COLLECTIVE FLOW ANALYSIS FROM MULTIPARTICLE CORRELATIONS

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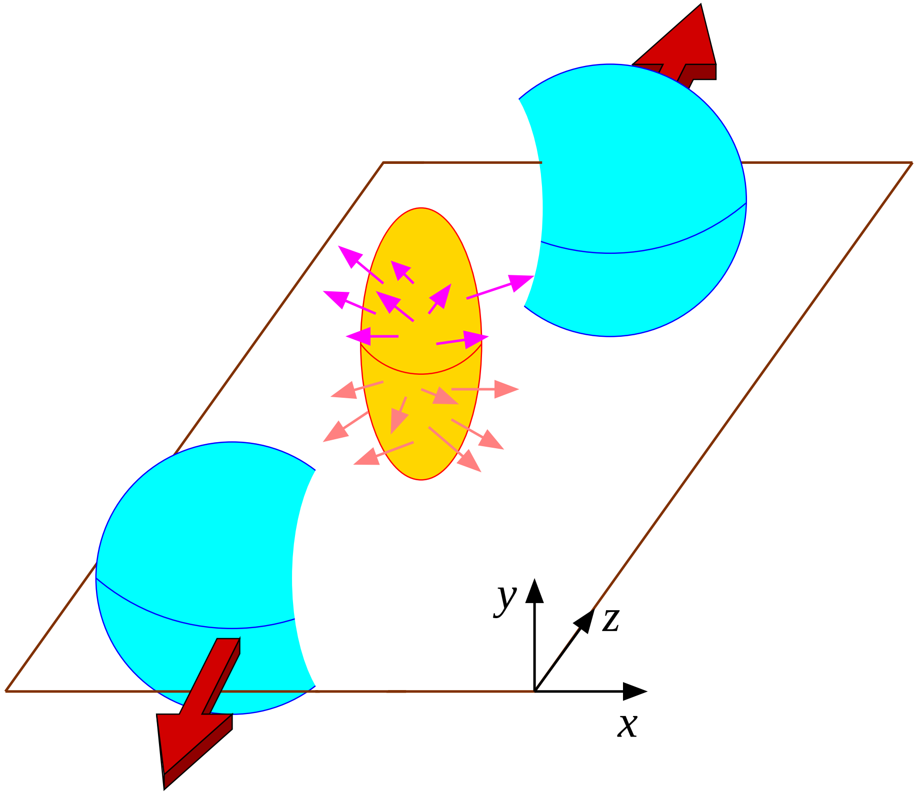
- Collective flow: v_1, v_2
- Methods of flow analysis
  - ♠ two-particle method(s): any v_n
  - ♡ four-, six-, eight-particle method: any v_n
  - ♡ three-particle method: v_1 only
- Application to experimental (NA49, STAR) data

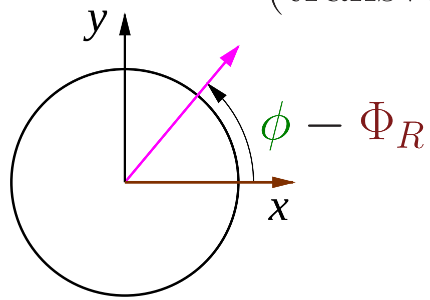
ANISOTROPIC FLOW



Source anisotropy

$\Rightarrow$ anisotropic emission of particles:

(transverse) FLOW



$$\frac{\mathrm{d}N}{\mathrm{d}\phi} \propto 1 + 2\, v_1 \cos(\phi - \Phi_R) + 2\, v_2 \cos 2(\phi - \Phi_R) + \ldots$$

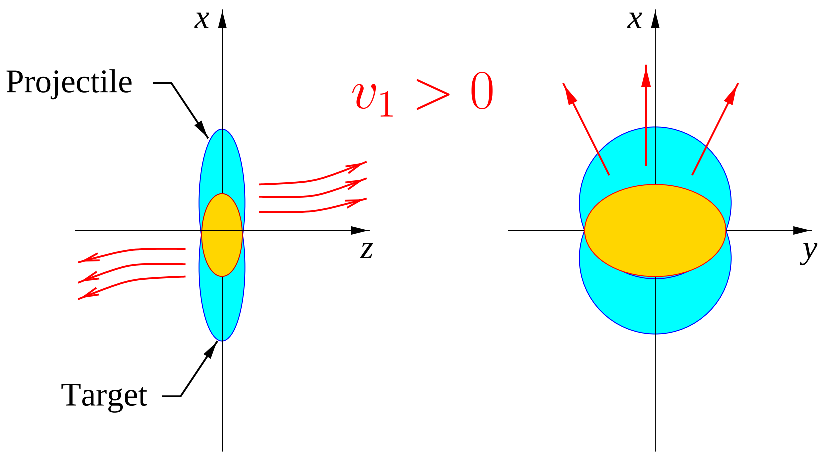
v_1 "directed", v_2 "elliptic"

$$v_n = \langle \cos n(\phi - \Phi_R) \rangle$$

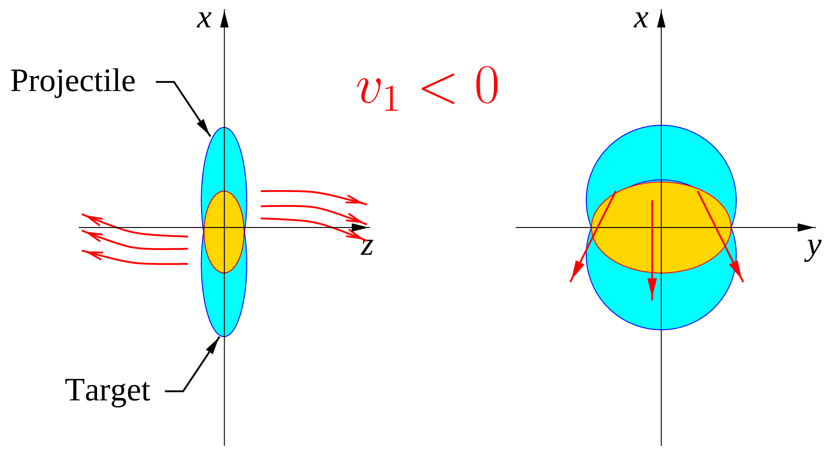
v_1, v_2 $\Rightarrow$ source equation of state

v_1 PHYSICS (ultrarelativistic energies)

Projectile protons "bounce" off target:



Pions flow opposite from protons:

