Does RHIC data indicate the formation of a perfect fluid?

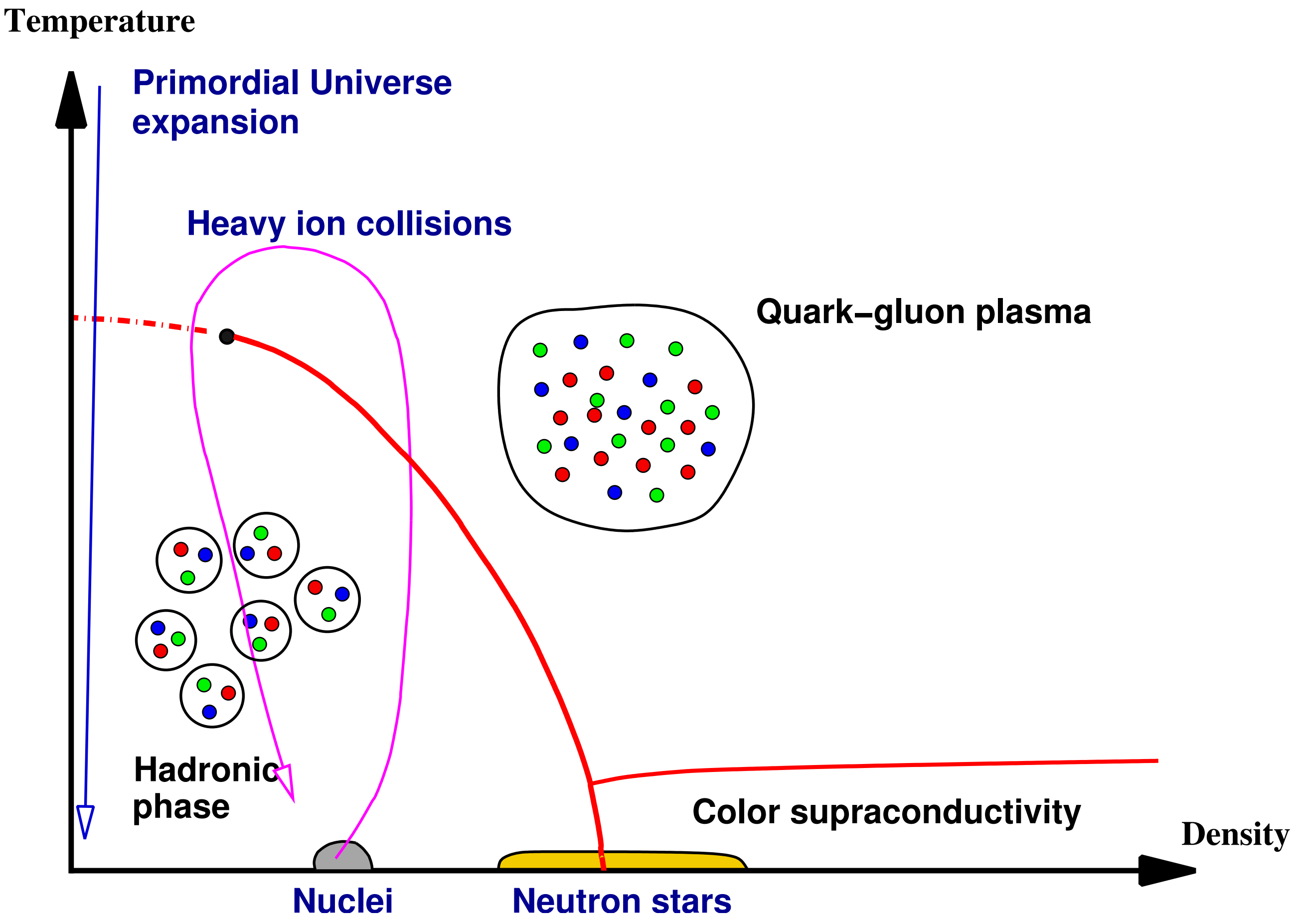
Nicolas BORGHINI

in collaboration with

R.S. BHALERAO J.-P. BLAIZOT J.-Y. OLLITRAULT

CERN

Why heavy ion collisions?



RHIC Au–Au results: the fashionable view

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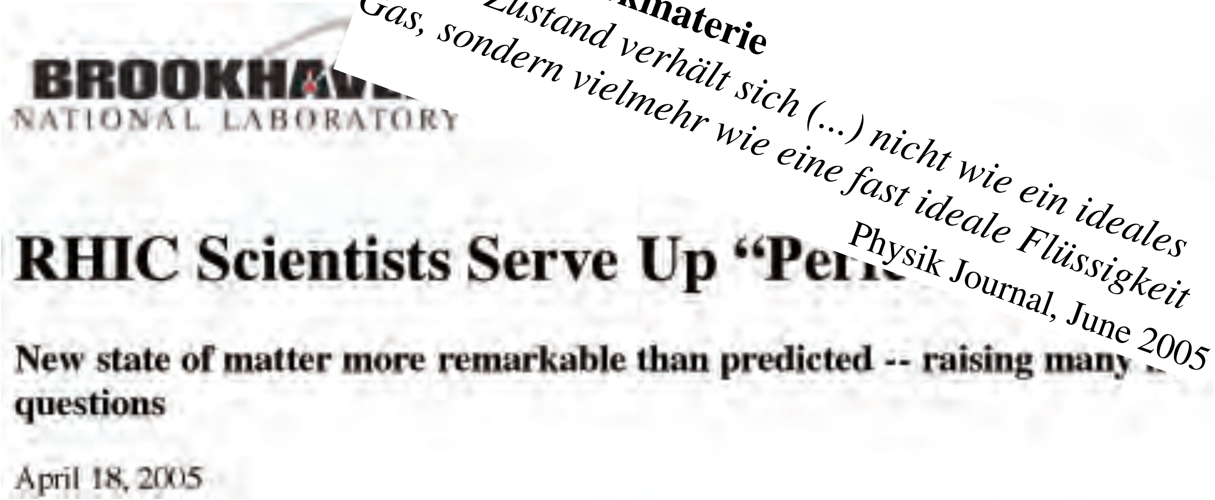
RHIC Scientists Serve Up "Perfect" Liquid

New state of matter more remarkable than predicted -- raising many new questions

April 18, 2005

RHIC Au–Au results: the fashionable view



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RHIC Scientists Serve Up “Per

New state of matter more remarkable than predicted -- raising many
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April 18, 2005

Flüssige Quarkmaterie

Dieser Zustand verhält sich (...) nicht wie ein ideales Gas, sondern vielmehr wie eine fast ideale Flüssigkeit

Physik Journal, June 2005

RHIC Au–Au results: the fashionable view

Flüssige Quarkmateri

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RHIC Sci

New st than predicted -- raising many

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April 18, 20

Liquid universe hints at strings

Physics in Action; June 2005

Researchers at RHIC have seen convincing new evidence for a quark-gluon plasma. But it looks more like a perfect liquid than a gas, which could have implications for string theory

RHIC Au–Au results: the fashionable view

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Physics in Action; June 2
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Ideal fluid dynamics reproduce both p_t spectra and $v_2(p_t)$ of soft ($p_t \lesssim 2$ GeV/c) identified particles for minimum bias collisions, near central rapidity.

This agreement necessitates a soft equation of state, and very short thermalization times: $\tau_{\rm thermalization} < 0.6$ fm/c.

$\Rightarrow$ **strongly interacting Quark-Gluon Plasma**

Ideal fluid dynamics in heavy-ion collisions

- A few reminders on fluid dynamics
- Fluid dynamics and heavy ion collisions: theory
    - Overall scenario
    - General predictions of ideal fluid dynamics
        - Momentum spectra
        - Anisotropic flow
- Fluid dynamics and heavy ion collisions: theory vs. data
- Reconciling data and theory 

Fluid dynamics: physical quantities

- Microscopic parameters
  - λ = mean free path between two collisions
  - v_{thermal} = average velocity of particles
- Macroscopic parameters
  - L = system size
  - v_{fluid} = fluid velocity
- Micro and macro are connected: kinetic theory
  - c_s = sound velocity $\sim v_{\text{thermal}}$
  - η = viscosity $\sim \lambda\, v_{\text{thermal}}$

Fluid dynamics: various types of flow

- **Thermodynamic equilibrium?** ☞ Knudsen number $Kn = \frac{\lambda}{L}$
  - $Kn \gg 1$: Ballistic (free-streaming) limit
  - $Kn \ll 1$: Thermalization : Fluid (hydro) limit
- **Viscous or Ideal?** ☞ Reynolds number $Re = \frac{L v_{\text{fluid}}}{\eta}$
  - $Re \gg 1$: Ideal (non-viscous) fluid
  - $Re \leq 1$: Viscous fluid
- **Compressible or Incompressible?** ☞ Mach number $Ma = \frac{v_{\text{fluid}}}{c_s}$
  - $Ma \ll 1$: Incompressible fluid
  - $Ma > 1$: Compressible (supersonic) fluid

Fluid dynamics: various types of flow

Three numbers:

$$Kn = \frac{\lambda}{L}, \qquad Re = \frac{L v_{\rm fluid}}{\eta}, \qquad Ma = \frac{v_{\rm fluid}}{c_s}$$

$\Rightarrow$ **an important relation:**

$$Kn \times Re = \frac{\lambda\, v_{\rm fluid}}{\eta} \sim \frac{v_{\rm fluid}}{c_s} = Ma$$

Compressible fluid: Thermalized means Ideal

Viscosity $\equiv$ departure from equilibrium

General scenario of a heavy-ion collision

⓪ Creation of a dense gas of particles

① At some time τ_0, the mean free path λ is much smaller than *all* dimensions in the system

$\Rightarrow$ thermalization (T_0), ideal fluid dynamics applies

② The fluid expands: density decreases, λ increases (system size also)

③ At some time, the mean free path is of the same order as the system size: ideal fluid dynamics is no longer valid

"(kinetic) freeze-out"

Freeze-out usually parameterized in terms of a temperature $T_{\rm f.o.}$

If the mean free path varies smoothly with temperature, consistency requires $T_{\rm f.o.} \ll T_0$

General scenario of a heavy-ion collision

To build a model (for comparison with experimental data), one needs

- an equation of state:

energy density

$$\text{pressure} \longrightarrow P = f(\epsilon), \qquad \frac{\mathrm{d}P}{\mathrm{d}\epsilon} = \underbrace{c_s}_{\text{speed of sound}}{}^2$$

- initial conditions:

  on a space-like hypersurface, one specifies
  - energy density
  - transverse velocity
  - longitudinal velocity

- a freeze-out temperature $T_{\rm f.o.}$ (or a freeze-out criterion)

☞ computation of several observables

Heavy-ion collisions: Momentum spectra

At freeze-out, particles are emitted according to thermal distributions (Bose–Einstein, Fermi–Dirac) boosted with the fluid velocity:

$$E\frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{p}} = C\int_{\Sigma} \exp\left(-\frac{p^\mu u_\mu(x)}{T_{\rm f.o.}}\right) p^\mu\, \mathrm{d}\sigma_\mu$$

freeze-out hypersurface

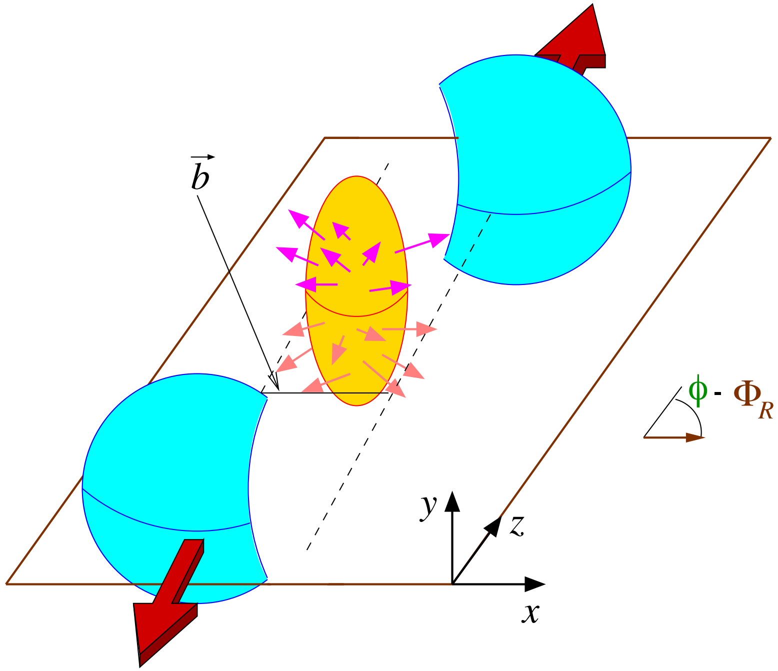
particle momentum

Remark: In the following, I shall use Boltzmann distributions!

