (quark or gluon) as it propagates in the vacuum.

MLLA: main ingredients

Modified Leading Logarithmic Approximation

- Resummation of double- and single-logarithms in $\ln \frac{1}{x}$ and $\ln \frac{E_{\rm jet}}{\Lambda_{\rm eff}}$
- Intra-jet colour coherence:
  - *independent* successive branchings $g \to gg$, $g \to q\bar{q}$, $q \to qg$
  - with angular ordering of the sequential parton decays:

    at each step in the evolution, the angle between father and offspring partons decreases



- Includes in a systematic way next-to-leading-order corrections

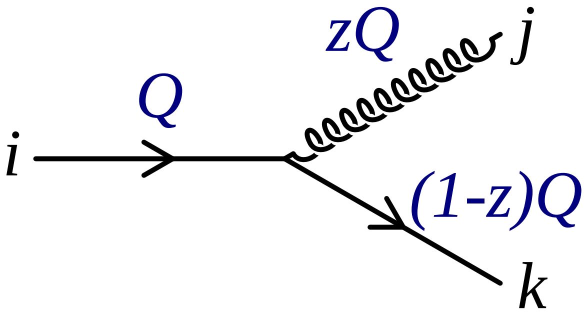
  $\mathcal{O}(\sqrt{\alpha_s(\tau)})$!
- Hadronization through "Local Parton-Hadron Duality" (LPHD)

MLLA:
generating functional

Central object : generating functional $Z_i[Q,\Theta;u(k)]$

☞ generates the various cross sections ($\to ggg, \to ggq\bar{q}\ldots$) for a jet coming from a parton i ($= g, q, \bar{q}$) with energy Q in a cone of angle Θ

$$\begin{aligned} Z_i[Q,\Theta;u(k)] \;=\; & \mathrm{e}^{-w_i(Q,\Theta)}\,u(Q) \\ & +\sum_j \int^{\Theta} \frac{\mathrm{d}\Theta'}{\Theta'} \int_0^1 \mathrm{d}z\; \mathrm{e}^{w_i(Q,\Theta')-w_i(Q,\Theta)}\, \frac{\alpha_s(k_\perp)}{2\pi} \\ & \quad \times P_{ji}(z)\, Z_j[zQ,\Theta';u]\, Z_k[(1-z)Q,\Theta';u] \end{aligned}$$



MLLA: generating functional

Central object : generating functional $Z_i[Q,\Theta;u(k)]$

☞ generates the various cross sections ($\to ggg$, $\to gg q\bar{q}\ldots$) for a jet coming from a parton i ($= g, q, \bar{q}$) with energy Q in a cone of angle Θ

$$Z_i[Q,\Theta;u(k)] = \boxed{e^{-w_i(Q,\Theta)}}\,u(Q) + \sum_j \int^{\Theta} \frac{d\Theta'}{\Theta'} \int_0^1 dz\, \boxed{e^{w_i(Q,\Theta')-w_i(Q,\Theta)}}\, \frac{\alpha_s(k_\perp)}{2\pi} \times P_{ji}(z)\, Z_j[zQ,\Theta';u]\, Z_k[(1-z)Q,\Theta';u]$$

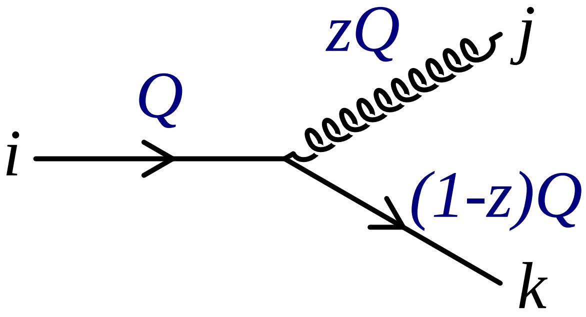
probability to have
no branching with angle $< \Theta$

between Θ and Θ'

angular ordering

splitting function $i \to jk$

$k_\perp \approx z(1-z)Q$

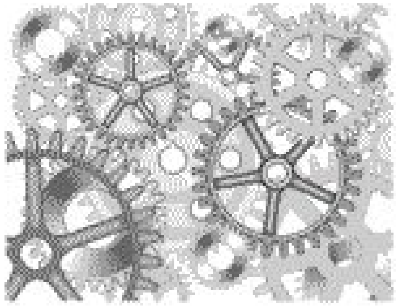


MLLA: limiting spectrum

The parton distribution in a jet with “energy” $\tau \equiv \ln \frac{Q}{\Lambda_{\text{eff}}}$ is given by

$$\bar{D}_i(x,\tau) \equiv Q \frac{\delta}{\delta u(xQ)} Z_i[\tau; u(k)]\Big|_{u\equiv 1}$$