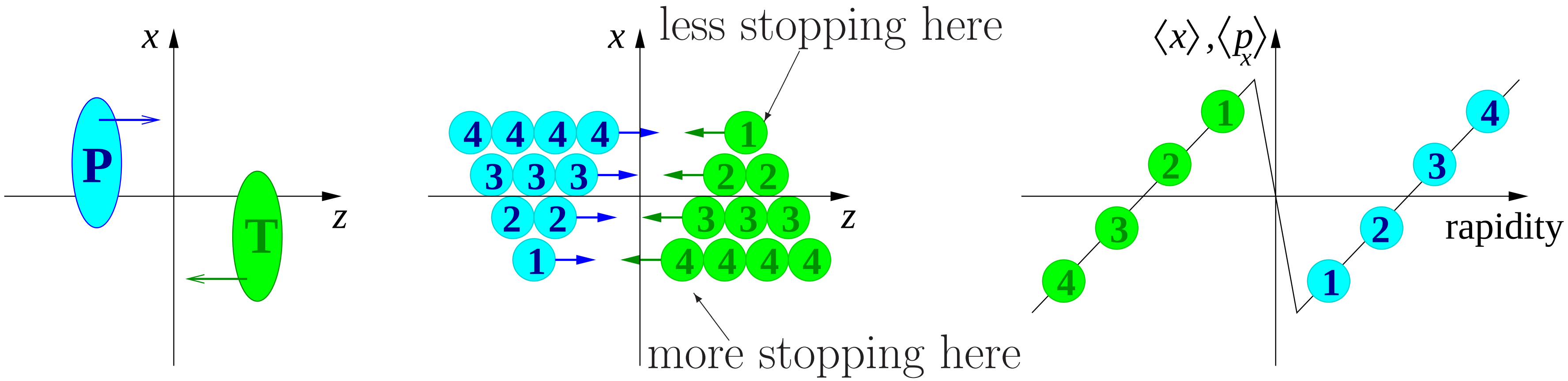


A priori, in the forward (projectile) region
- proton v_1 positive
- pion v_1 negative

v_1 PHYSICS (ultrarelativistic energies)

"Antiflow":

Assume space-momentum correlation & incomplete baryon stopping

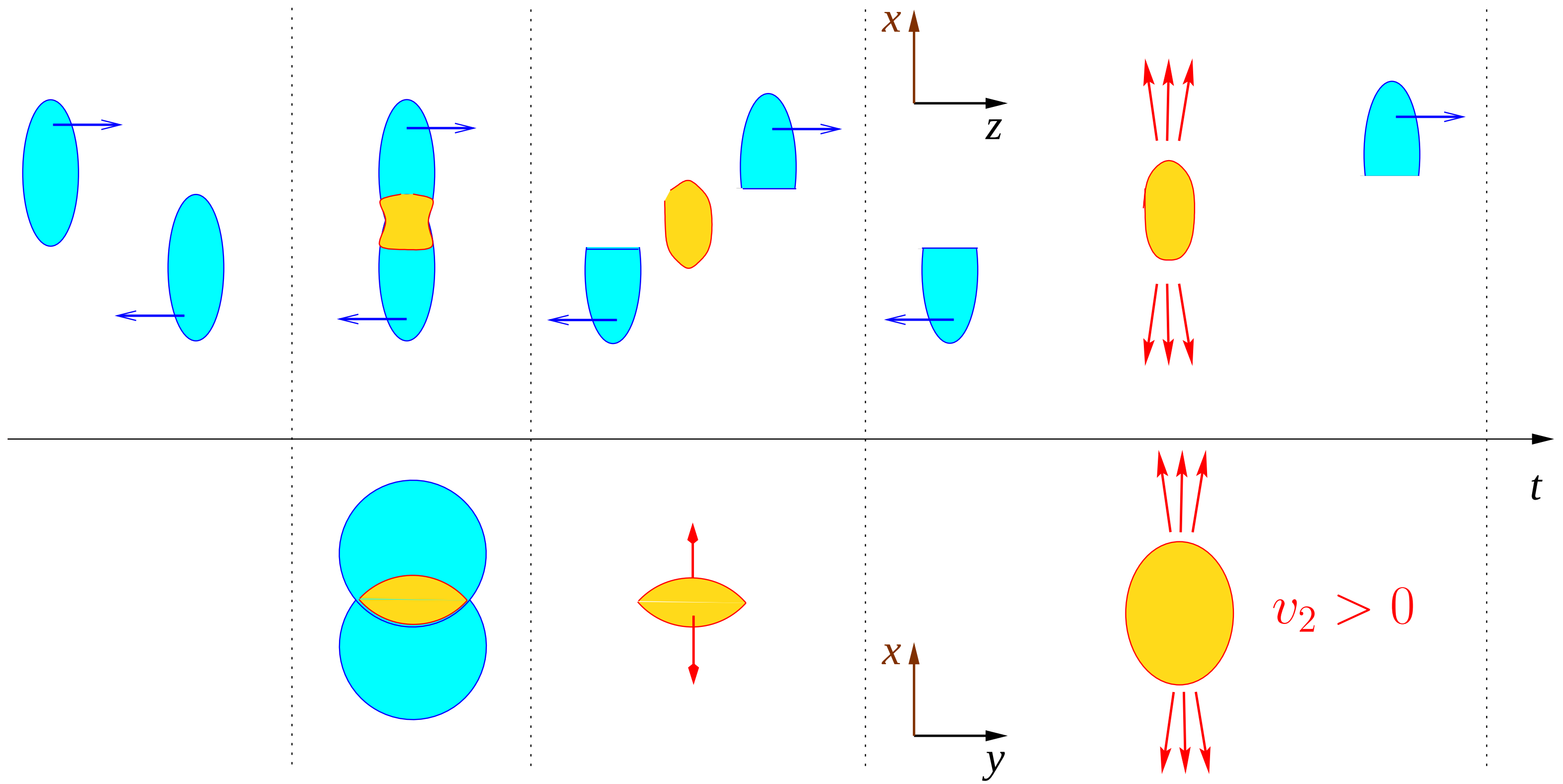


$\Longrightarrow$ negative proton v_1 just above midrapidity ("v_1 wiggle")

R. J. M. Snellings *et al.*, Phys. Rev. Lett., 2000

v_2 PHYSICS (ultrarelativistic energies)

Time evolution of the collision:

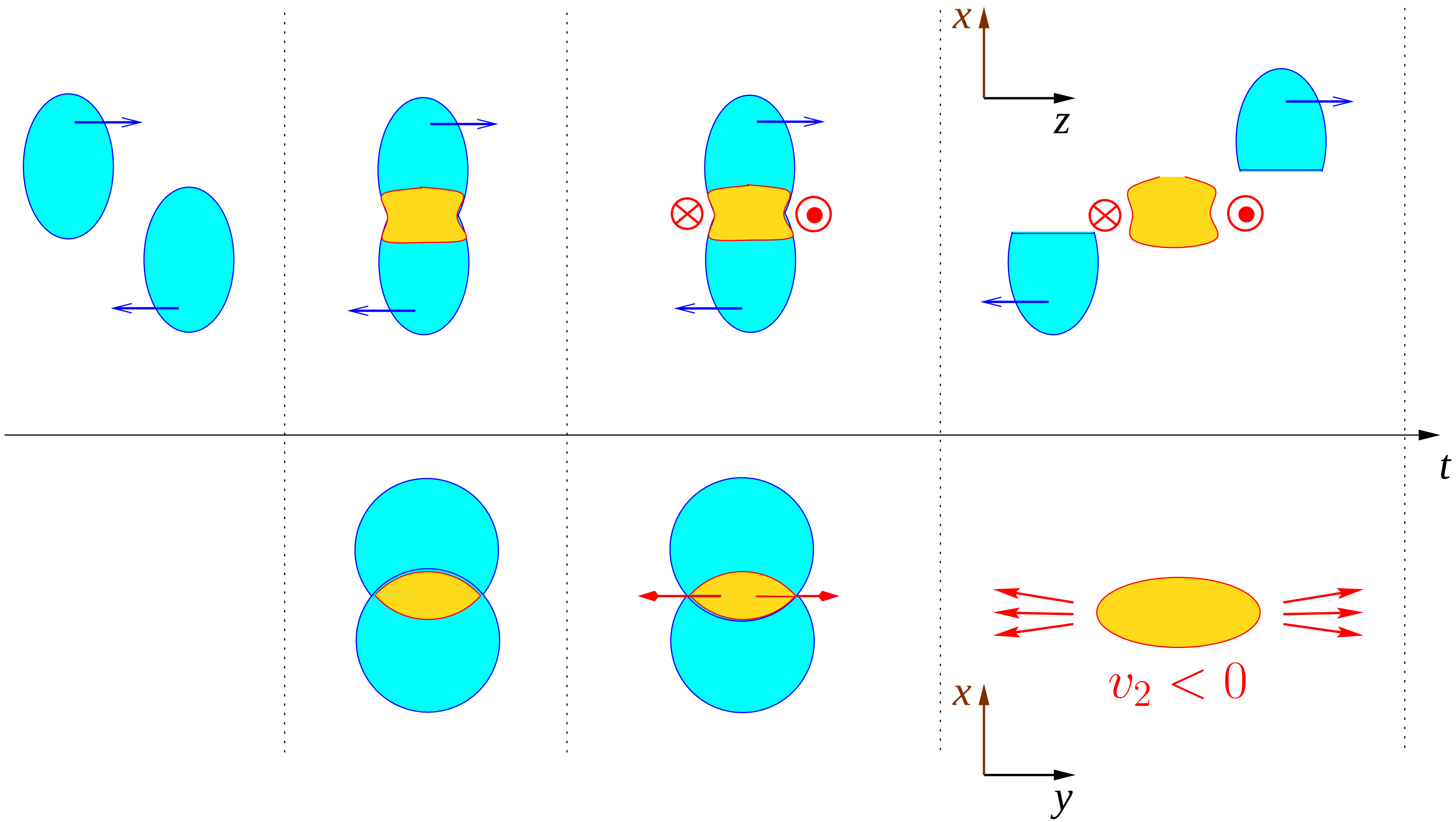


"In-plane" elliptic flow, $v_2 > 0$

J.-Y. Ollitrault, Phys. Rev. **D46** (1992) 229

v_2 PHYSICS (lower energies)

Time evolution of the collision:



"Out-of-plane" elliptic flow, "Squeeze-out"

METHODS OF FLOW ANALYSIS, part I

Flow = correlation with the impact parameter direction

$$v_n = \langle \cos n(\phi - \Phi_R) \rangle = \left\langle e^{in(\phi - \Phi_R)} \right\rangle$$

Φ_R reaction plane azimuth... unknown! 

$\Rightarrow$ solution (?): extract v_n from 2-particle correlations

$$\left\langle e^{in(\phi_1 - \phi_2)} \right\rangle = \left\langle e^{in(\phi_1 - \Phi_R)} \, e^{in(\Phi_R - \phi_2)} \right\rangle$$
$$\approx \left\langle e^{in(\phi_1 - \Phi_R)} \right\rangle \left\langle e^{in(\Phi_R - \phi_2)} \right\rangle = (v_n\{2\})^2$$

GANIL, GSI, AGS, SPS, RHIC...

Assumption: the correlation between two particles is due to the correlation of each one with the reaction plane

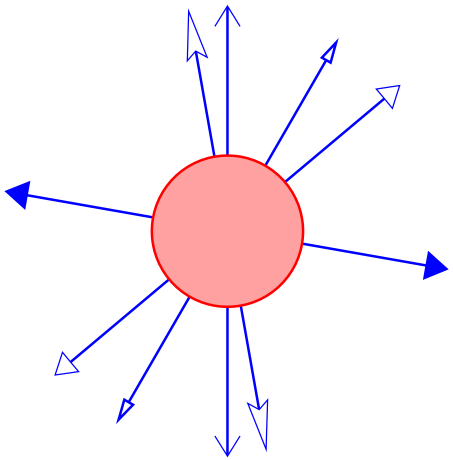
Problem: there exist other correlation sources.

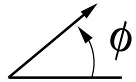
TWO-PARTICLE NONFLOW CORRELATIONS

a simple example

Central collision $\longrightarrow$ NO flow, $v_n = 0$.

Strong direct back-to-back correlations: particles emitted by pairs





$\Rightarrow \langle \cos 2(\phi_1 - \phi_2) \rangle > 0.$

The standard analysis measures $v_2\{2\} \equiv \sqrt{\langle \cos 2(\phi_1 - \phi_2) \rangle} \neq 0$...

Many sources of nonflow correlations:

- quantum correlations (HBT)
- momentum conservation
- resonance decays
- strong and Coulomb interactions
- (mini)jets, etc.