("Quantum" effects may only affect pions at very low transverse momentum, where their spectrum is anyway contaminated by decay products)

Heavy-ion observable: Anisotropic flow

Non-central collision:



Initial anisotropy of the source (in the transverse plane)

⇒ anisotropic pressure gradients, larger along the impact parameter $\vec{b}$

⇒ anisotropic emission of particles:

anisotropic (collective) flow

$$E\frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{p}} \propto \frac{\mathrm{d}N}{p_t\,\mathrm{d}p_t\,\mathrm{d}y}\left[1 + 2\underline{v_1}\cos(\phi-\Phi_R) + 2\underline{v_2}\cos 2(\phi-\Phi_R) + \ldots\right]$$

measure pressure effects ⇒ equation of state

Ideal fluid dynamics: general predictions

Consistent ideal fluid dynamics picture requires $T_{\rm f.o.} \ll T_0$

$$\Leftrightarrow$$

Ideal-fluid limit = $T_{\rm f.o.} \to 0$ limit

☞ one can compute in a model-independent way

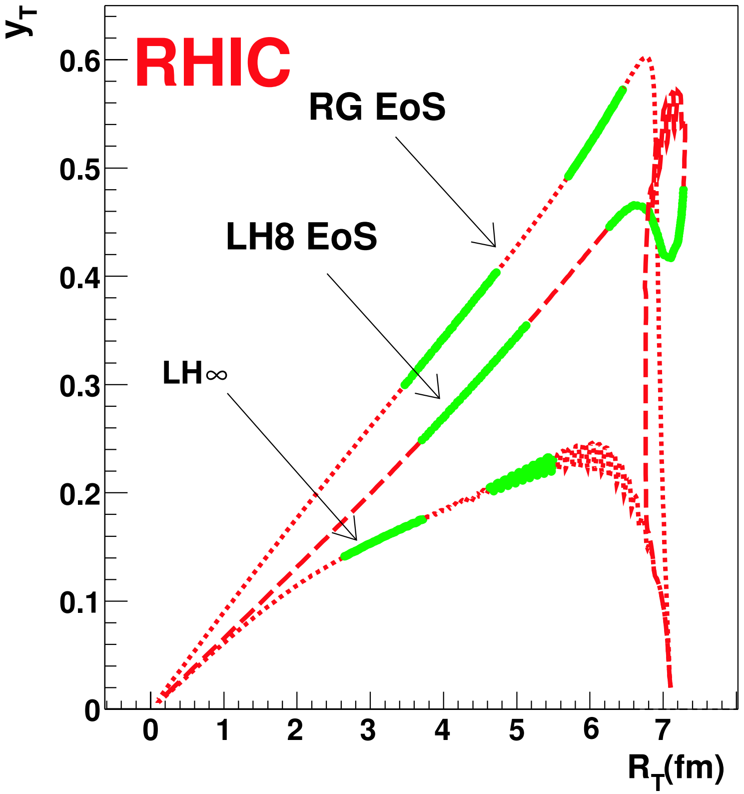
- the spectrum $E\dfrac{\mathrm{d}N}{\mathrm{d}^3\mathbf{p}} = C\displaystyle\int_\Sigma \exp\left(-\frac{p^\mu u_\mu(x)}{T_{\rm f.o.}}\right) p^\mu \, \mathrm{d}\sigma_\mu$
- the anisotropic flow $v_n = \dfrac{\displaystyle\int_0^{2\pi} \frac{\mathrm{d}\phi}{2\pi}\, E\frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{p}} \cos n\phi}{\displaystyle\int_0^{2\pi} \frac{\mathrm{d}\phi}{2\pi}\, E\frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{p}}}$

using saddle-point approximations (or the steepest-descent method)

N.B. & J.-Y. Ollitrault, **nucl-th/0506045**

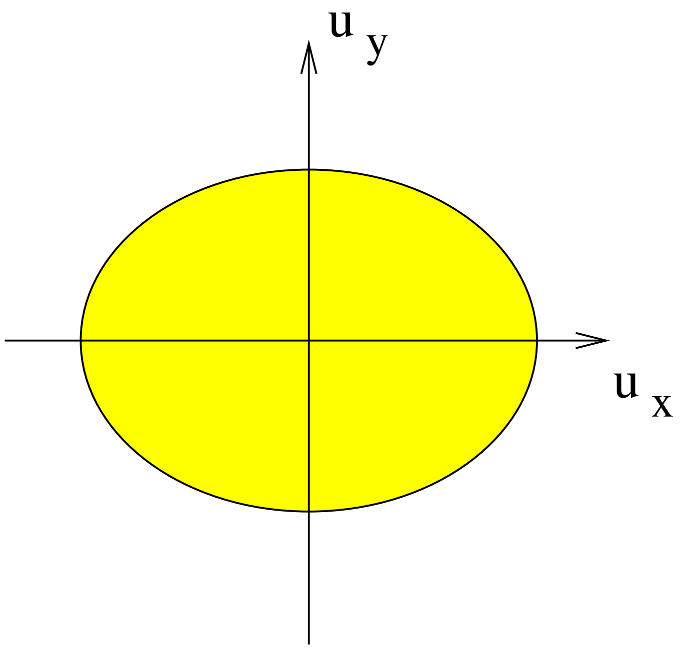
Ideal fluid dynamics: general predictions

Fluid rapidity profiles



Kolb & Heinz, `nucl-th/0305084`

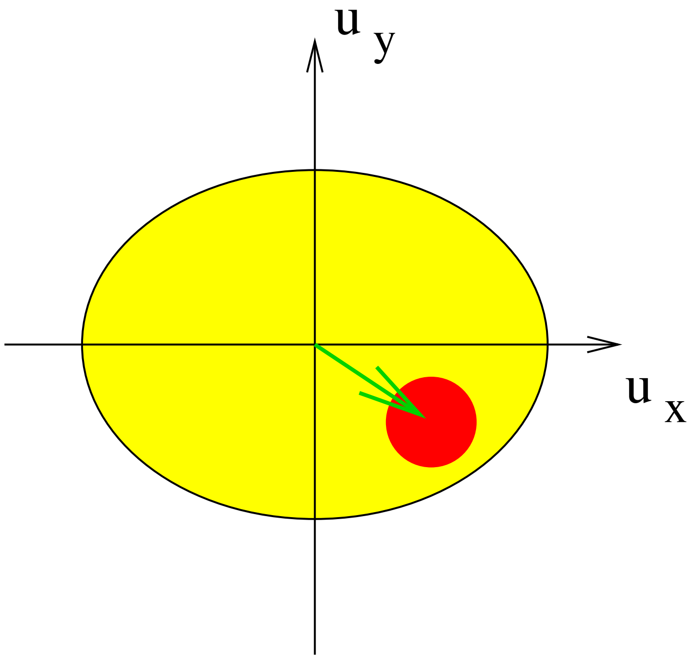
Profile in non-central collisions



(velocity larger along the direction of impact parameter)

Ideal fluid dynamics: general predictions

Slow particles ($p_t/m < u_{\rm max}(\frac{\pi}{2})$) move together with the fluid



There is a point where the fluid velocity equals the particle velocity

☞ Integrand in the momentum spectrum is $\underline{\text{Gaussian}}$, with width $(p^\mu u_\mu)^{1/2}_{\rm min} = \sqrt{m}$
↳ saddle-point approximation!

- Similar spectra for different hadrons:

$$E\frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{p}} = c^h(m)\, f\left(\frac{p_t}{m}, y, \phi\right)$$

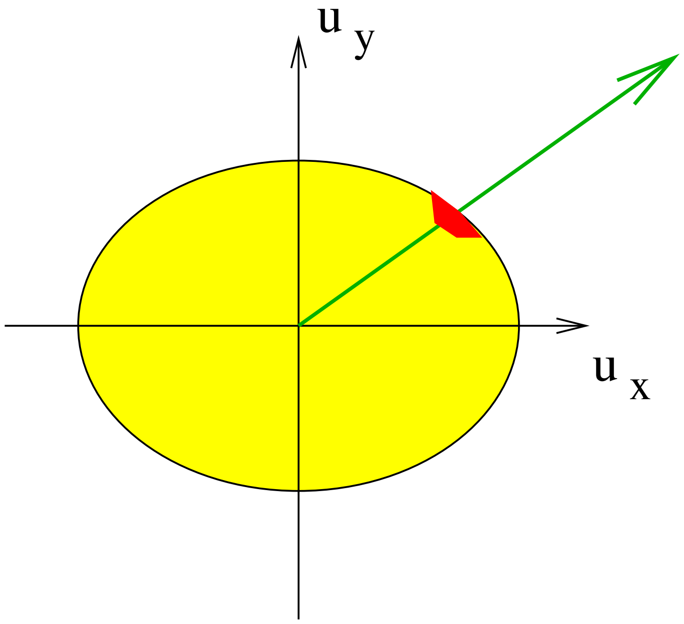
- $v_n\left(\frac{p_t}{m}, y\right)$ universal!

$\Rightarrow$ mass-ordering of $v_2(p_t, y)$

Ideal fluid dynamics: general predictions

Fast particles ($p_t/m > u_{\max}(0)$) move faster than the fluid

Particle comes from where the fluid is fastest along the direction of its velocity:



$$(p^\mu u_\mu)_{\min} = \underbrace{m_t\sqrt{1+u_{\max}(\phi)^2}} > m$$

Saddle-point approximation around

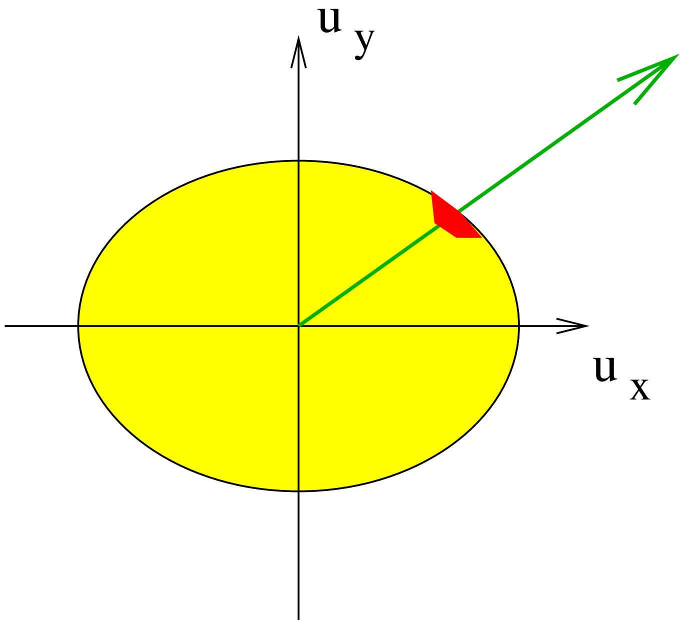
- $$E\frac{\mathrm{d}N}{\mathrm{d}^2\mathbf{p}_t\,\mathrm{d}y} \propto \frac{1}{\sqrt{p_t - m_t v_{\max}}}\exp\left(\frac{p_t u_{\max} - m_t u^0_{\max}}{T_{\text{f.o.}}}\right)$$

(If the point where the minimum is reached lies on the border of Σ, use the steepest-descent method ⇒ no $\sqrt{\ }$)

Ideal fluid dynamics: general predictions

Fast particles ($p_t/m > u_{\max}(0)$) move faster than the fluid

Particle comes from where the fluid is fastest along the direction of its velocity:



$$(p^\mu u_\mu)_{\min} = \underbrace{m_t\sqrt{1+u_{\max}(\phi)^2}}_{} > m$$

Saddle-point approximation around

- $$E\frac{\mathrm{d}N}{\mathrm{d}^2\mathbf{p}_t\,\mathrm{d}y} \propto \frac{1}{\sqrt{p_t - m_t v_{\max}}} \exp\left(\frac{p_t u_{\max} - m_t u^0_{\max}}{T_{\mathrm{f.o.}}}\right)$$
- $$v_2(p_t) \propto \frac{u_{\max}}{T_{\mathrm{f.o.}}}(p_t - m_t v_{\max})$$
  $\Rightarrow$ mass-ordering of $v_n(p_t)$
- $$v_4(p_t) = \frac{v_2(p_t)^2}{2}$$

Ideal fluid dynamics: general predictions

What can be found in **nucl-th/0506045**?