infrared cutoff



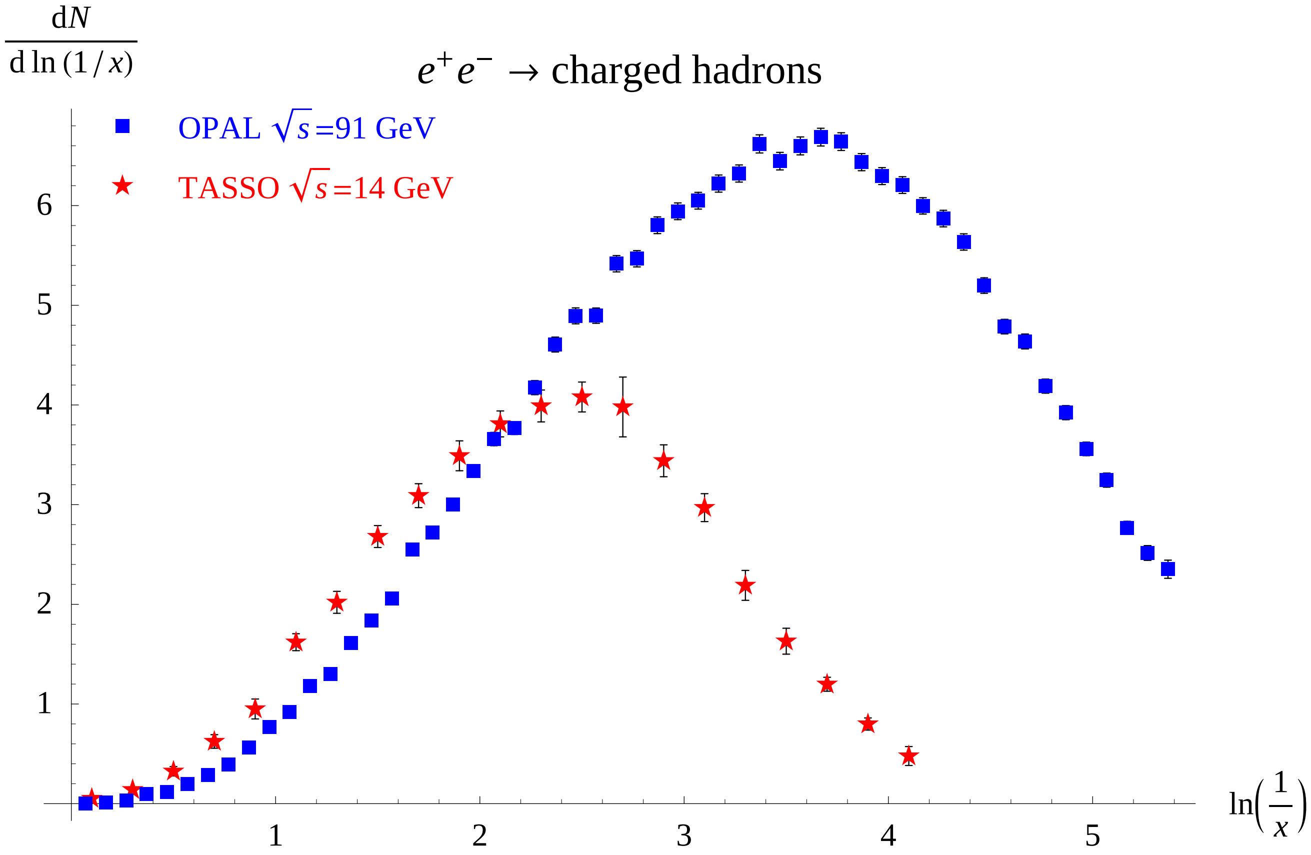
“Limiting spectrum”:

$$\bar{D}^{\text{lim}}(x,\tau,\Lambda_{\text{eff}}) = \frac{4N_c\tau}{bB(B+1)} \int_{\epsilon-\mathrm{i}\infty}^{\epsilon+\mathrm{i}\infty} \frac{\mathrm{d}\nu}{2\pi\mathrm{i}}\, x^{-\nu}\Phi(-A+B+1, B+2; -\nu\tau)$$