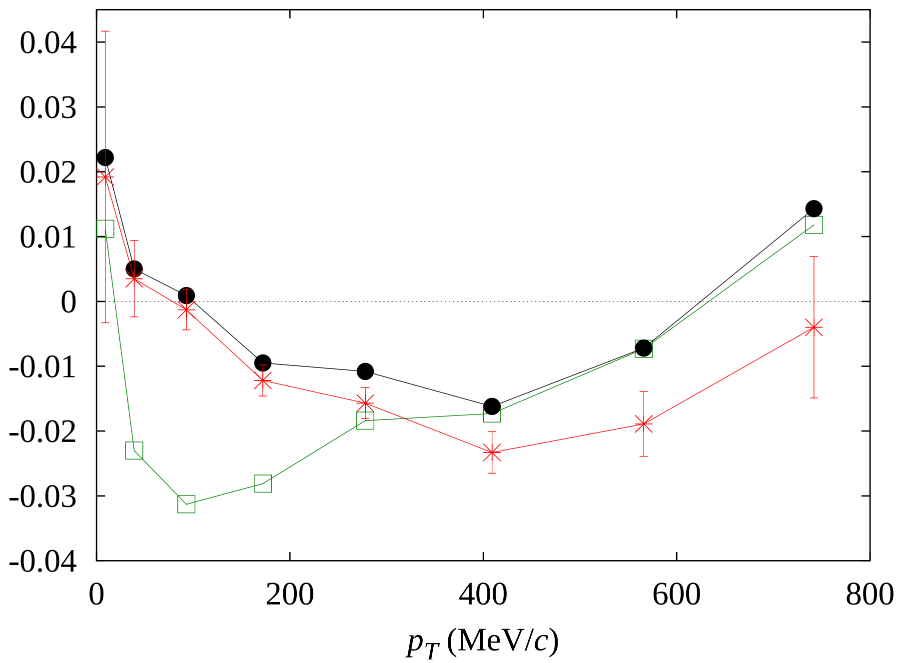
Unwanted correlations of the same magnitude as the flow correlations!

$$\mathcal{O}\left(\frac{1}{N}\right) \sim (v_n)^2$$

Solution (?):
compute & subtract the correlations

Problem: are all sources known?

charged pion v_1 at SPS



□ : "data" (NA49, PRL 1998)

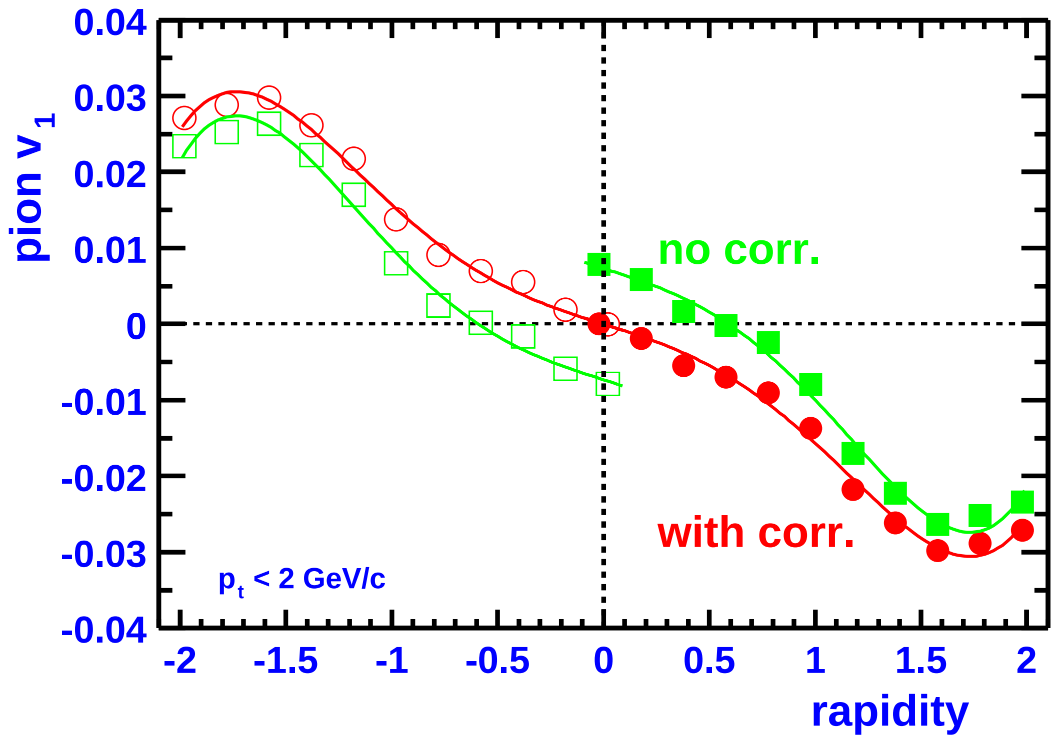
● : data − HBT

× : data − (HBT & p_T conservation)

N.B., P.M. Dinh, J.-Y. Ollitrault, Phys. Lett. **B477** (2000) 51, Phys. Rev. **C62** (2000) 034902

NONFLOW CORRELATIONS (continued)

Influence of momentum conservation (NA49 π^+, π^-, 158A GeV, min. bias)



N.B., P.M. Dinh, J.-Y. Ollitrault, A.M. Poskanzer, S.A. Voloshin, Phys. Rev. **C66** (2002) 014901

METHODS OF FLOW ANALYSIS, part II

Nonflow two-particle correlations are a nuisance... let us eliminate them!

$\Rightarrow$ cumulant of the four-particle correlation:

$$c_n\{4\} \equiv \left\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)}\right\rangle - \left\langle e^{in(\phi_1-\phi_2)}\right\rangle\left\langle e^{in(\phi_3-\phi_4)}\right\rangle - \left\langle e^{in(\phi_1-\phi_4)}\right\rangle\left\langle e^{in(\phi_3-\phi_2)}\right\rangle$$

$$= -(v_n\{4\})^4 + \underbrace{O\left(\frac{1}{N^3}\right)}$$

nonflow FOUR-particle correlations, negligible.

Increased sensitivity: analysis valid if $\boxed{v_n \gg \frac{1}{N^{3/4}}}$, better than $v_n \gg \frac{1}{N^{1/2}}$.

systematic error $\delta(v_n\{4\}^4) \simeq \frac{1}{N^3}$

N.B., P.M. Dinh, J.-Y. Ollitrault, Phys. Rev. **C63** (2001) 054906, **C64** (2001) 054901

"CUMULANT" METHOD OF FLOW ANALYSIS (continued)

Why only 4 particles?