- The first ever distinction between *slow* ($p_t/m < u_{\max}$) and *fast* ($p_t/m > u_{\max}$) particles

☞ $u_{\max}$ maximum transverse four-velocity of the expanding fluid

⇒ depends on the model: equation of state

- Various model-independent scaling laws, derived within ideal fluid dynamics, for the momentum spectra and anisotropic flow coefficients ($v_2, v_4, \ldots$) of both classes of particles

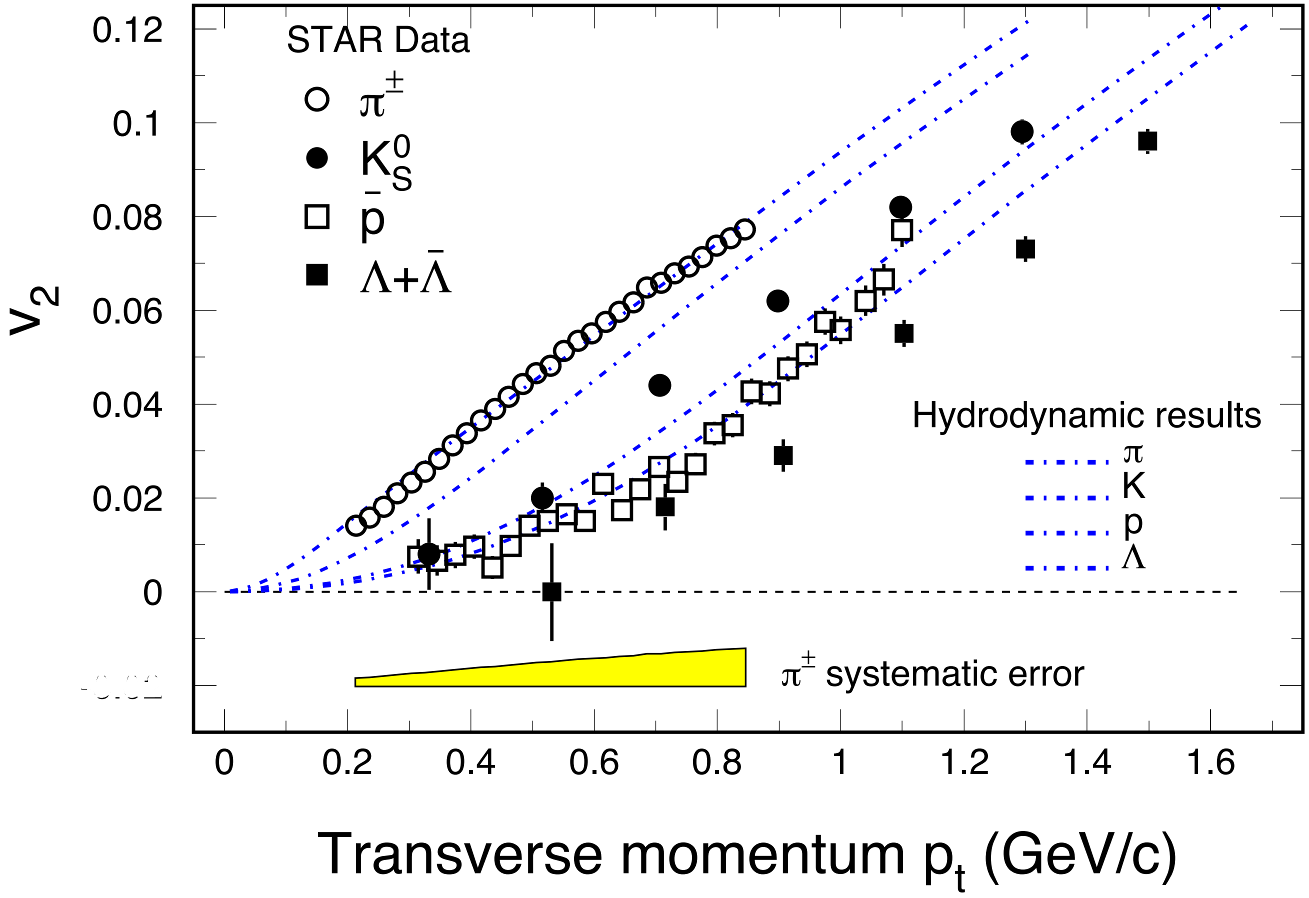
☞ these scaling laws can be used

  - to test if applying ideal hydro to experimental data is relevant
  - to check ideal fluid dynamics-based ~~black boxes~~ models

RHIC data: a personal choice [1/4]

$v_2(p_t)$ at midrapidity, minimum bias collisions:

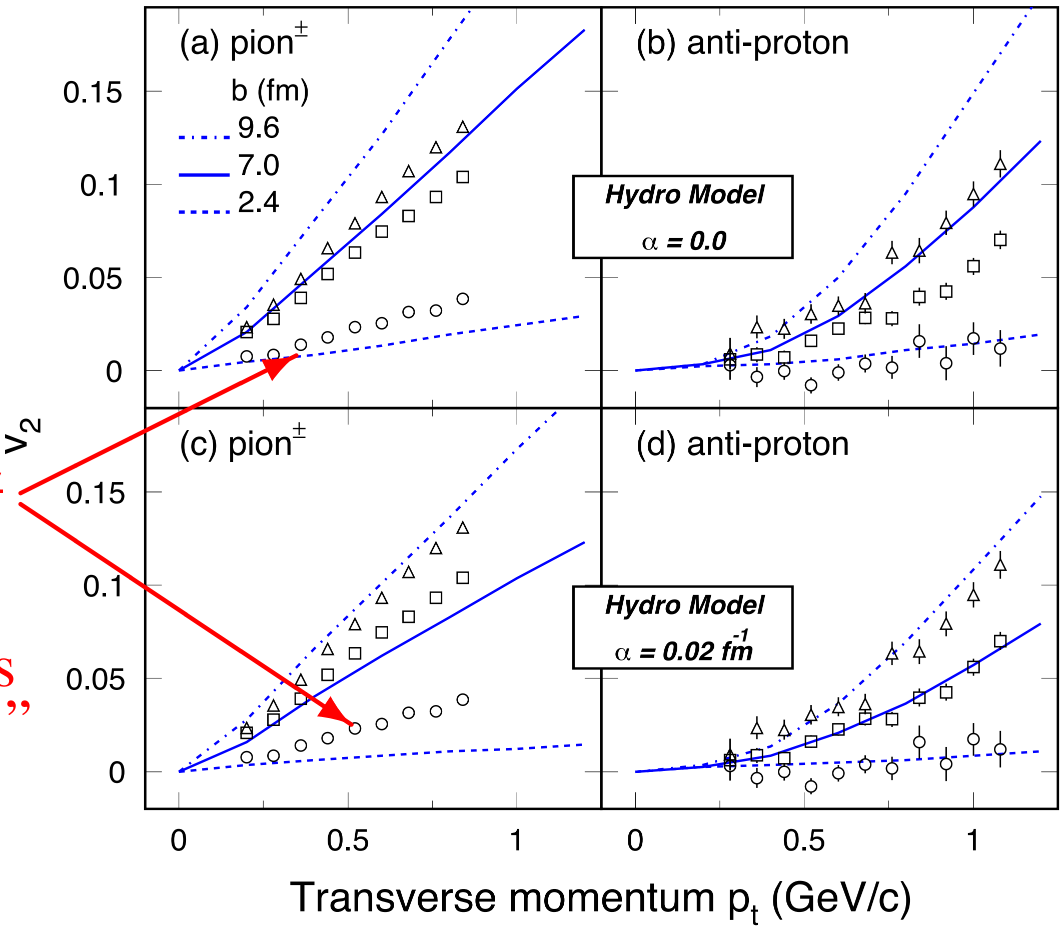
STAR Collaboration, `nucl-ex/0409033`



RHIC data:
a personal choice [2/4]

$v_2(p_t)$ for various centralities (impact parameters):

STAR Collaboration, `nucl-ex/0409033`



data above ideal fluid

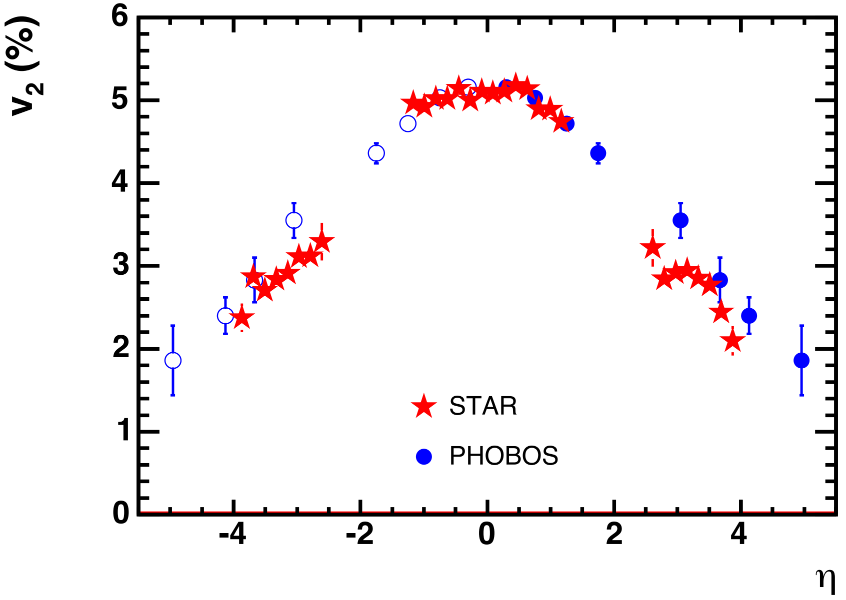
≠

"hydro yields maximum v_2"

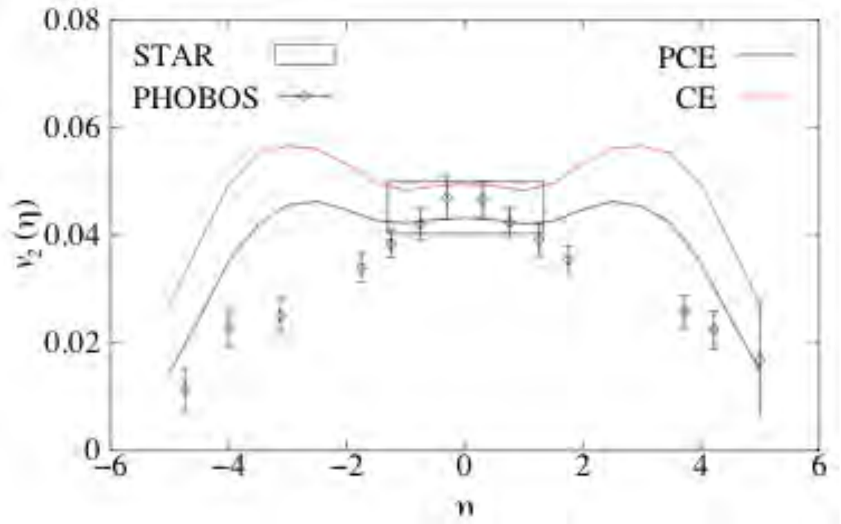
RHIC data: a personal choice [3/4]

(Pseudo)rapidity dependence of v_2

STAR Collaboration, `nucl-ex/0409033`



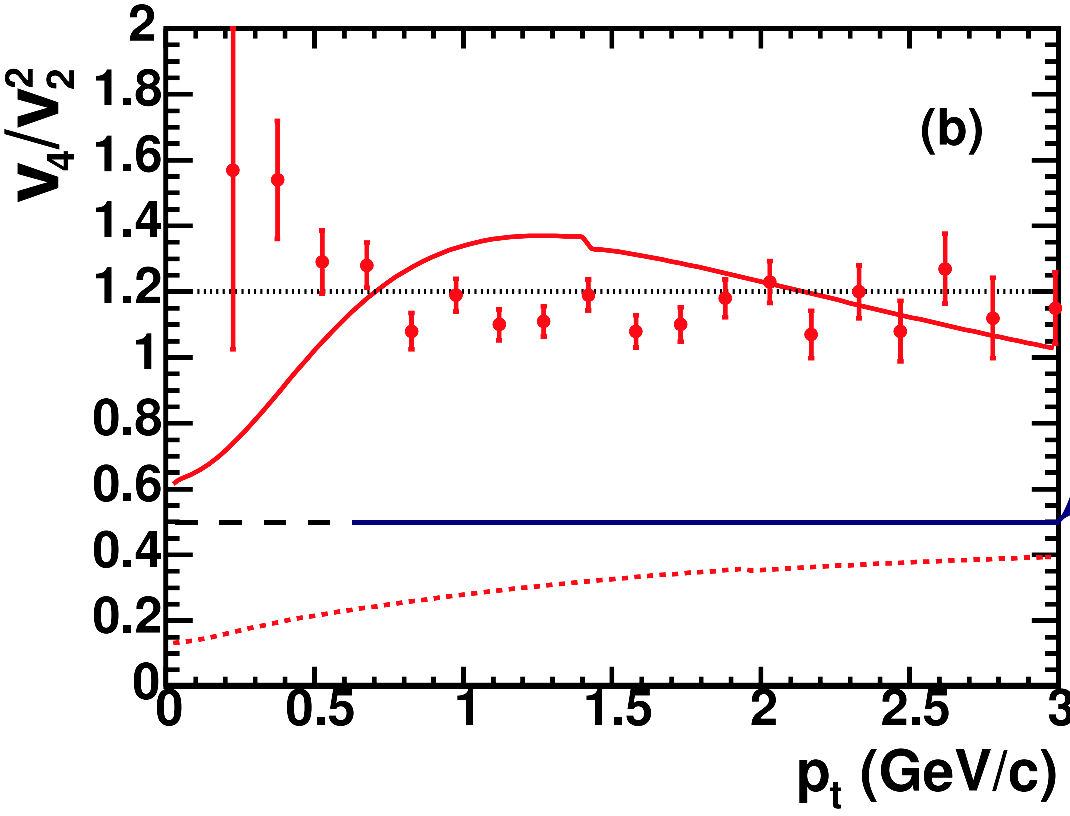
Hirano & Tsuda, Phys. Rev. C **66** (2002) 054905