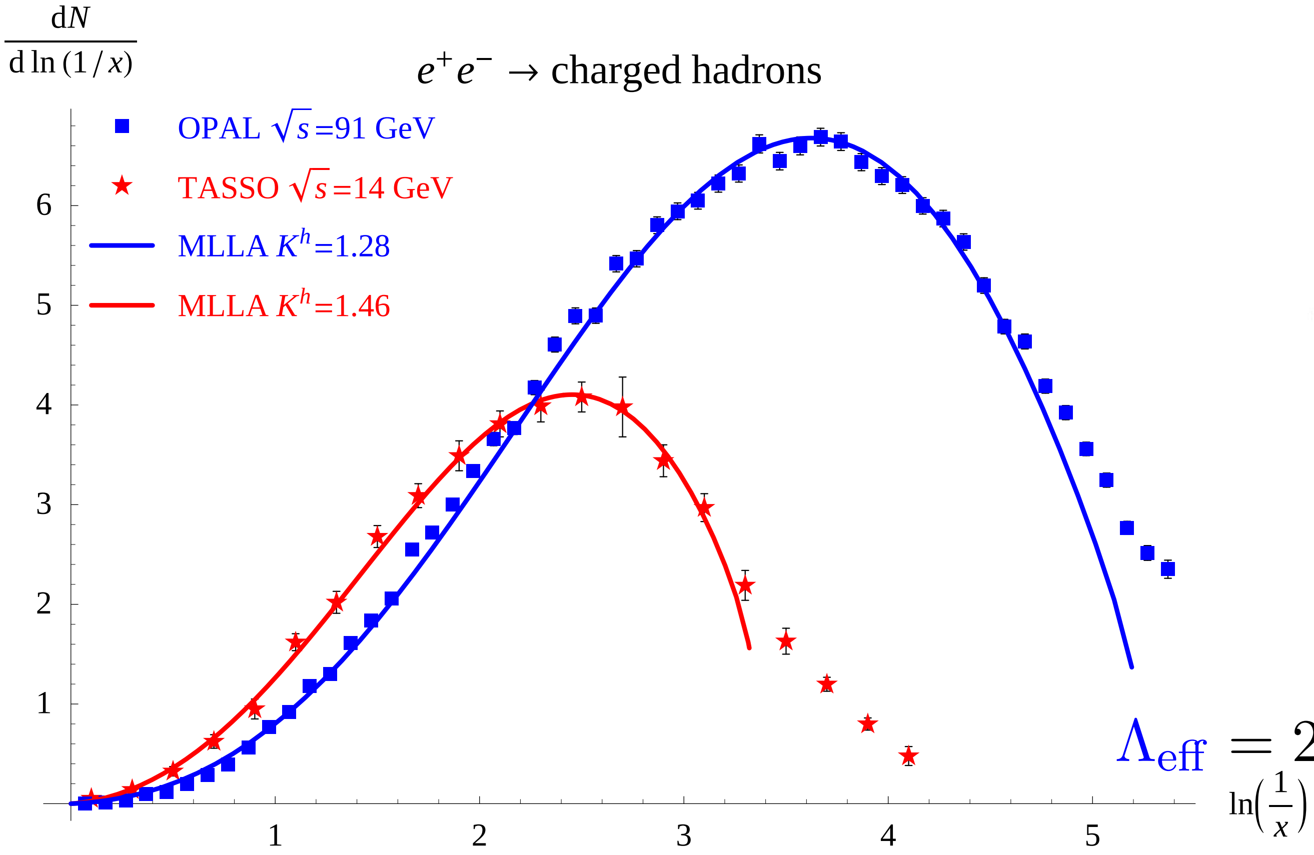
with

$$A \equiv \frac{4N_c}{b\nu}, \qquad B \equiv \frac{a}{b}, \qquad a \equiv \frac{11}{3}N_c + \frac{2N_f}{3N_c^2}, \qquad b \equiv \frac{11}{3}N_c - \frac{2}{3}N_f$$

Jets in elementary collisions: MLLA vs. data



Jets in elementary collisions: MLLA vs. data



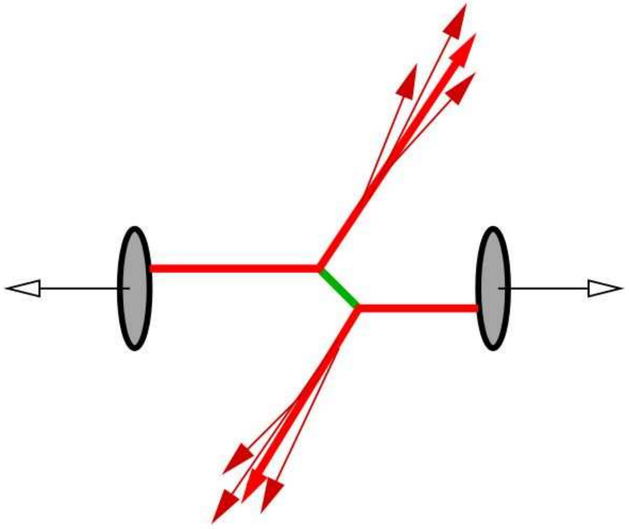
$\Lambda_{\rm eff} = 253$ MeV

Good description of the data also in $p\bar{p}$ collisions (CDF...)