- Measured six-particle correlations $\Longrightarrow$ cumulant of the six-particle correlation:

$$c_n\{6\} = 4\,(v_n\{6\})^6 + O\left(\frac{1}{N^5}\right)$$

- Measured eight-particle correlations $\Longrightarrow$ cumulant of the eight-particle correlation:

$$c_n\{8\} = -33\,(v_n\{8\})^8 + O\left(\frac{1}{N^7}\right)$$

Systematic error decreases!

$$\delta(v_n\{6\}^6) \simeq \frac{1}{N^5}, \qquad \delta(v_n\{8\}^8) \simeq \frac{1}{N^7} \cdots$$

“CUMULANT” METHOD OF FLOW ANALYSIS (continued)

Problem: the method is statistics-consuming, requires many multiplets:

$$\frac{\delta v_n\{4\}}{v_n} \simeq \frac{1}{2\sqrt{N_{\rm evts}}} \frac{1}{(v_n\sqrt{N})^4}$$

statistical uncertainty depends on v_n

$(v_n\sqrt{N})^2$ with 2-particle m.

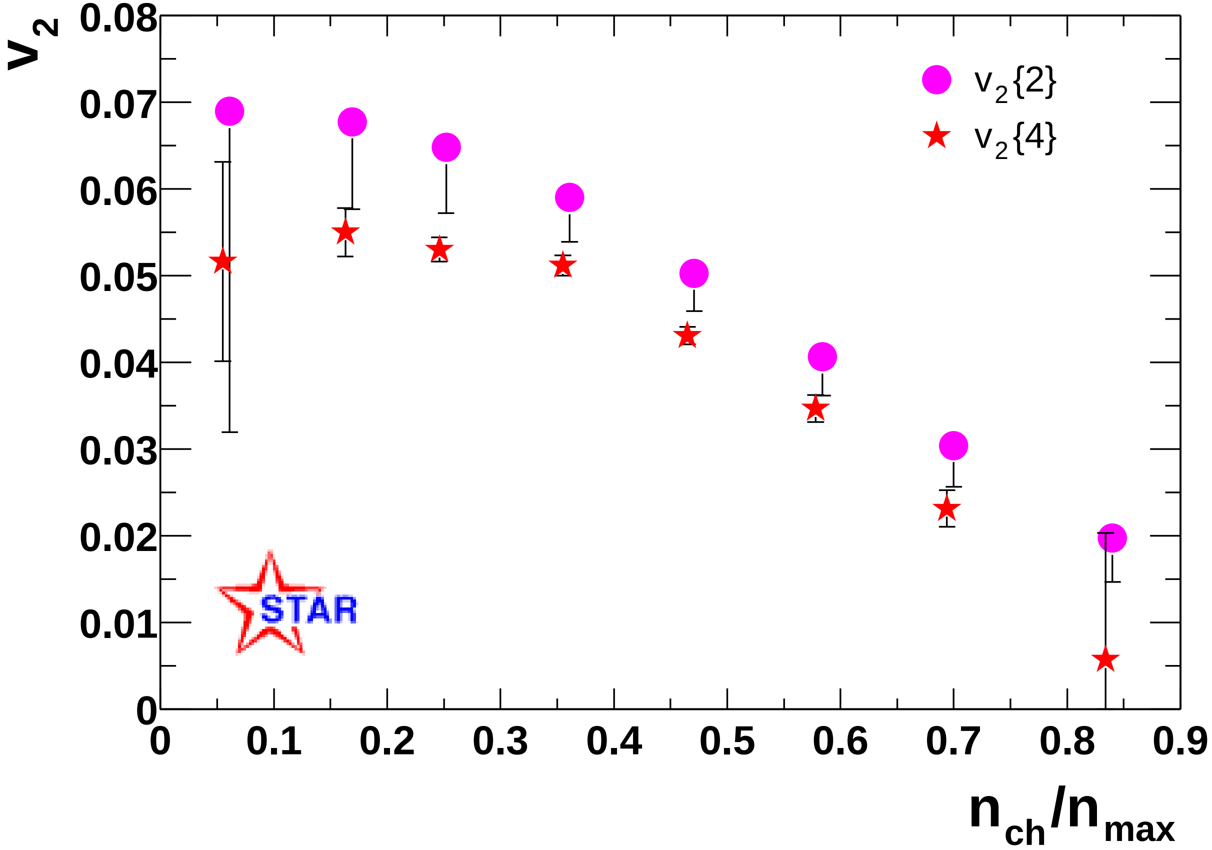
$\Rightarrow$ <u>smaller v_n requires more statistics!</u>

Higher order (6-, 8-particle) cumulants need even more statistics

Problem (at ultrarelativistic energies) for v_1, not for v_2

RESULTS FROM THE "CUMULANT" METHOD!

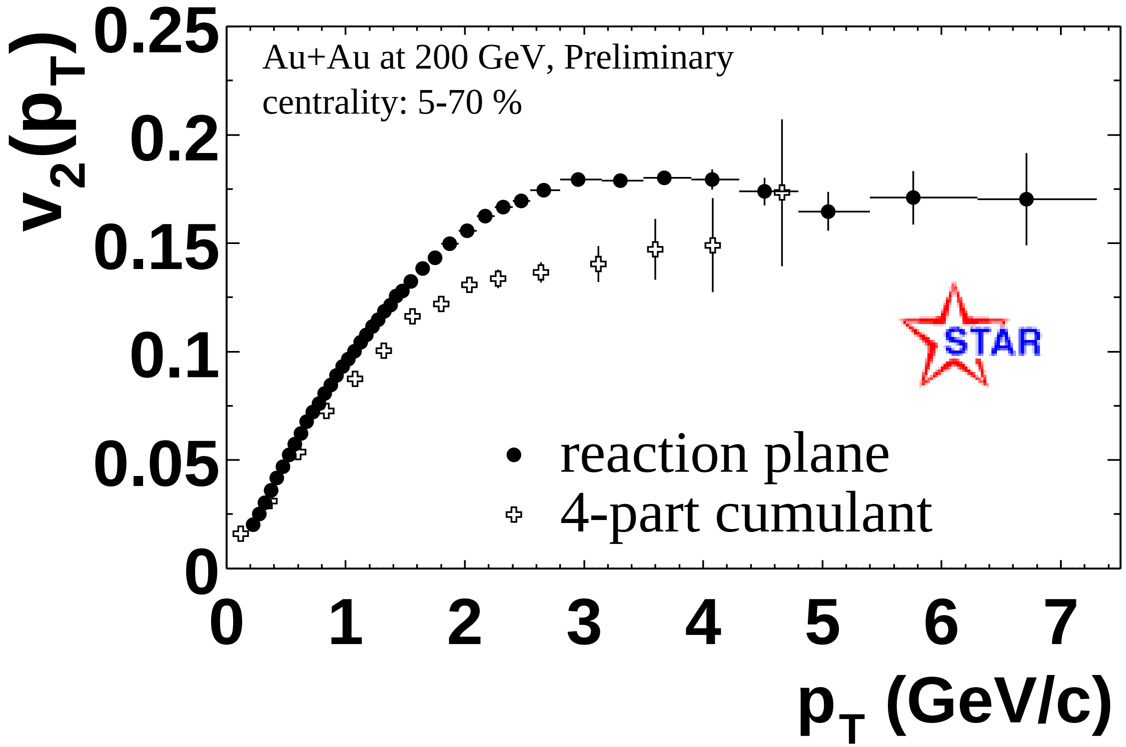
Au-Au collisions, $\sqrt{s_{NN}} = 130$ GeV, v_2 vs. centrality:



STAR Collaboration, Phys. Rev. **C66** (2002) 034904

RESULTS FROM THE "CUMULANT" METHOD!