RHIC data: a personal choice [4/4]

Transverse momentum dependence of $\frac{v_4}{(v_2)^2}$

STAR Collaboration, `nucl-ex/0409033`



ideal fluid prediction

Ideal fluid dynamics vs. RHIC data

$$\left.\begin{array}{l} \spadesuit\ v_2(p_t)\ \text{hydro} \ < \ \text{data} \\ \spadesuit\ v_2(y)\ \text{hydro} \ \neq \ \text{data} \\ \spadesuit\ \dfrac{v_4}{(v_2)^2}\ \text{hydro} \ < \ \text{data} \end{array}\right\}\ \text{is the ideal fluid assumption valid?}$$

Ideal fluid dynamics vs. RHIC data

$$\left.\begin{array}{l} \spadesuit\ v_2(p_t)\ \text{hydro} \ < \ \text{data} \\ \spadesuit\ v_2(y)\ \text{hydro} \ \neq \ \text{data} \\ \spadesuit\ \dfrac{v_4}{(v_2)^2}\ \text{hydro} \ < \ \text{data} \end{array}\right\} \ \text{what is wrong with ideal fluid scenario?}$$

⓪ Creation of a dense gas of particles

① At some time τ_0 (~ 0.6 fm/c in hydro models), the mean free path λ is much smaller than *all* dimensions in the system

$\Rightarrow$ thermalization, ideal fluid dynamics applies

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Ideal fluid dynamics vs. RHIC data

$$\left.\begin{array}{l} \spadesuit\ v_2(p_t)\ \text{hydro} < \text{data} \\ \spadesuit\ v_2(y)\ \text{hydro} \neq \text{data} \\ \spadesuit\ \dfrac{v_4}{(v_2)^2}\ \text{hydro} < \text{data} \end{array}\right\}\ \text{what is wrong with ideal fluid scenario?}$$

⓪ Creation of a dense gas of particles

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$\Rightarrow$ thermalization, ideal fluid dynamics applies

Is this really true?

What are the length scales in the system at time τ_0?

Heavy ion collisions: length scales

At time τ_0, two possible choices for the system size L which enters Kn

- $L = c\tau_0$ longitudinal size (strong Lorentz contraction!)
- $L = R$ transverse size (R "reduced" radius, $\frac{1}{R} = \sqrt{\frac{1}{\sigma_x^2} + \frac{1}{\sigma_y^2}}$)

At short times, $\tau_0 \lesssim 1$ fm/c, there are several possibilities:

1. $\lambda \ll c\tau_0$: early thermalization (preferred by most)
2. $\lambda \sim c\tau_0$
3. $c\tau_0 \ll \lambda \ll R$: only "transverse" thermalization
4. $\lambda \sim R$
5. $\lambda \gg R$: "initial state" dominates

☞ Anisotropic flow cannot resolve 1–3
RHIC data favor 4

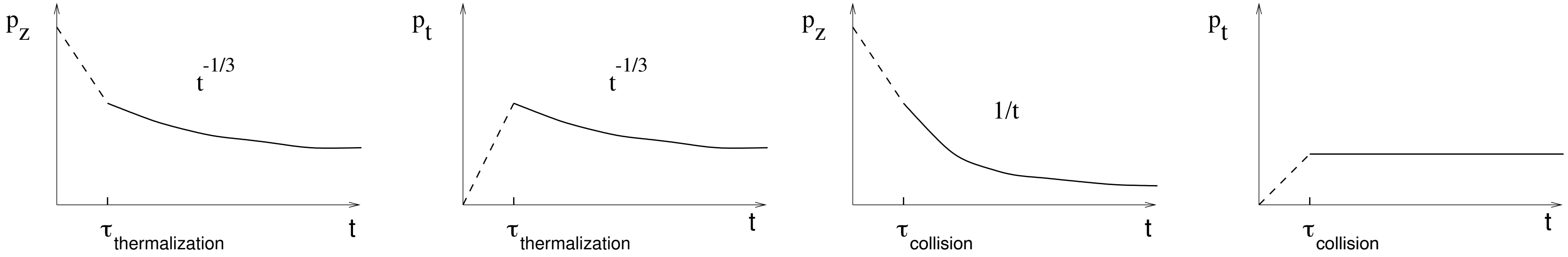
Full thermalization vs. transverse thermalization

Full equilibrium (case 1):

- First, make momenta isotropic: produce huge p_t
- Then, longitudinal expansion decreases p_z: need to decrease p_t also: cooling

No longitudinal equilibrium, transverse equilibrium (cases 3–5):

- Fast longitudinal expansion: longitudinal pressure ~ 0
- Transverse momenta do not change

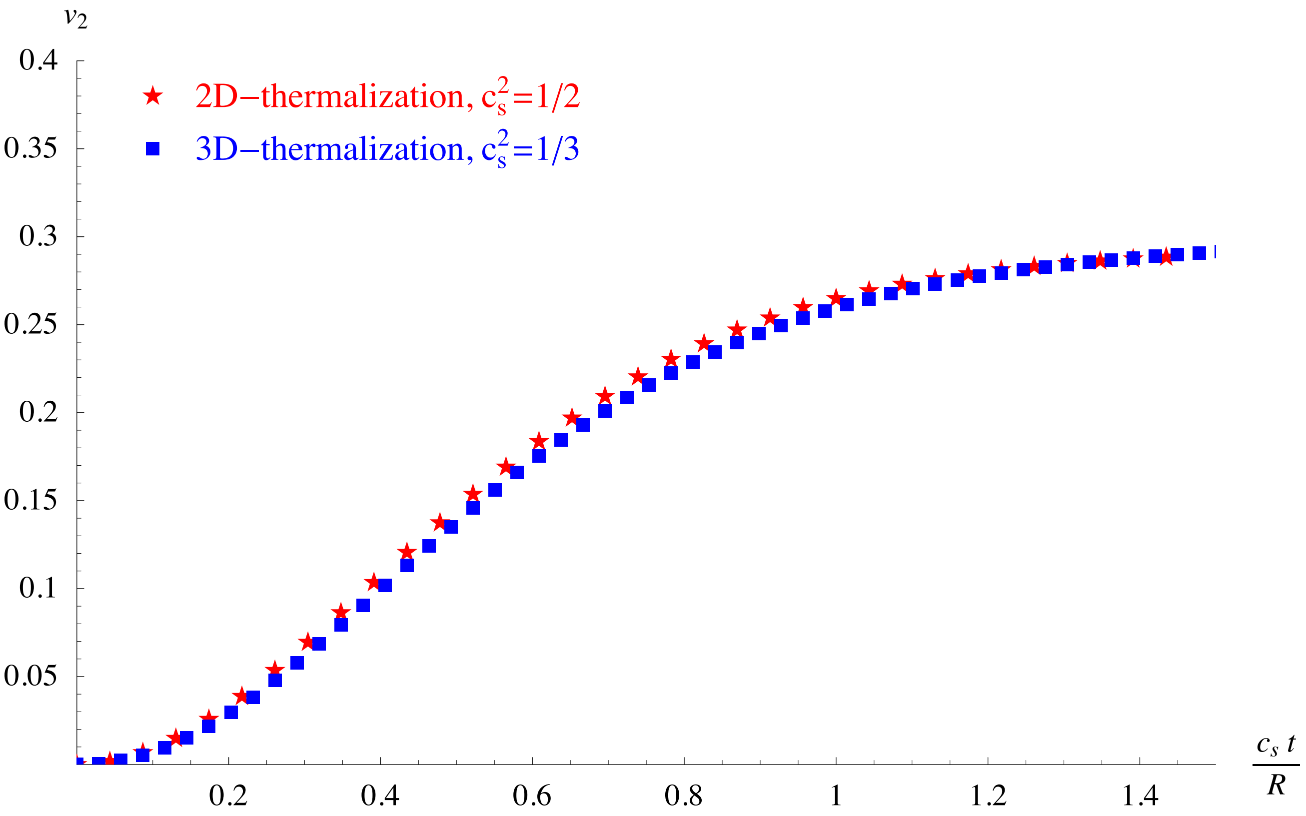


But only the final p_t are measured, not their time evolution

Full thermalization vs. transverse thermalization

v_2 cannot distinguish between full and only transverse equilibration

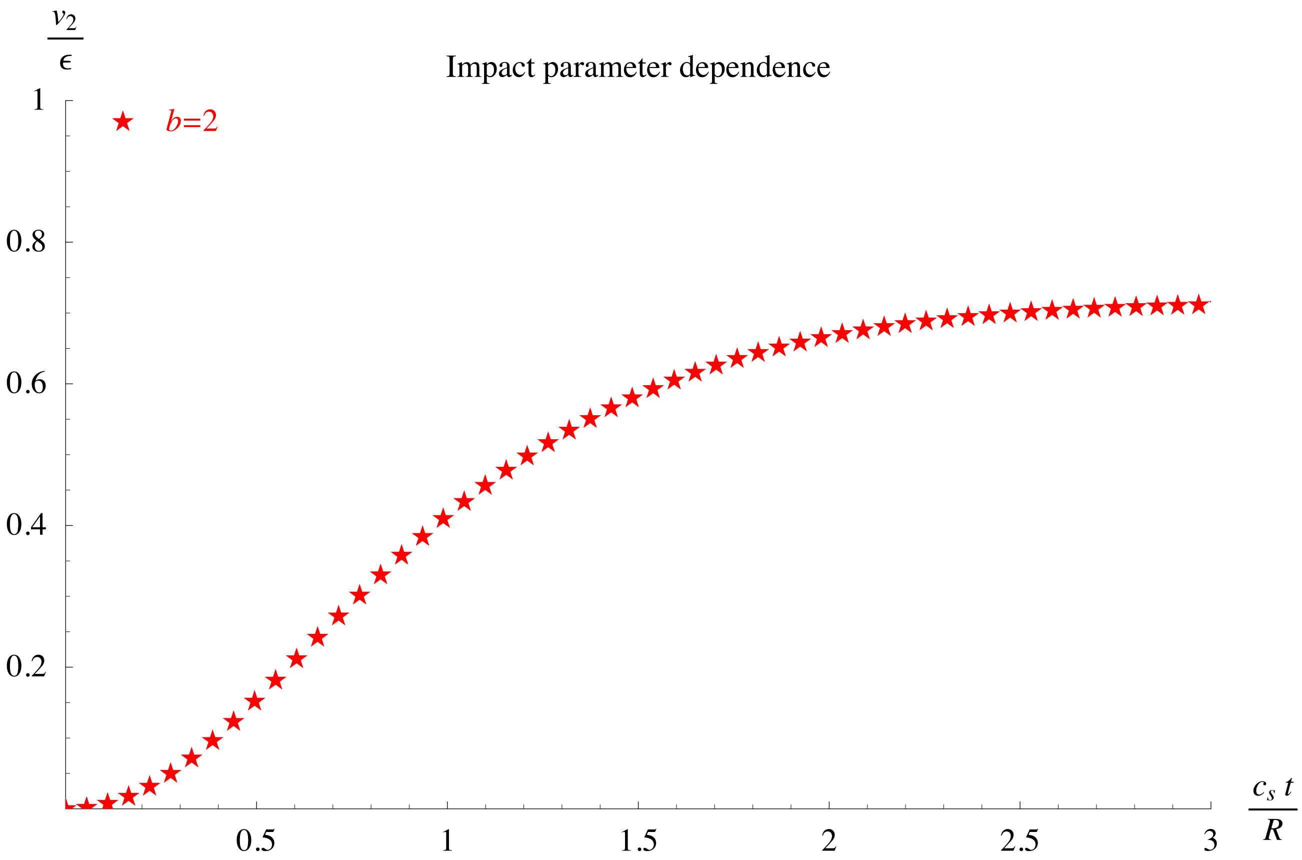
$P_z = P_x = P_y$ ↵ (full) — only transverse ↳ $P_z = 0$: free streaming