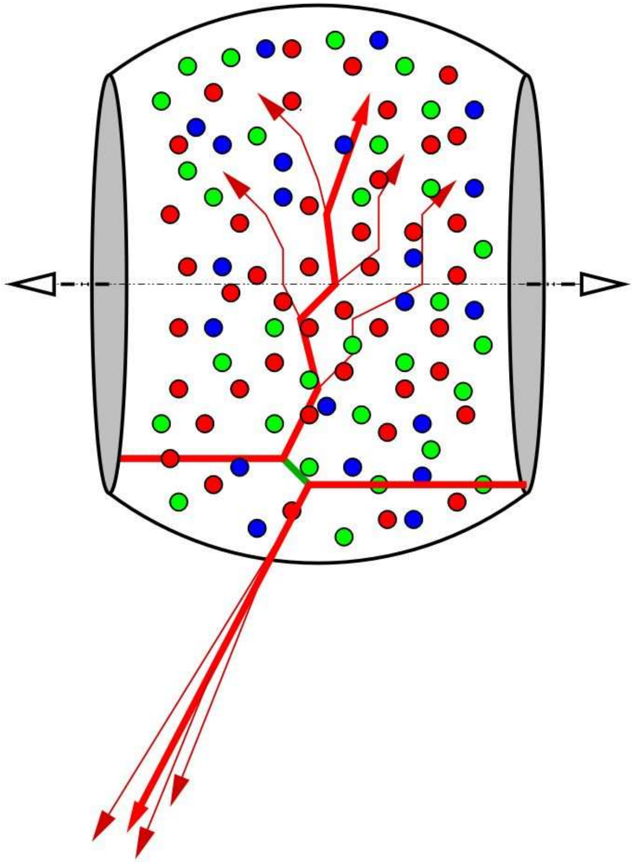
MLLA is reliable! (even for large x)

Influence of the medium: the emerging view

pp collisions:



Au–Au collisions:



Fast partons dissipate their energy while traversing the medium; only those created close to the edge can escape and emerge as jets

Influence of the medium: a possibility

- The hump of the limiting spectrum is mostly due to the singular parts of the splitting functions
- In medium, the emission of soft gluons by a fast parton increases

☞ One can model medium-induced effects by modifying the parton splitting functions $P_{ji}(z)$…

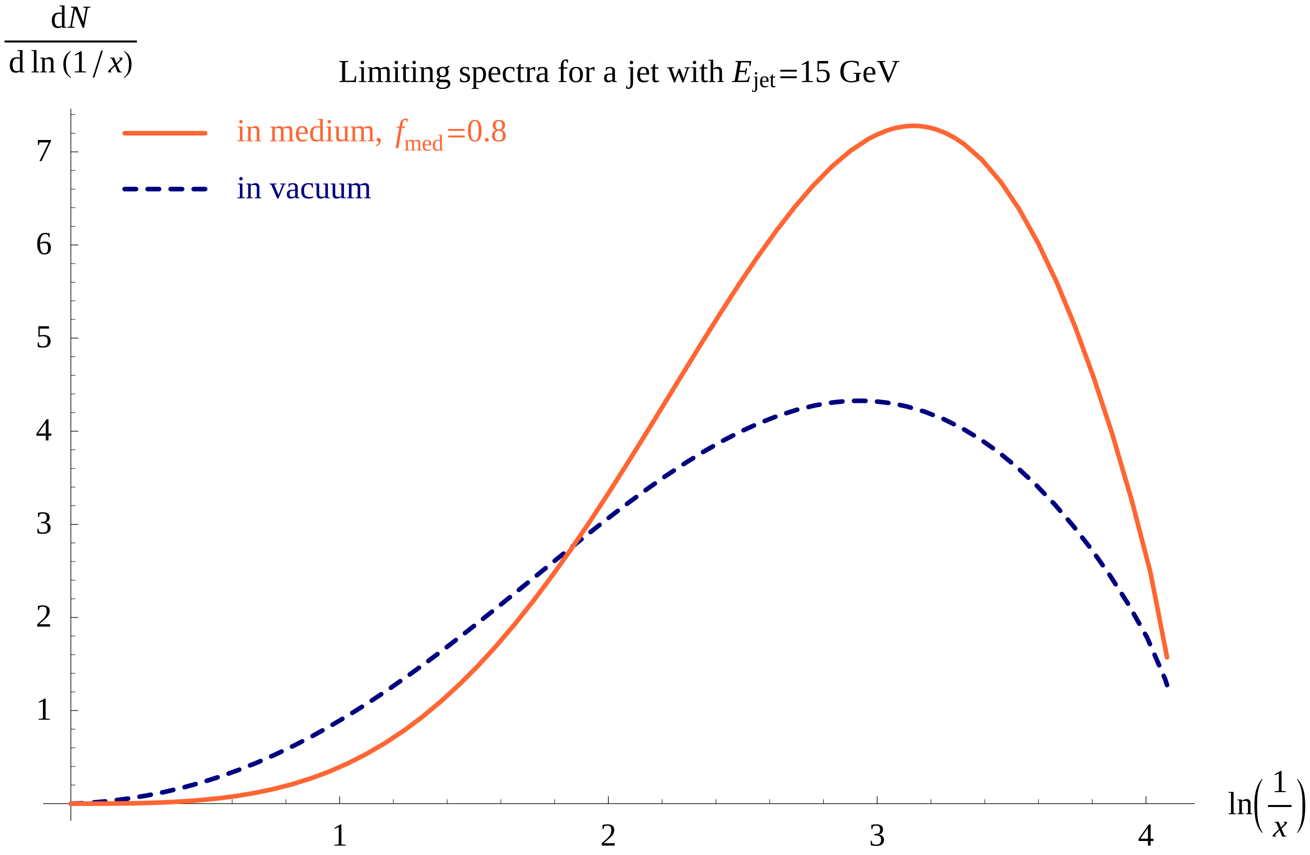
… and especially their singular parts:

$$P_{qq}(z) = \frac{4}{3}\left[\frac{2(1+f_{\rm med})}{1-z} - (1+z)\right]$$

$f_{\rm med} > 0 \Rightarrow$ Bremsstrahlung increases

N.B. & U.A. Wiedemann, **hep-ph/0506218**

Influence of the medium on the parton spectrum



f_{med} fixed to reproduce R_{AA} ☞ redistribution of radiated partons:

high p_T (large x) $\longrightarrow$ low p_T (small x)

Medium-induced modification of the associated multiplicity

Ideal case: photon + jet

☞ photon gives jet energy E_T

- Count how many jet particles have a momentum larger than some given cut $P_T^{\rm cut}$ after propagating through the medium:

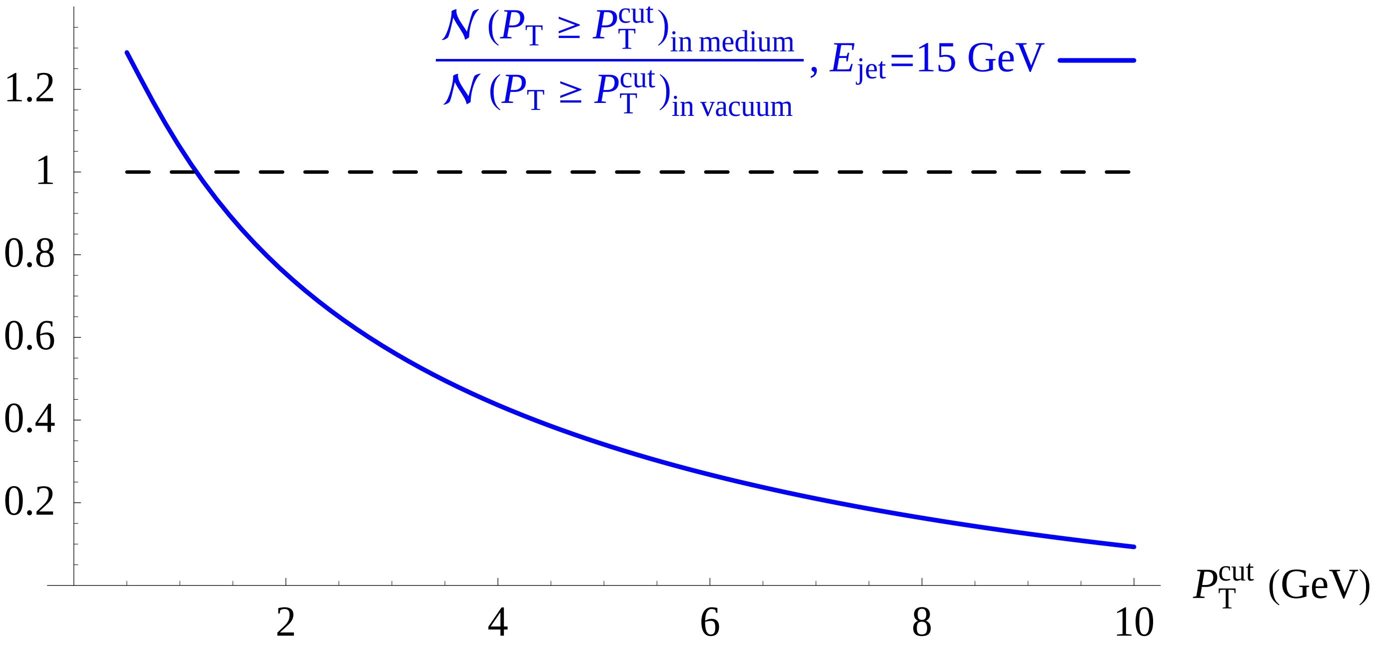
$$\mathcal{N}(P_T \geq P_T^{\rm cut})_{\rm medium}$$