Au-Au collisions, $\sqrt{s_{NN}} = 200$ GeV, charged hadron v_2 vs. p_T:



STAR Collaboration, `nucl-ex/0210027`

METHODS OF FLOW ANALYSIS, part III:
v_1 from THREE-PARTICLE CORRELATIONS NEW

Idea: consider the mixed three-particle correlation:

$$\left\langle e^{i(\phi_1+\phi_2-2\phi_3)} \right\rangle = (v_1\{3\})^2\, v_2 + \underbrace{\mathcal{O}\left(\frac{1}{N^2}\right)}$$

nonflow **3**-particle correlation

v_2 measured in separate analysis $\Rightarrow$ value of v_1

♡ Statistical uncertainty moderate: $\dfrac{\delta v_1\{3\}}{v_1} \simeq \dfrac{1}{2\sqrt{N_{\rm evts}}} \dfrac{1}{(v_1\sqrt{N})^2\,(v_2\sqrt{N})}$

especially if v_2 is "large" (SPS, RHIC)

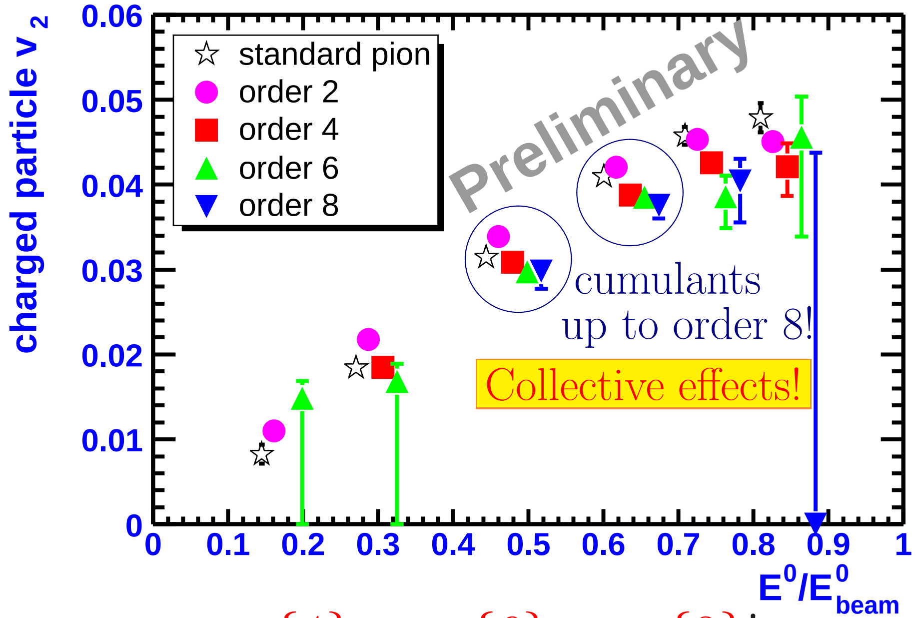
♡ Error due to nonflow correlations negligible

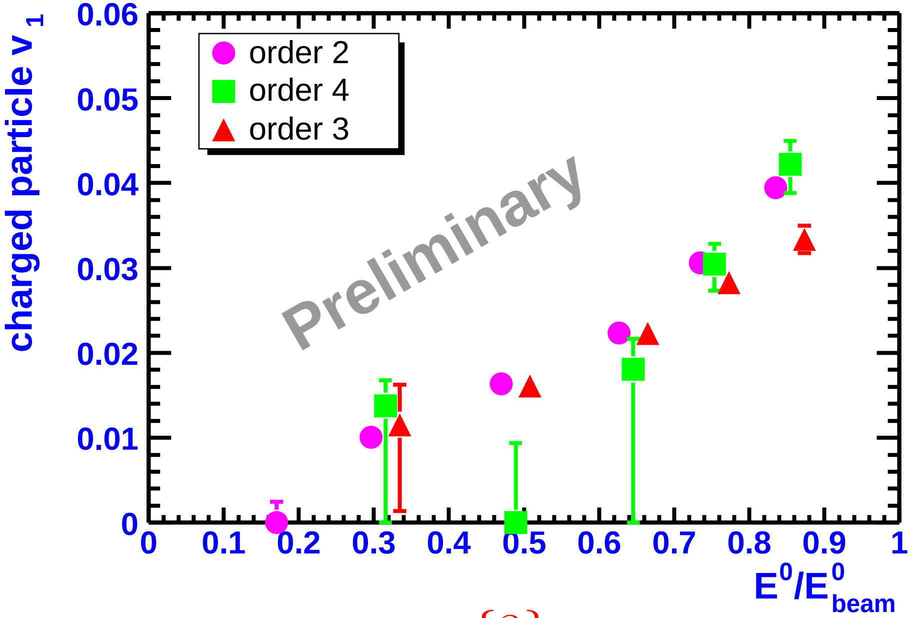
N.B., P.M. Dinh, J.-Y. Ollitrault, Phys. Rev. **C66** (2001) 014905

v_1 from THREE-PARTICLE CORRELATIONS

Integrated v_2 and v_1 vs. centrality, 158A GeV Pb-Pb

(1.) Pick a reference (integrated) v_2 (2.) and deduce the integrated v_1:

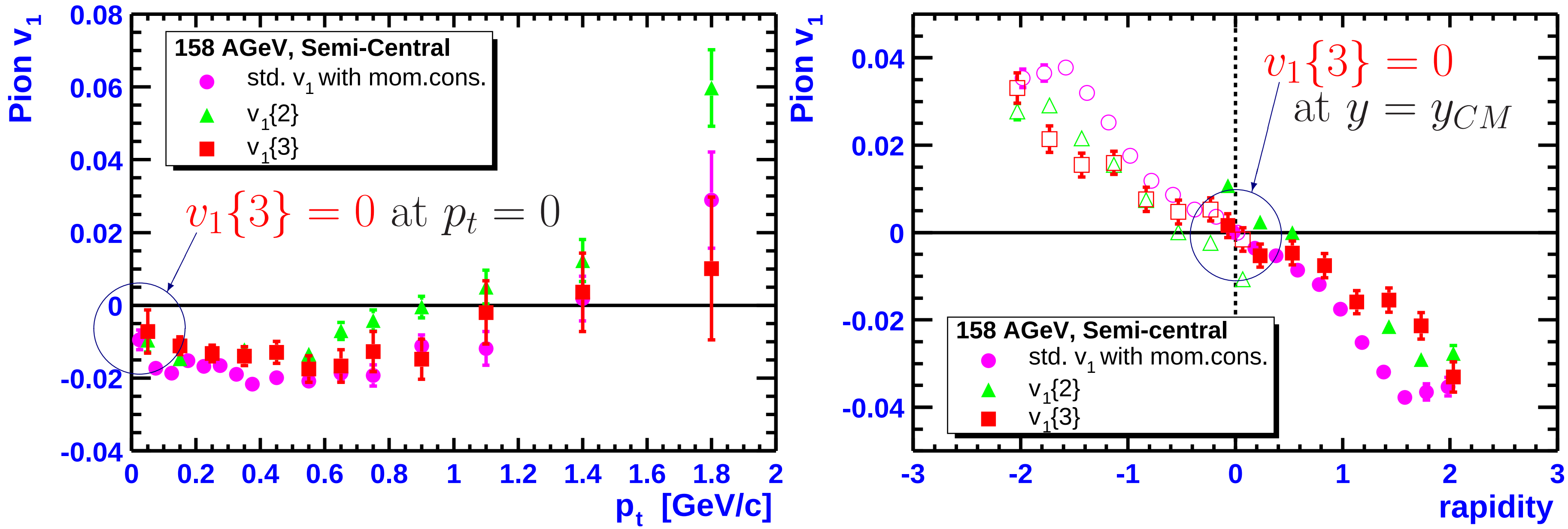




$v_2\{4\} = v_2\{6\} = v_2\{8\}$! $\Longrightarrow$ $v_1\{3\}$

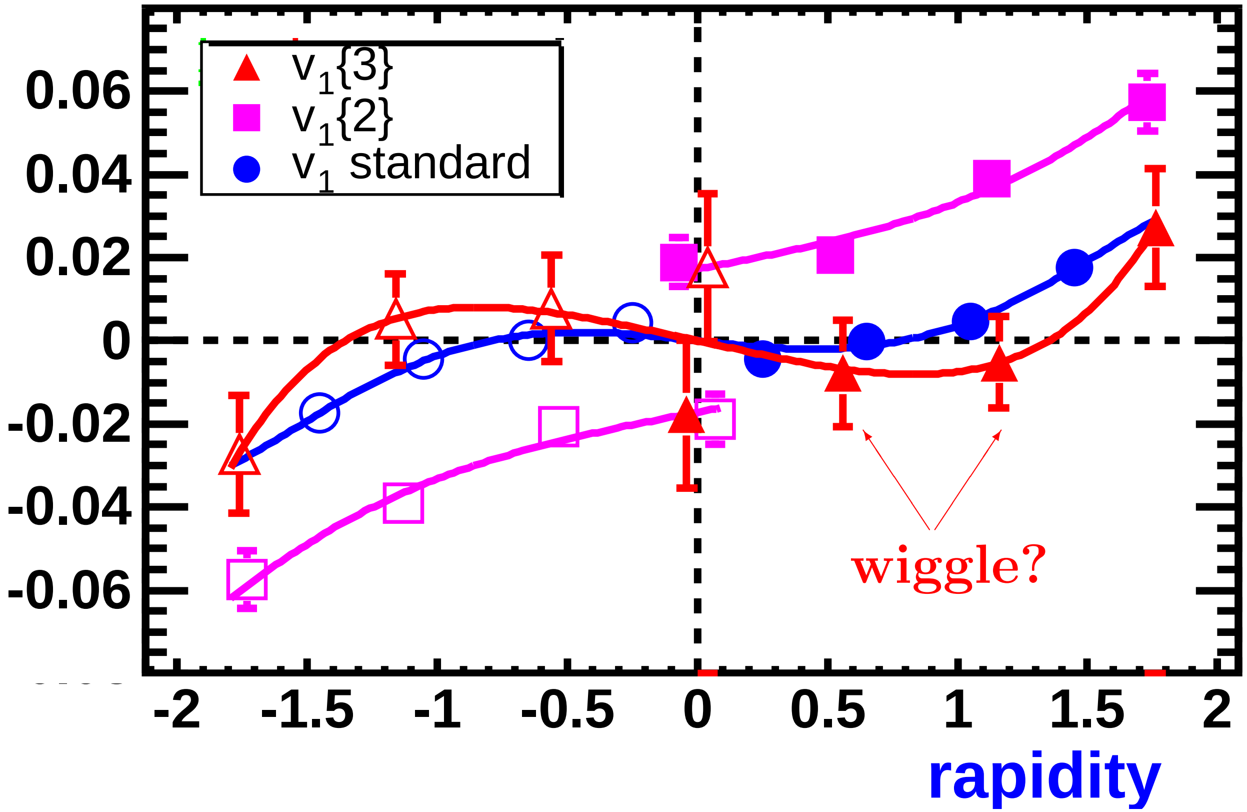
v_1 from THREE-PARTICLE CORRELATIONS

③ Reference v_2 + integrated $v_1\{3\}$ yield differential $v_1\{3\}$:
$v_1(p_t)$ and $v_1(y)$ for charged pions, semicentral 158A GeV Pb-Pb