Dependence of v_2 on centrality

The natural time scale for v_2 is R/c_s: massless particles

$$c_s^2 = \frac{1}{3}$$



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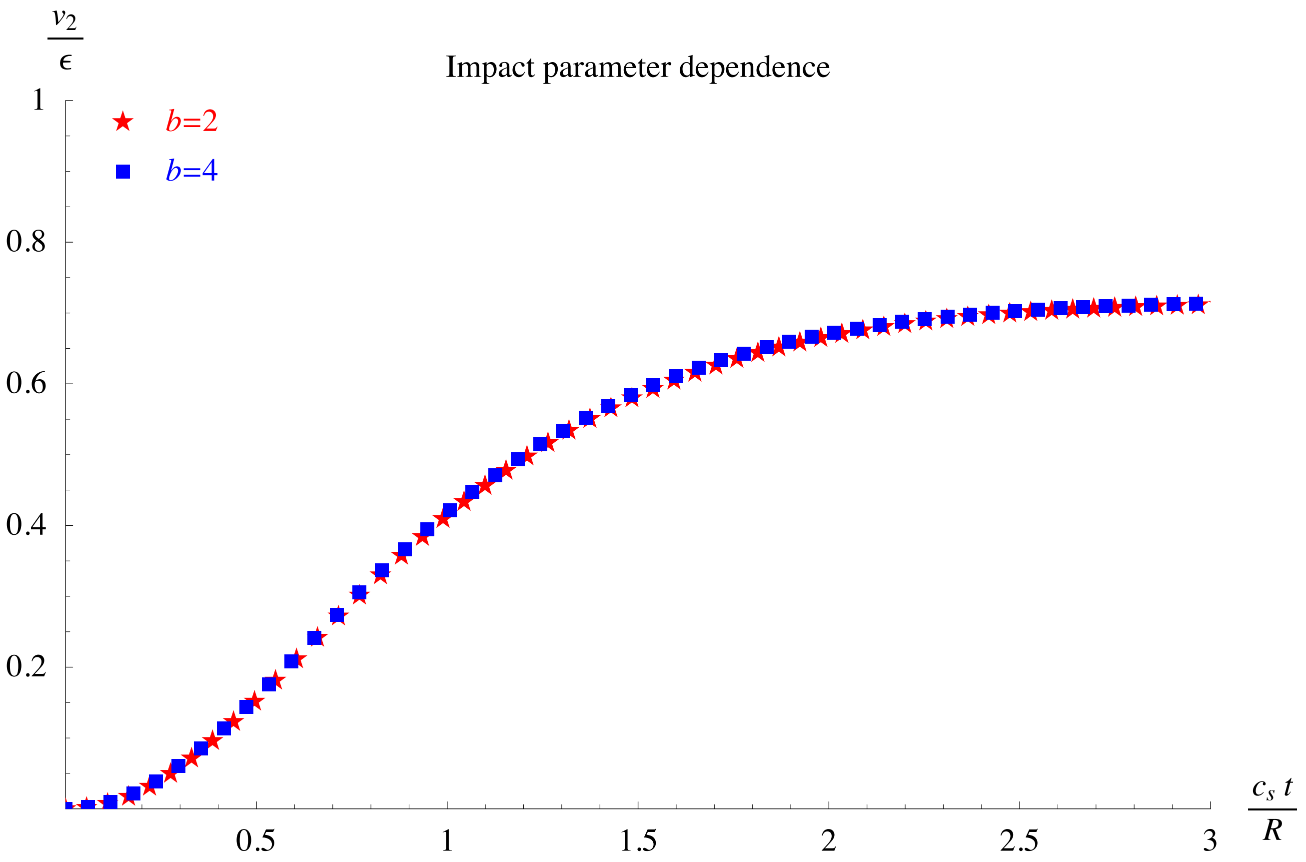

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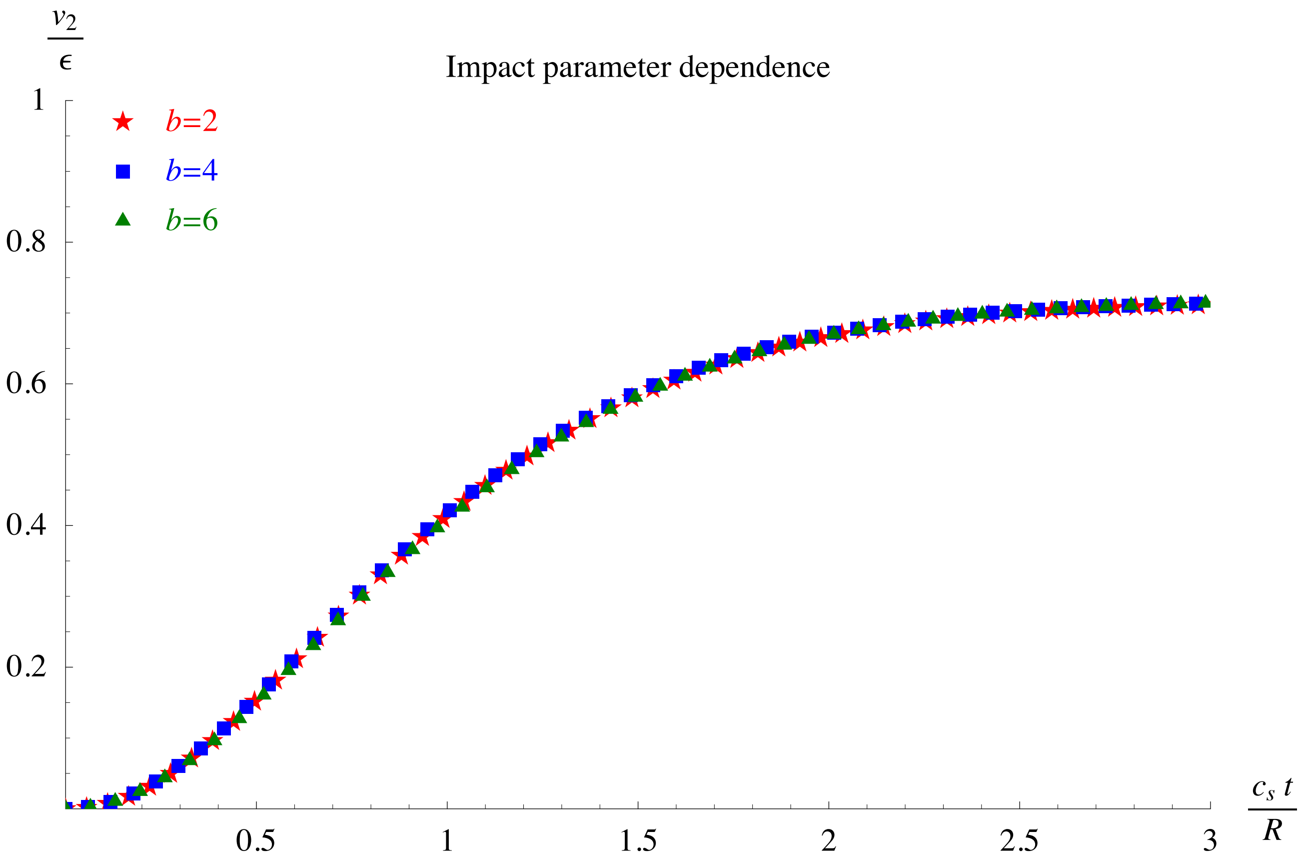

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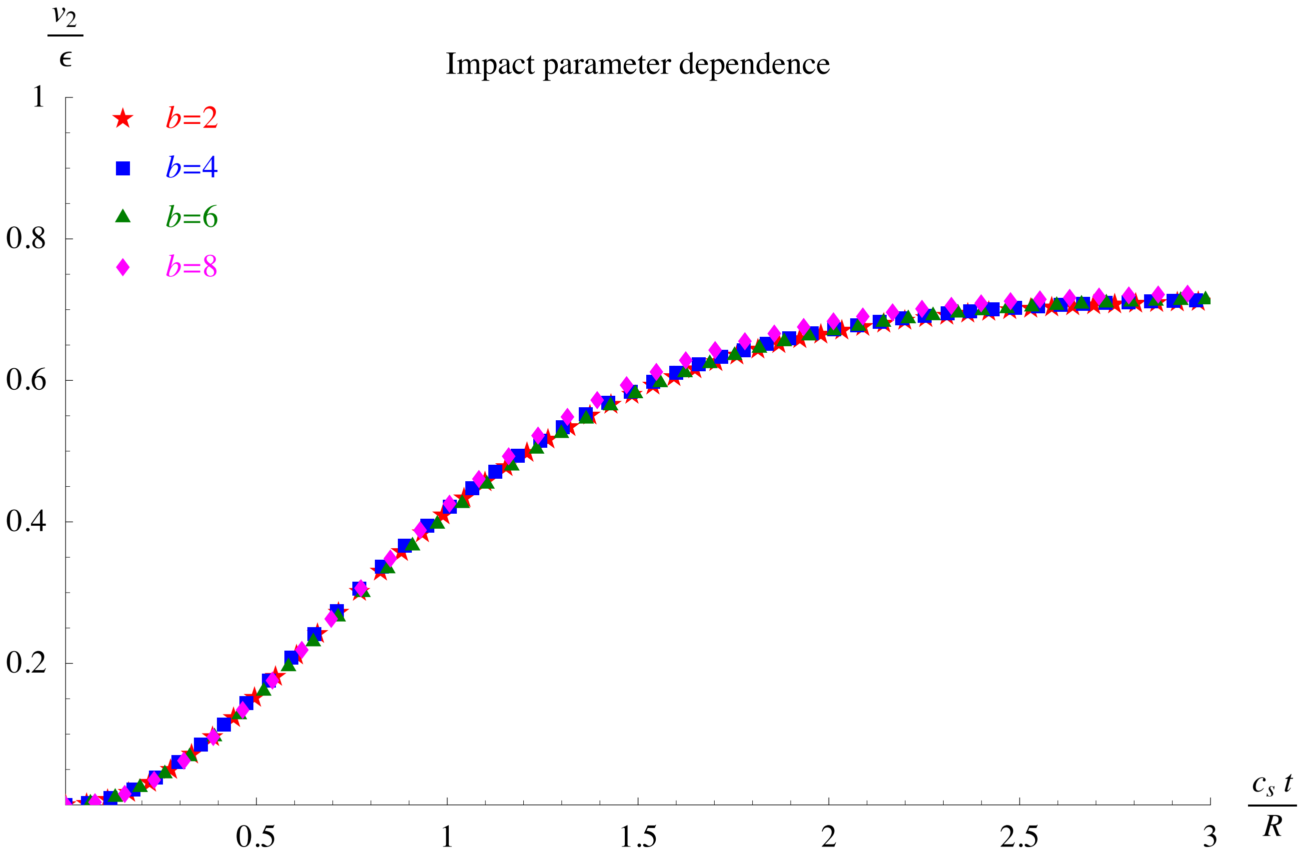

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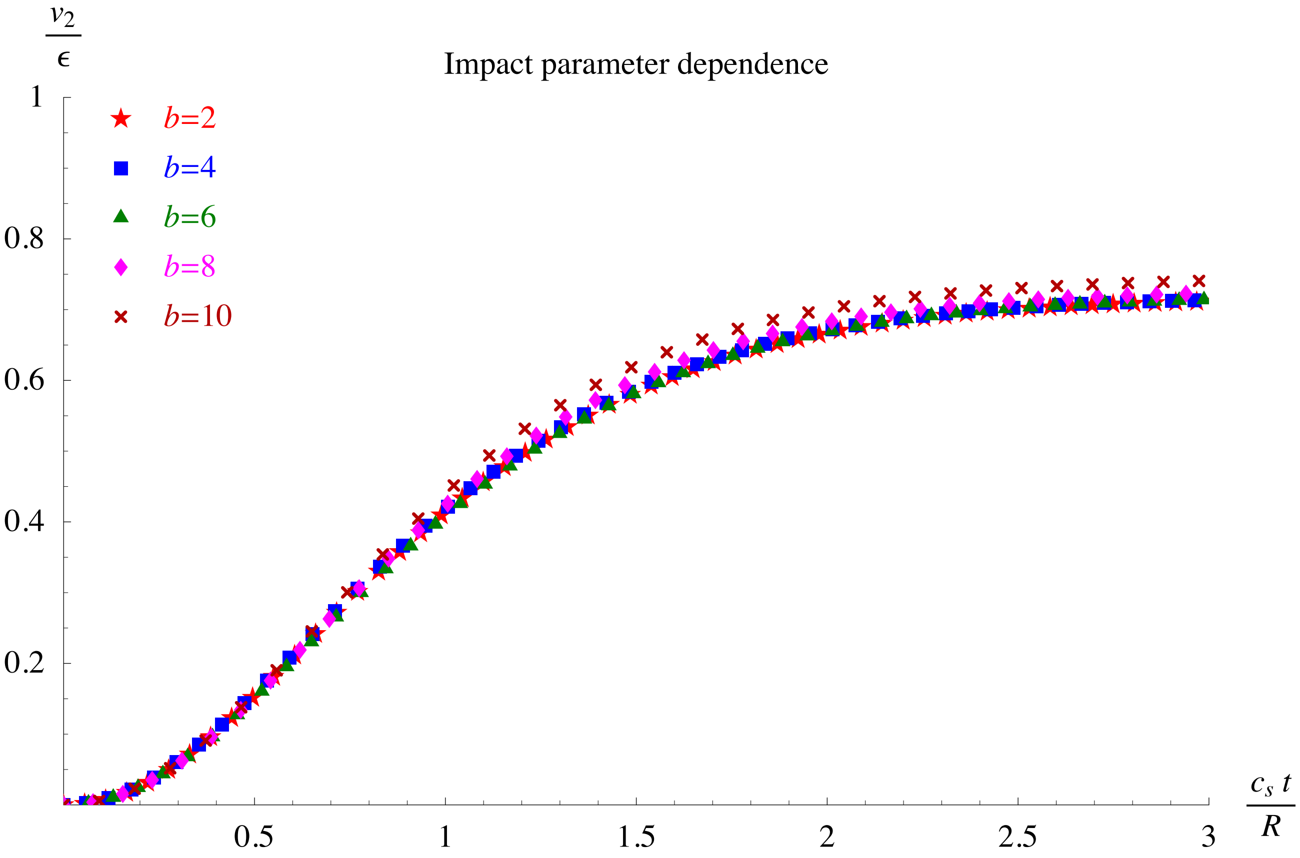