- For a jet *in vacuum* with energy E_T, the spectrum is known $\Rightarrow$ one knows (measurement / *in vacuum* MLLA)

$$\mathcal{N}(P_T \geq P_T^{\rm cut})_{\rm vacuum}$$

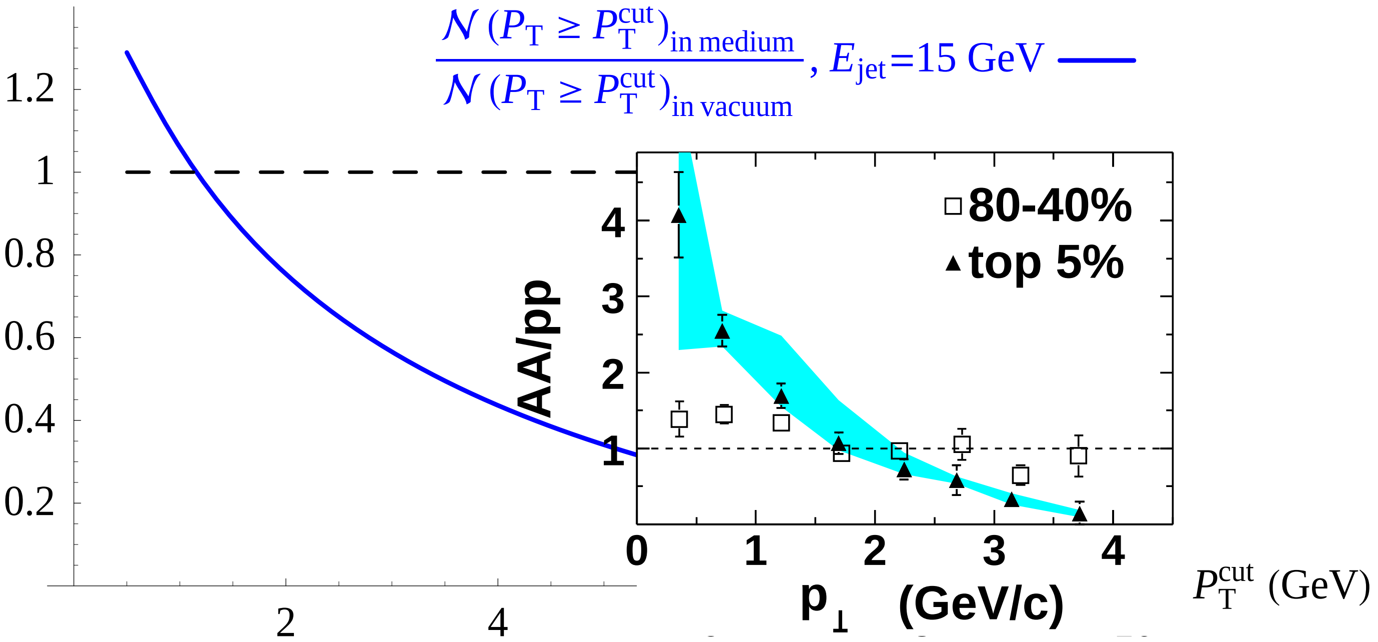
- Compare $\mathcal{N}(P_T \geq P_T^{\rm cut})_{\rm medium}$ with $\mathcal{N}(P_T \geq P_T^{\rm cut})_{\rm vacuum}$

Medium-induced modification of the associated multiplicity



In the presence of a medium, less particles for $P_T \gtrsim 1.5$ GeV (particle excess for $P_T \lesssim 1.5$ GeV!)

Medium-induced modification of the associated multiplicity



In the presence of a medium, less particles for $P_T \gtrsim 1.5$ GeV (particle excess for $P_T \lesssim 1.5$ GeV!)