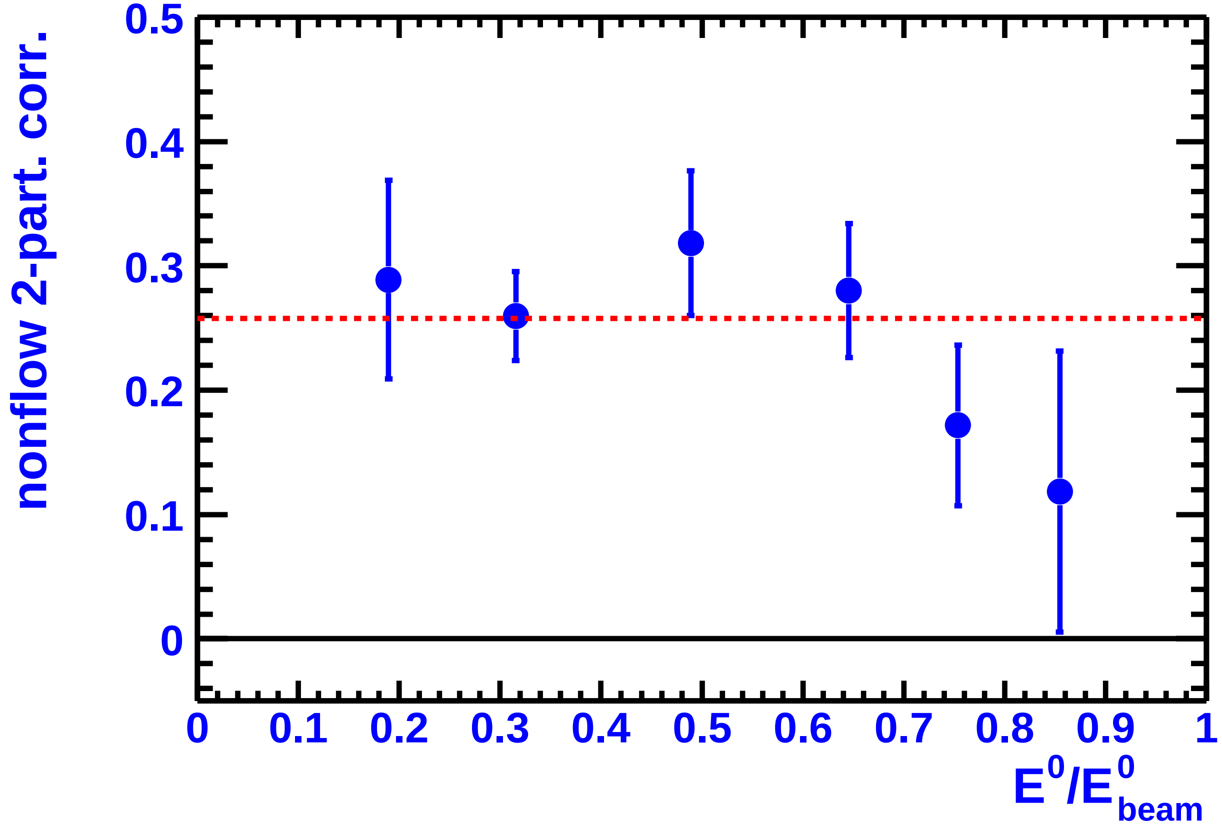
v_1 from THREE-PARTICLE CORRELATIONS

$v_1(y)$ of protons, semicentral 158A GeV Pb-Pb



TWO-PARTICLE NONFLOW EFFECTS

$$\left.\begin{array}{r}(v_2\{2\})^2 - {v_2}^2 = \mathcal{O}(1/N) \\ {v_2}^2 \simeq (v_2\{k>2\})^2\end{array}\right\} \begin{array}{l} N\left[(v_2\{2\})^2 - (v_2\{4\})^2\right] = \mathcal{O}(1), \\ \text{independent of centrality}\end{array}$$



COLLECTIVE FLOW ANALYSIS FROM MULTIPARTICLE CORRELATIONS

♡ Transverse flow interesting subject!

♡ Connection with other topics (HBT) ⇒ complementary picture

cf. E895, STAR

♡ Reliable methods of analysis are available!

♡ Experimental results are available too!

(E895), NA49, STAR

PRACTICAL ANALYSIS: GENERATING FUNCTIONS

1. For each event, compute the generating functions

$$G_n(z) = \prod_{k=1}^{M} \left(1 + \frac{z^* e^{in\phi_k} + z e^{-in\phi_k}}{M}\right),$$

and

$$\mathcal{G}(z_1, z_2) = \prod_{k=1}^{M} \left(1 + \frac{z_1^* \, e^{i\phi_k} + z_1 \, e^{-i\phi_k} + z_2^* \, e^{2i\phi_k} + z_2 \, e^{-2i\phi_k}}{M}\right).$$

Why?

2. Then average them over events:

$$\langle G_n(z)\rangle = 1 + \cdots + \frac{|z|^2}{M^2}\left\langle \sum_{j\neq k} e^{in(\phi_j-\phi_k)} \right\rangle + \cdots + \frac{|z|^4}{4M^4}\left\langle \sum_{j,k,l,m} e^{in(\phi_j+\phi_k-\phi_l-\phi_m)} \right\rangle + \cdots$$

and

$$\langle \mathcal{G}(z_1, z_2)\rangle = \cdots + \frac{{z_1^*}^2 z_2}{M^3}\left\langle \sum_{j,k,l} e^{i(\phi_j+\phi_k-2\phi_l)} \right\rangle + \cdots$$

③ Deduce the cumulants, taking

$$M\left(\langle G_n(z)\rangle^{1/M}-1\right)=|z|^2\left\langle\!\left\langle e^{i(\phi_1-\phi_2)}\right\rangle\!\right\rangle+\cdots+\frac{|z|^4}{4}\left\langle\!\left\langle e^{i(\phi_1+\phi_2-\phi_3-\phi_4)}\right\rangle\!\right\rangle+\cdots$$

and

$$M\left(\langle \mathcal{G}(z_1,z_2)\rangle^{1/M}-1\right)=\cdots+\frac{z_1^{*2}z_2}{2}\left\langle\!\left\langle e^{i(\phi_1+\phi_2-2\phi_3)}\right\rangle\!\right\rangle+\cdots$$

The generating function yields ALL CUMULANTS at once!! 

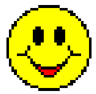
④ Extract the flow:

- for v_n, n arbitrary (especially for v_2) use $M(\langle G_n(z)\rangle^{1/M}-1)=\ln I_0(2\,v_n|z|)$,

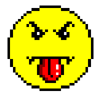
and/or perform the appropriate acceptance corrections

- with the above-determined reference value of v_2, extract $v_1\{3\}$, using

$$\left\langle\!\left\langle e^{i(\phi_1+\phi_2-2\phi_3)}\right\rangle\!\right\rangle=\begin{cases}(v_1\{3\})^2\,v_2 & \text{(good acceptance)}\\ \alpha\,(v_1\{3\})^2\,v_2 & \text{(~~PHENIX~~ uneven acceptance)}\end{cases}$$

(good acceptance )

α ← depends on the detector

(~~PHENIX~~ uneven acceptance )

⑤ Post your paper on `nucl-ex` and collect citations.