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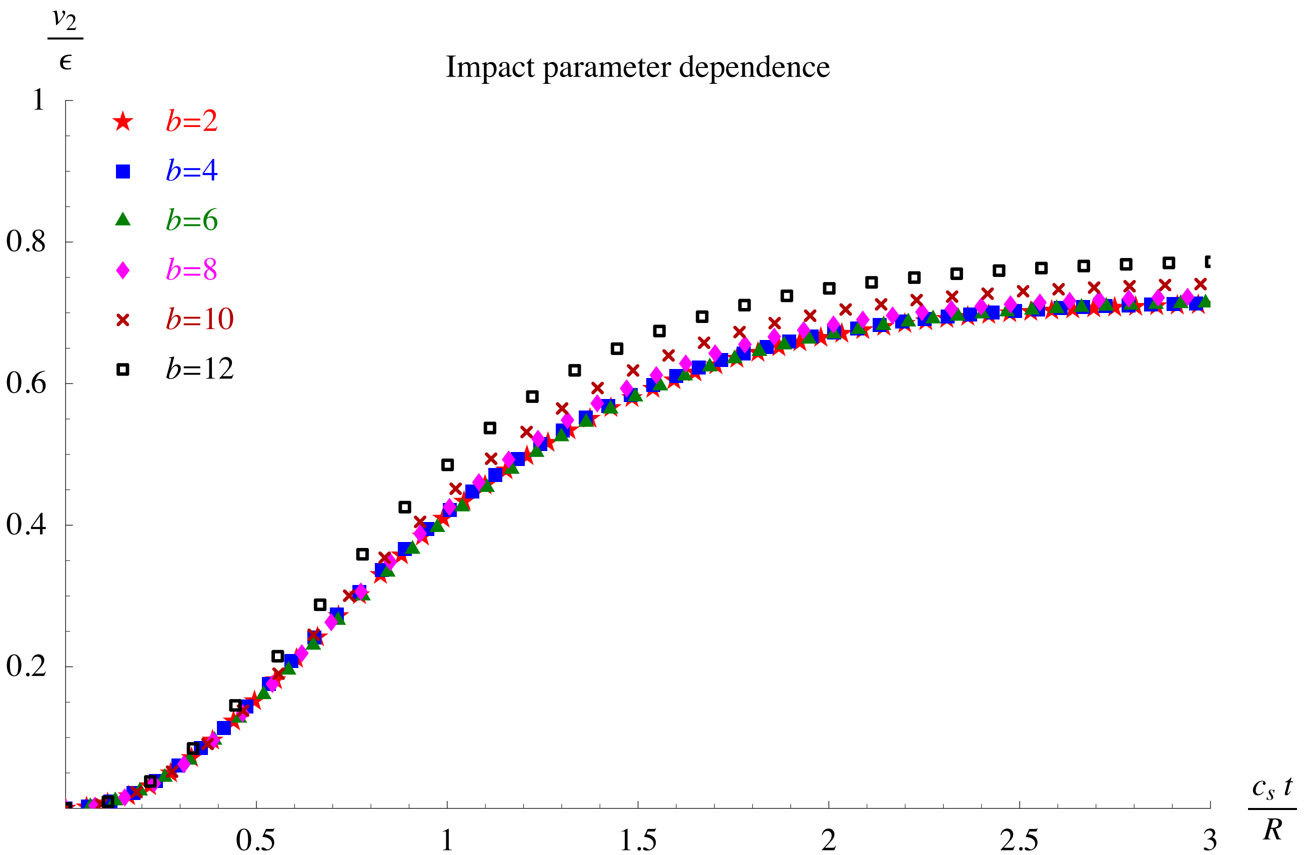


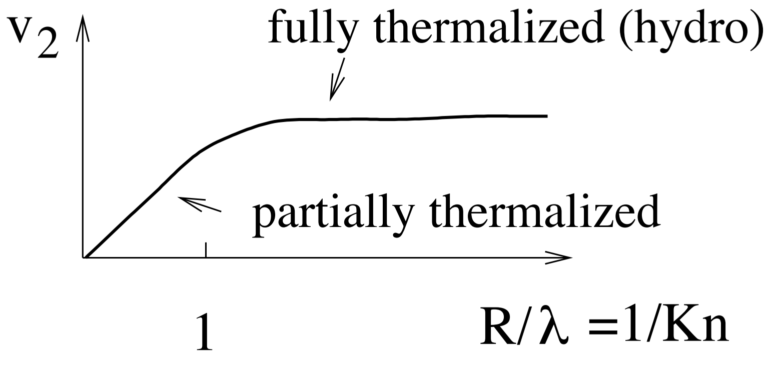
v_2 knows nothing about early times!

Anisotropic flow: a control parameter

The natural time scale for v_2 is R/c_s

$\Rightarrow$ number of collisions to build up v_2:

$$\frac{1}{Kn} \simeq \frac{R}{c_s\lambda} = \frac{R\sigma n(\tau)}{c_s}$$



σ interaction cross section, $n(\tau)$ particle density

Anisotropic flow: a control parameter

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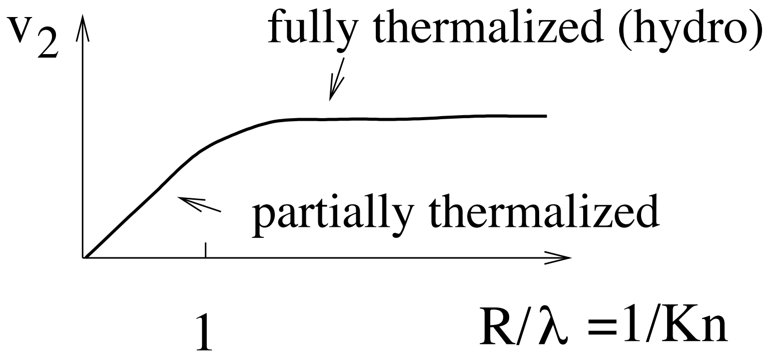


$$\frac{1}{Kn} \simeq \frac{R}{c_s\lambda} = \frac{R\sigma n(\tau)}{c_s} \simeq \frac{c}{c_s}\frac{\sigma}{S}\frac{\mathrm{d}N}{\mathrm{d}y}$$

σ interaction cross section, $n(\tau)$ particle density, S transverse surface

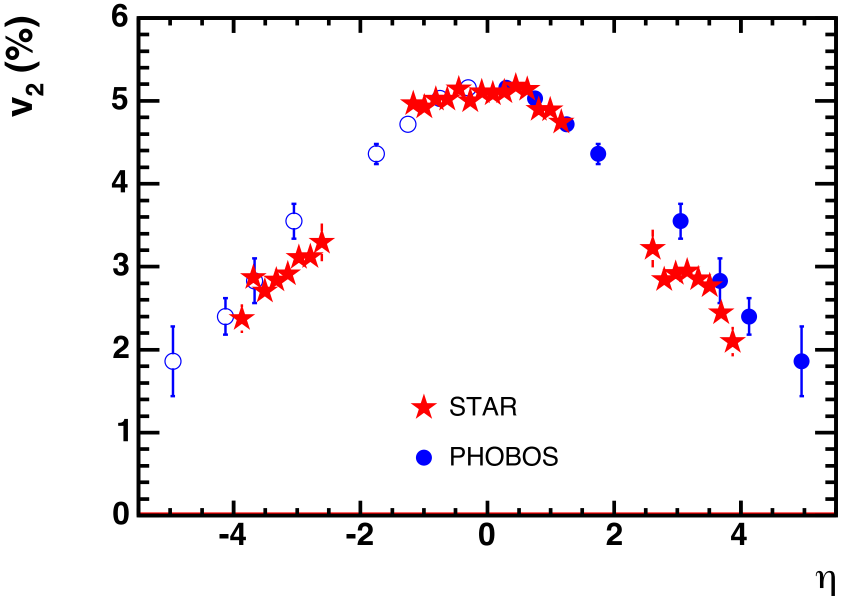
☞ $\frac{1}{S}\frac{\mathrm{d}N}{\mathrm{d}y}$ control parameter for v_2: to vary Kn, one can study

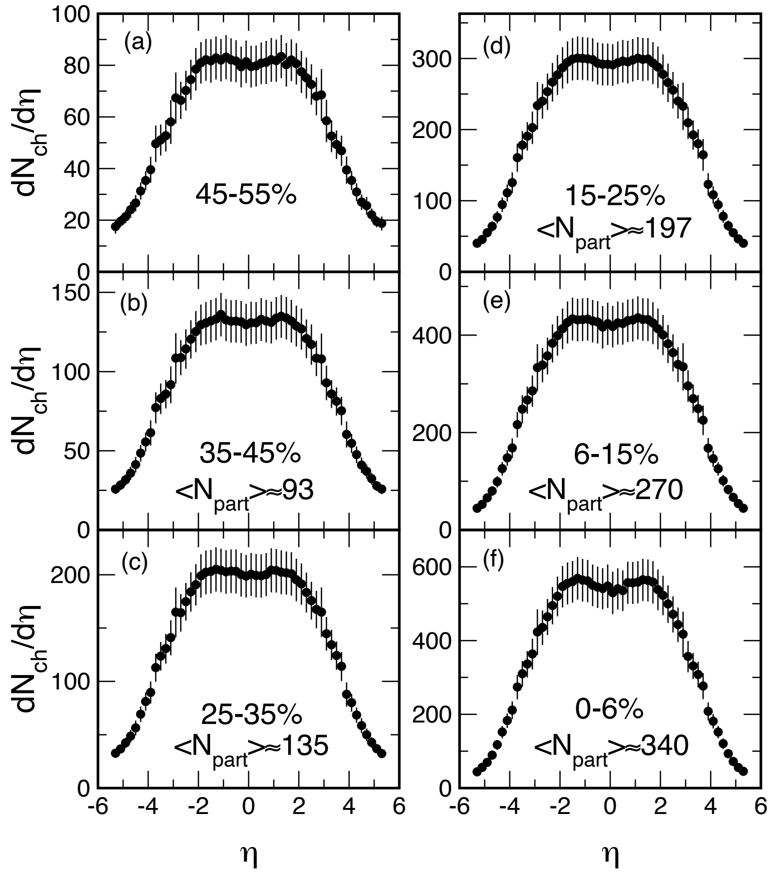
- rapidity dependence
- centrality dependence
- system-size dependence → importance of lighter systems!
- beam-energy dependence

RHIC data: incomplete thermalization

(Pseudo)rapidity dependence of v_2

STAR Collaboration, `nucl-ex/0409033`





☞ can be explained by incomplete thermalization

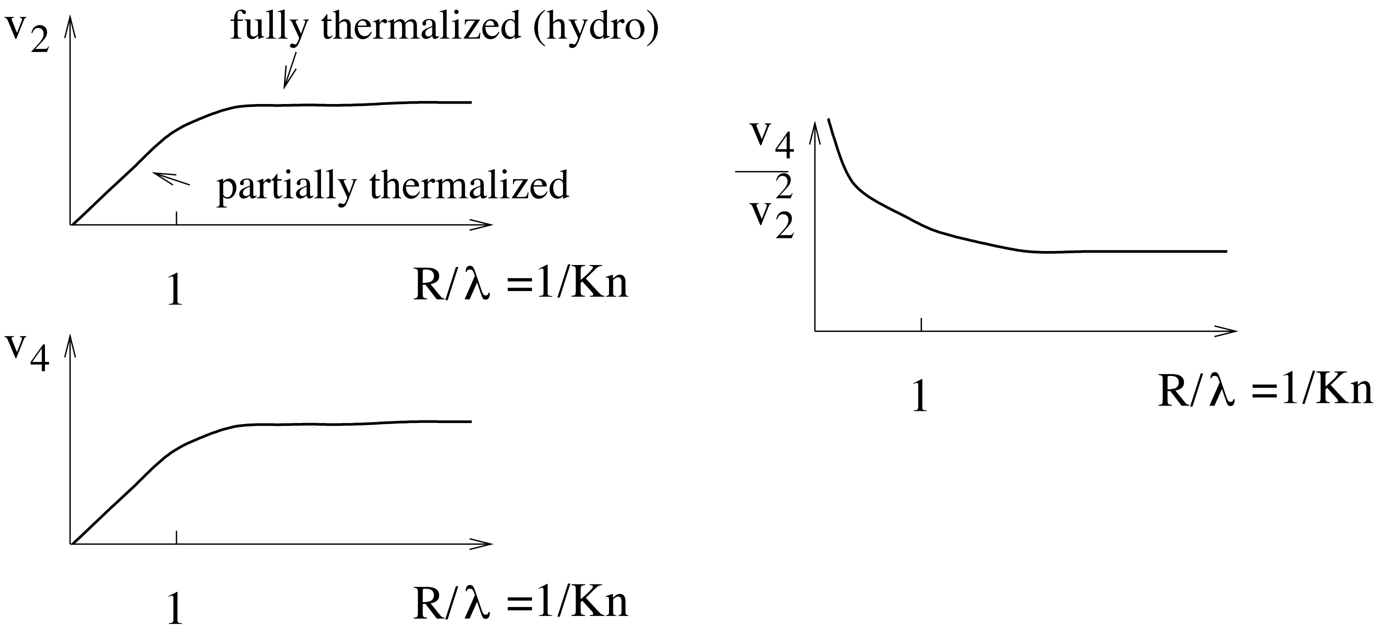
Hirano, Phys. Rev. C **65** (2002) 011901