cf. STAR PRL **95** (2005) 152301

Hadron spectra

What if the jet energy is unknown…

The measured hadron spectrum is the convolution of

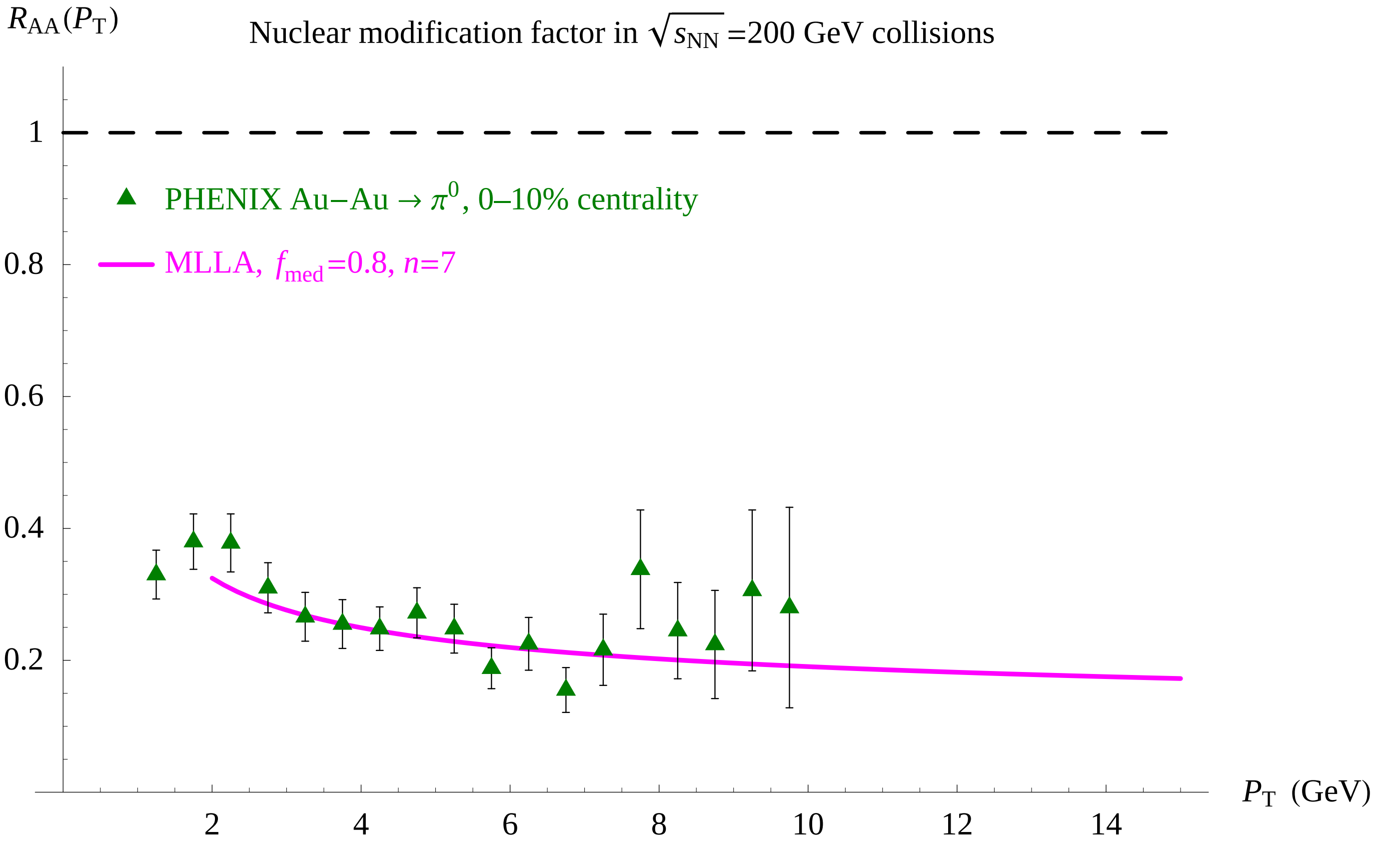
- a parton spectrum $\propto 1/(p_T)^n$
- the "fragmentation function" $\bar{D}^h(x,\tau)$

$$\frac{\mathrm{d}N}{\mathrm{d}P_T} \propto \int \frac{\mathrm{d}x}{x^2}\,\frac{1}{{p_T}^n}\,\bar{D}^h(x,p_T) = \int \frac{\mathrm{d}x}{x^2}\,\frac{x^n}{{P_T}^n}\,\bar{D}^h\!\left(x,\frac{P_T}{x}\right)$$

which can be computed within MLLA for both a jet in vacuum and a jet propagating through a medium

⇒ gives the nuclear modification factor R_{AA}

Nuclear modification factor



Reasonable agreement with PHENIX π^0 results:
Formalism can account for a factor 5 suppression of high-p_T spectra

Phenomenology of nucleus–nucleus collisions

Complementary observables yield alternative views of the physics involved in heavy ion collisions at ultrarelativistic energies

- collective flow: a mature observable, which provides information on the bulk: equilibration (kinetic and/or chemical)?

  ☞ macroscopic approaches (fluid dynamics, statistical physics)

  ...but not only: flow of rare or of high-p_T particles

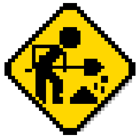
- jets: rare phenomena, but which involve processes that can be computed from first principles: reliable reference!

  Numerous jets at LHC, over a wide kinematic range

  ☞ new physics opportunities: intrajet multiparticle correlations...

  Monte-Carlo implementation(s) of the new formalism

Phenomenology of nucleus–nucleus collisions



(to-do list?)