RHIC data: incomplete thermalization

Ideal fluid dynamics predicts $\frac{v_4}{(v_2)^2} = \frac{1}{2}$, RHIC data is above ($\sim 1.2$)

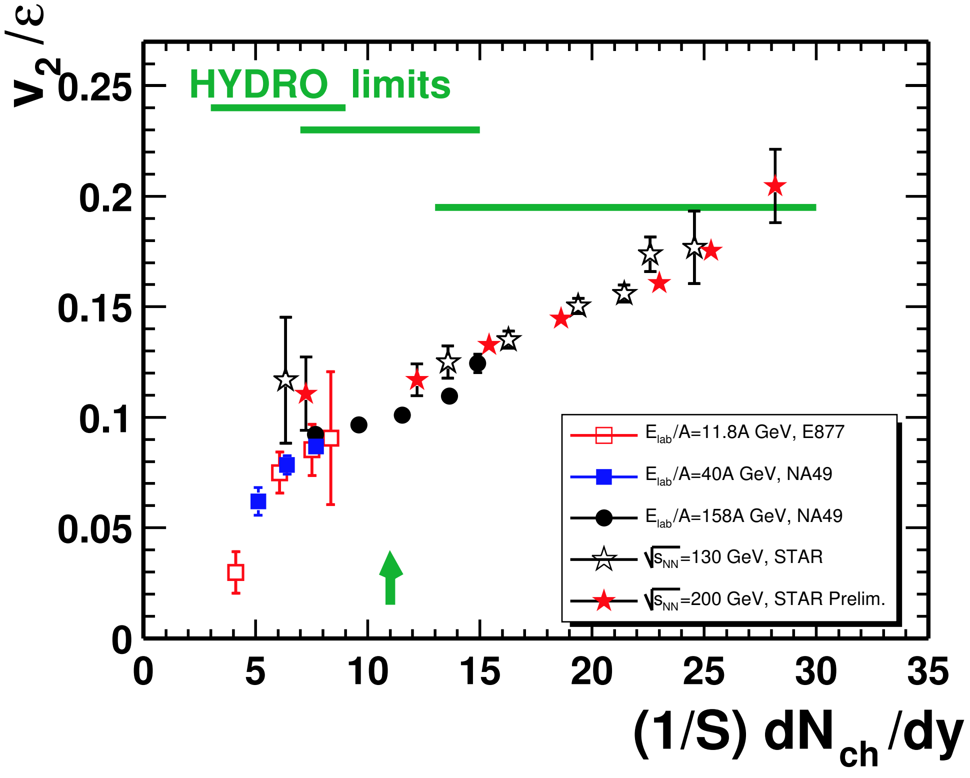
☞ increase can be explained by incomplete thermalization

In a “one-collision” model, one can show that $v_n \propto \sigma \quad \Rightarrow \quad \frac{v_4}{(v_2)^2} \propto \frac{1}{\sigma}$



Anisotropic flow: a control parameter

Beam-energy dependence:

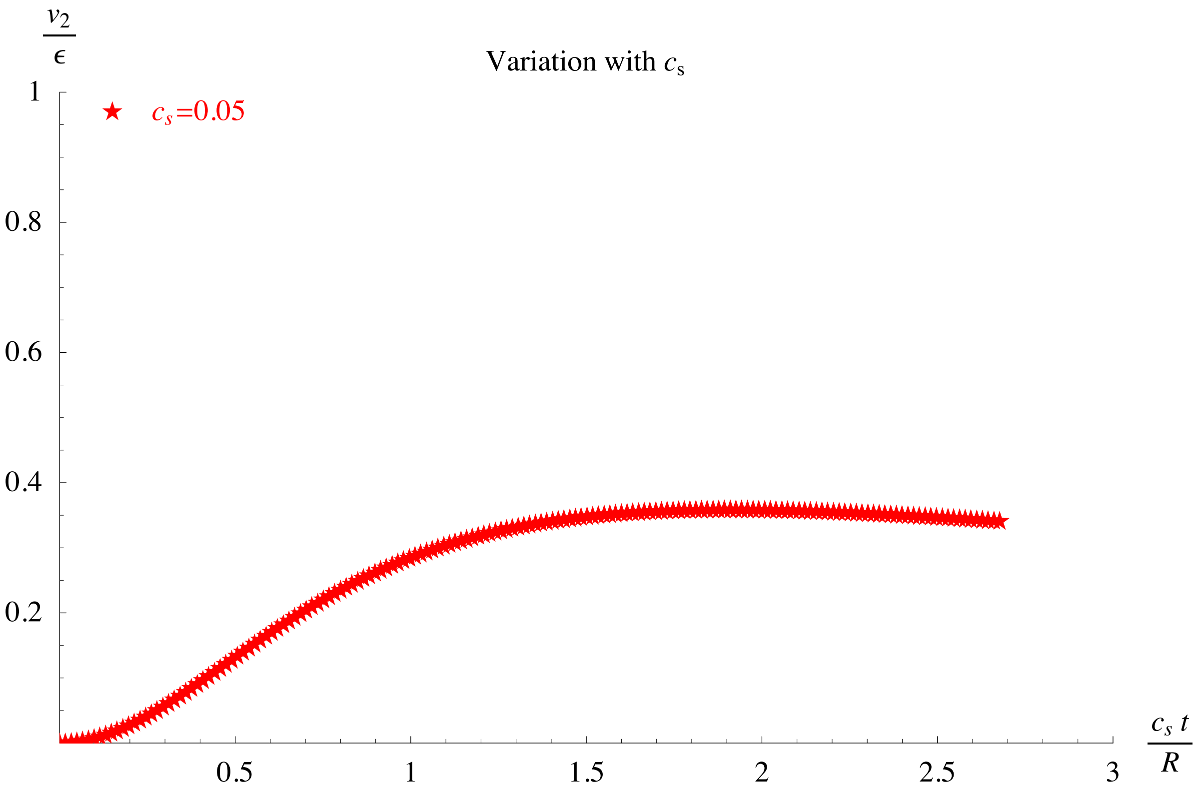


NA49 Collaboration, Phys. Rev. C **68** (2003) 034903

Scaling law seems to work but data alone does not point to a saturation of v_2 as expected from ideal fluid behaviour

Dependence of v_2 on the speed of sound

How can data overshoot the “ideal fluid limit”?



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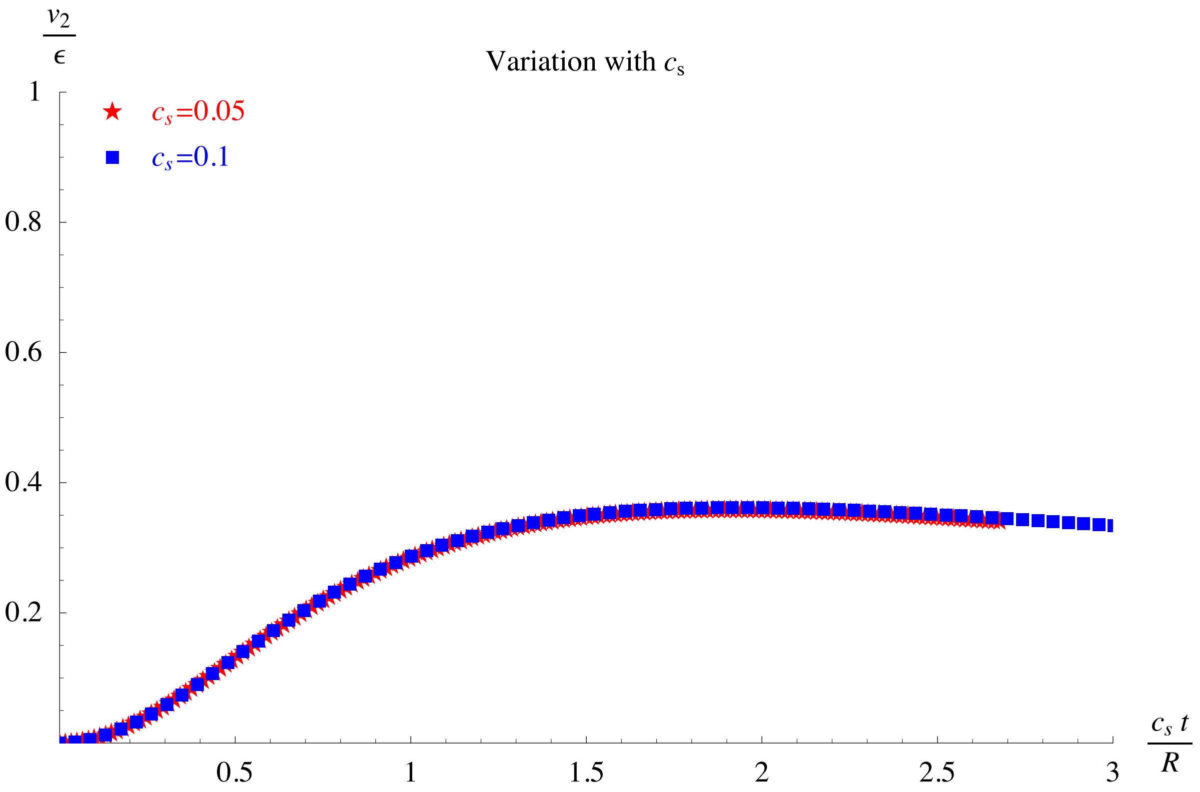

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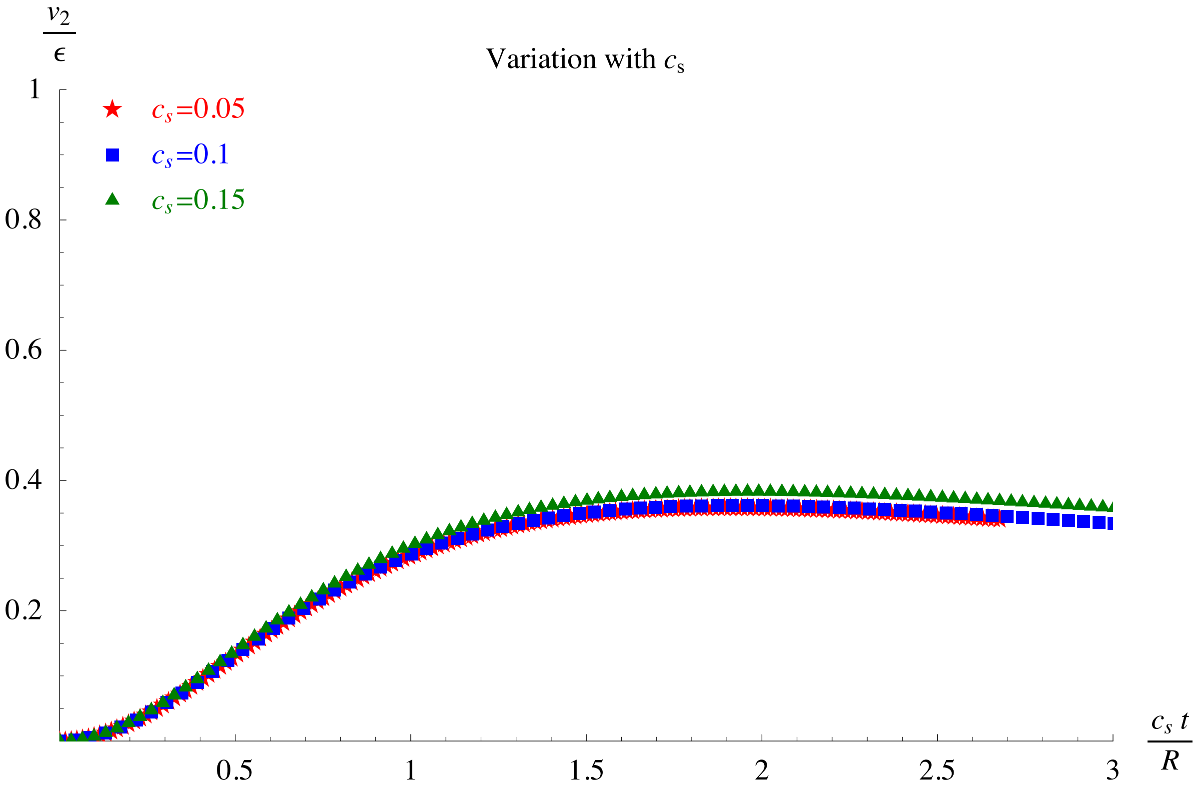

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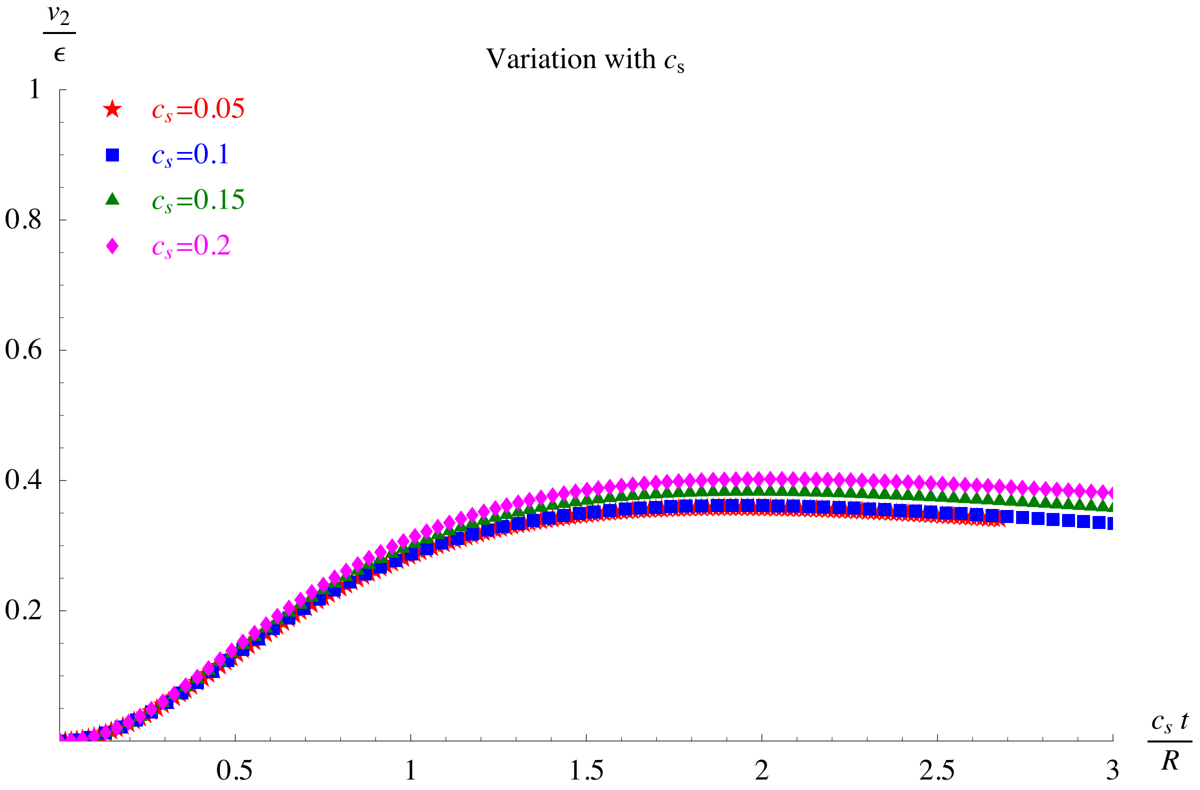

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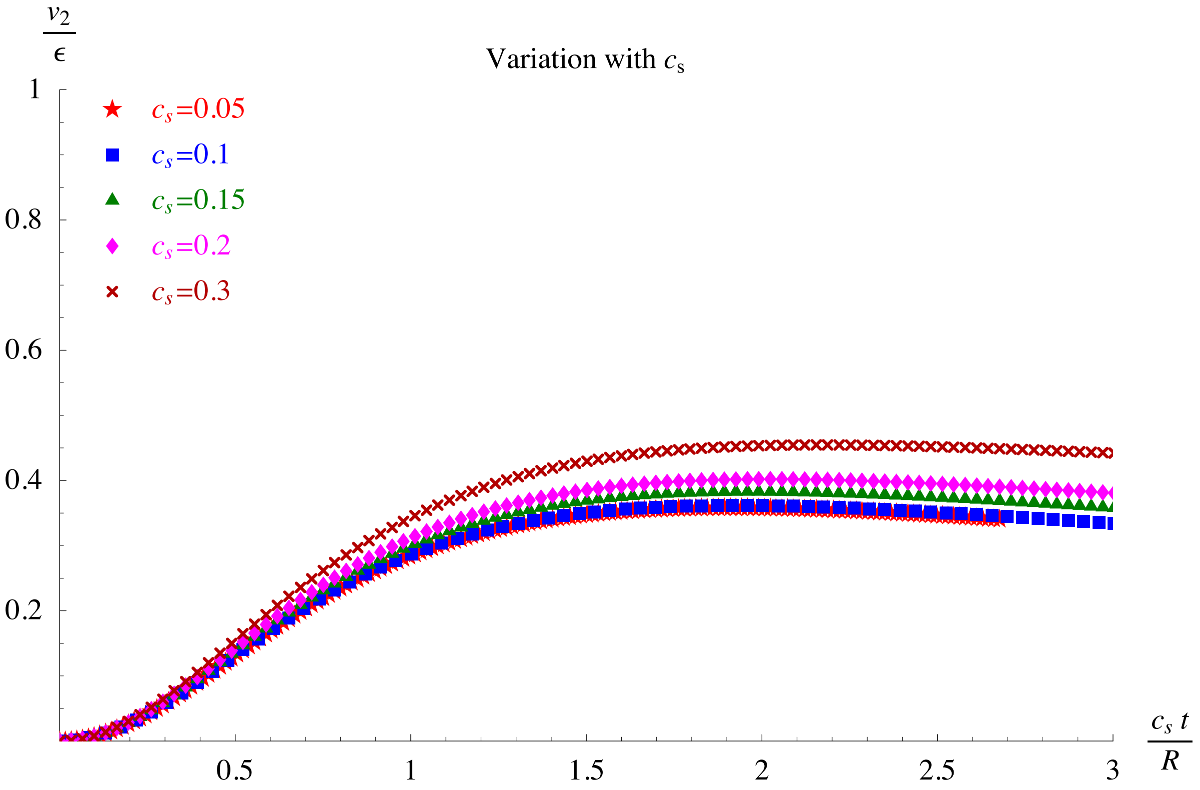

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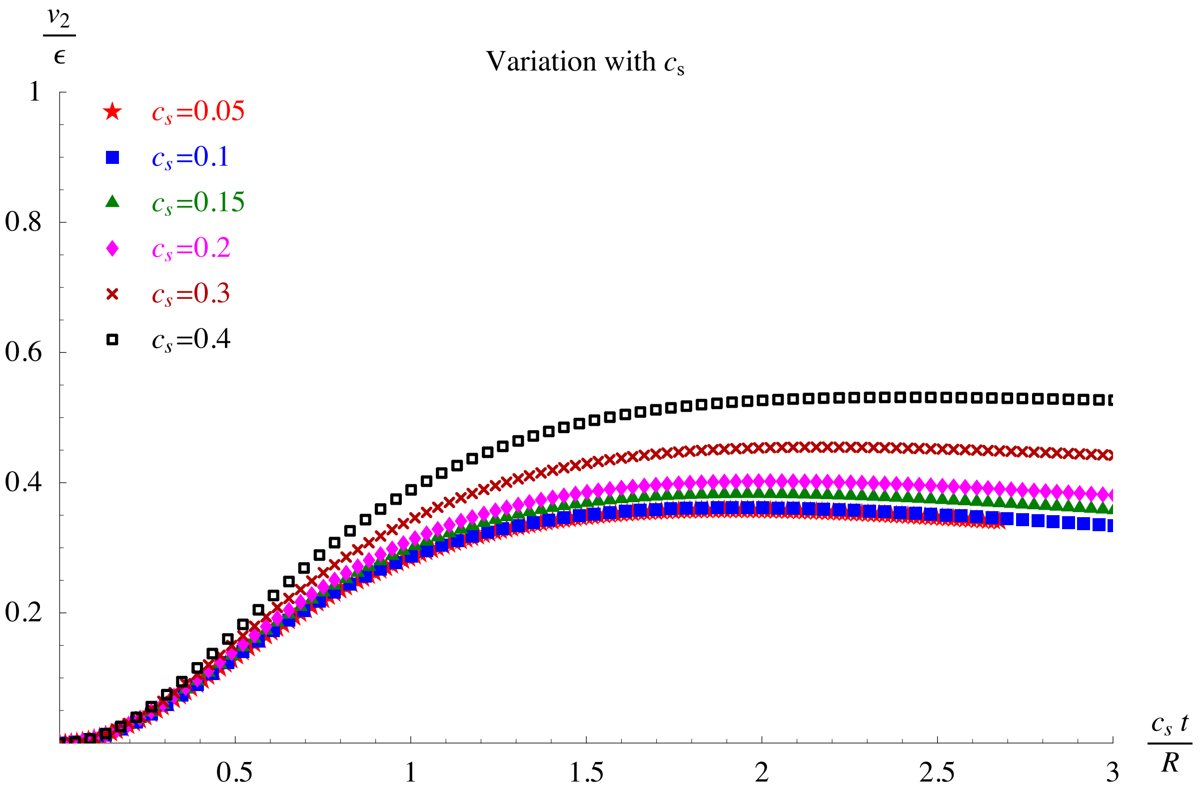

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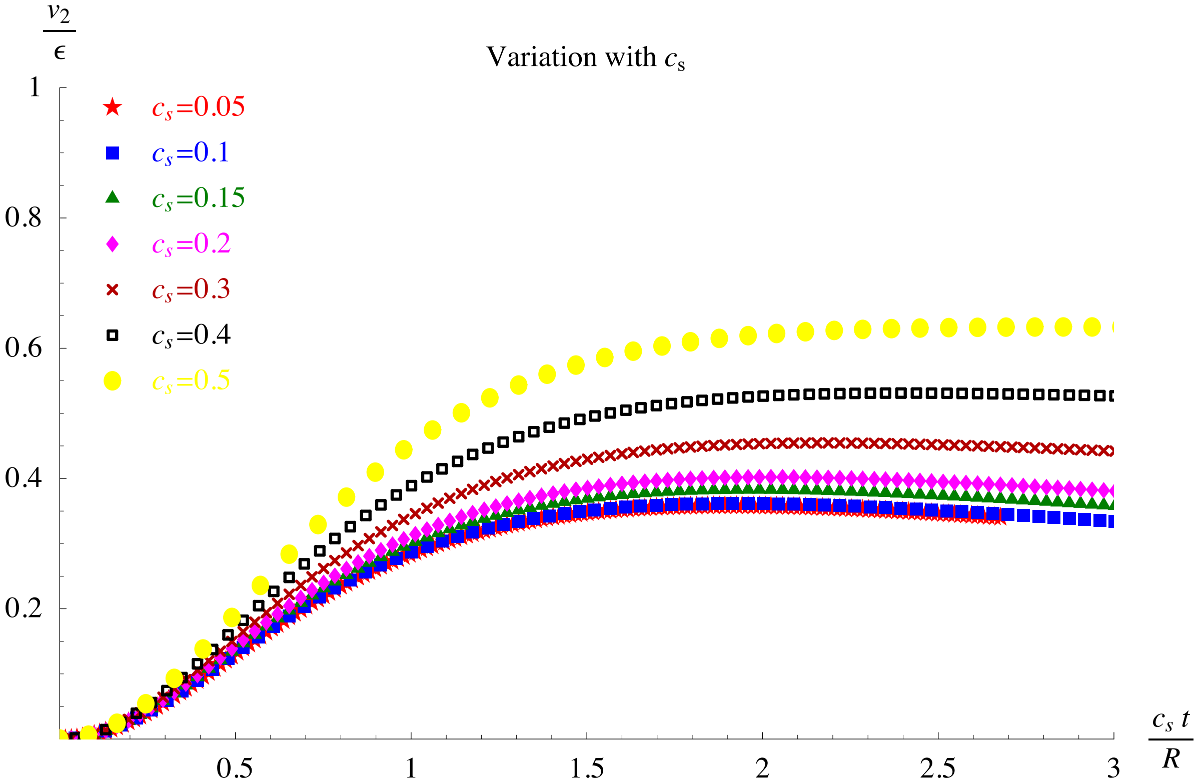

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☞ one can increase v_2 by increasing c_s!

Incomplete thermalization at RHIC

What you will (hopefully) find in the paper(s) in preparation by R.S. Bhalerao, J.-P. Blaizot, N.B. and J.-Y. Ollitrault

- A reminder: the natural time scale for anisotropic flow is $\dfrac{R}{c_s}$
  - no knowledge about early times
  - anisotropic flow cannot conclude on transverse equilibration, i.e., full thermalization
- Size of v_2 controlled by $\dfrac{1}{S}\dfrac{\mathrm{d}N}{\mathrm{d}y}$ but no hint at saturation in the data

  incomplete transverse equilibration: $\lambda \sim R$

  ☞ anisotropic flow tool to measure λ
- v_2 overshoots the hydro prediction... because there is a crossover, not a first-order phase transition
- Predictions for Cu–Cu collisions at RHIC (and for LHC?)