- Jet physics in the medium

- A bridge between micro- and macroscopic description:
  dissipative phenomena

  Microscopic energy redistribution, using a realistic Monte-Carlo code of medium-induced effects, vs. viscous fluid dynamics
  (gluon Bremsstrahlung vs. Mach cone)

  Interplay between the Yang–Mills fields invoked in mechanisms of fast-thermalization and prompt partons?

Phenomenology of nucleus–nucleus collisions

Extra slides

Methods of flow analysis

Anisotropic flow is usually measured using two-particle correlations:

$$\langle\cos 2(\phi_1 - \phi_2)\rangle \approx \langle\cos 2(\phi_1 - \Phi_R)\rangle\langle\cos 2(\Phi_R - \phi_2)\rangle = (v_2)^2$$

Assumption: all two-particle correlations are due to flow…

…which is obviously wrong!

"Non-flow" sources of correlations: jets, decays of short-lived particles, global momentum conservation, quantum effects between identical particles, etc. can bias the "standard" flow analysis

The bias is comparatively larger for smaller systems

☞ New methods for measuring flow have been developed

cumulants of multiparticle correlations, Lee–Yang zeroes

(N.B., P.M. Dinh, J.-Y. Ollitrault, R.S. Bhalerao, 2000–2004)

Measuring collective flow

Generating function $G_n(z) \equiv \left\langle \prod_{j=1}^{M} (1 + z \cos n\phi_j) \right\rangle$

- If no flow: system made of independent sub-systems
  $$G_n(z) = \prod_{\text{subsyst.}} G_{\text{sub.}}(z)$$
  $\Rightarrow$ the zeroes of G_n are unchanged when M increases
- In the presence of collective flow: the position of the zeroes is $\propto 1/M$

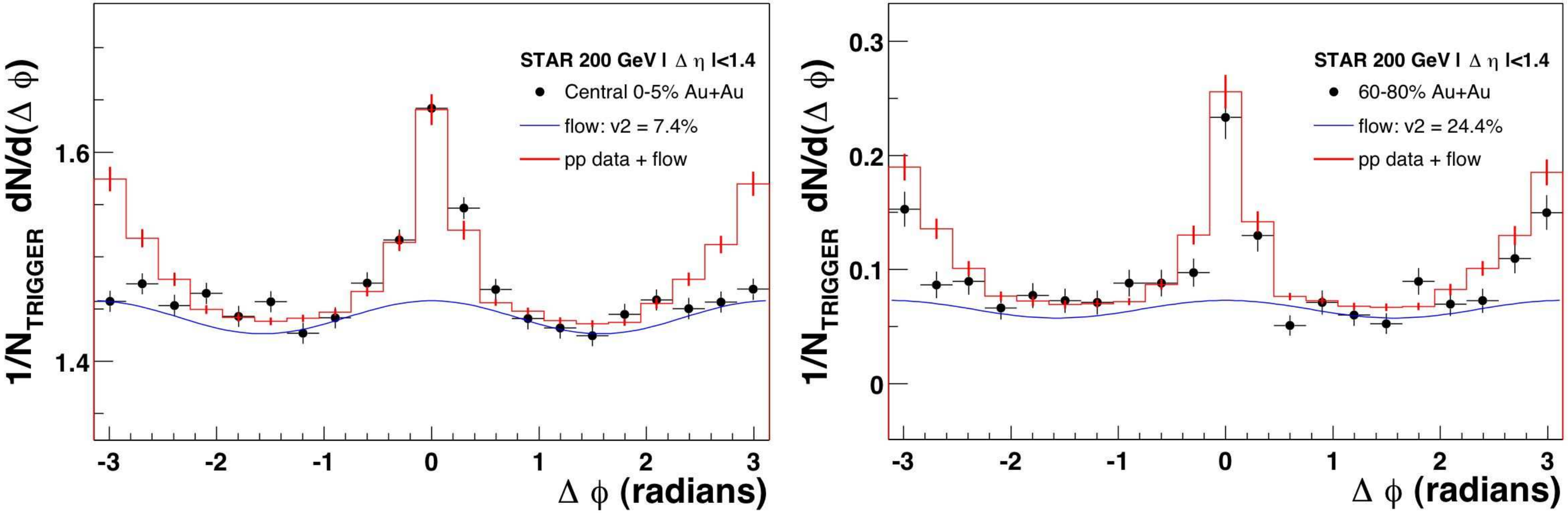
$\Rightarrow$ The first (“Lee–Yang”) zero of $G_n(z)$ gives v_n

Jets in Au–Au collisions at RHIC

Study of the azimuthal correlations between

① a "leading particle", momentum $P_{T\,\max}$, origin of azimuths, and

② "associated particles: momentum $P_{T\,\text{cut}} < P_T < P_{T\,\max}$, azimuth ϕ



☞ No recoil jet ($\phi \sim 180^{\rm o}$) in central Au–Au events