Non-trivial correlation patterns from trivial phenomena

… and how to avoid them

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Non-trivial correlation patterns from trivial phenomena

- Cooking up a fancy meal with plain ingredients… and a questionable recipe.

- Improving on the recipe…

- … and on the presentation of the meal.

First ingredient:

total momentum conservation

In the center-of-mass frame of the colliding nuclei, the N particles emitted in a Au-Au collision satisfy $\mathbf{p}_1 + \mathbf{p}_2 + \cdots + \mathbf{p}_N = \mathbf{0}$.

An old idea...

PHYSICAL REVIEW D VOLUME 6, NUMBER 11 1 DECEMBER 1972

Azimuthal Correlations of High-Energy Collision Products

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(Received 24 July 1972)

Experimental distributions of azimuthal angles between particles produced in pp and pd collisions at 28 GeV/c and K^-p collisions at 9 GeV/c are presented and studied.

The study of two-particle correlations is a natural step beyond the investigation of single-particle distributions.[1,2] Such a study could be very useful in clarifying our understanding of multiple-particle production in high-energy collisions.

In this paper we concentrate on azimuthal correlations, that is, distributions $d\sigma/d\phi_{ij}$ where ϕ_{ij} is the angle between transverse momenta $\vec{k}_i$ and $\vec{k}_j$ of two final-state particles.

The main goal of our study is to identify the correlations which arise simply from momentum conservation and the experimentally observed damping of transverse momenta.

An old idea...

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Azimuthal Correlations of High-Energy Collision Products

II. MOMENTUM-CONSERVATION CONSTRAINT

We consider the azimuthal distribution $d\sigma^n/d\phi$ in a general reaction with n particles in the final state. Transverse-momentum conservation imposes some constraints on this distribution. Denoting the transverse momentum of the ith particle by $\vec{k}_i$, we see that transverse momentum conservation gives the condition

$$\sum_i k_i^{\,2} + \sum_{i\neq j} \vec{k}_i \cdot \vec{k}_j = 0 \;.$$

Upon averaging over all particles, we find $n\langle k_i^2\rangle + n(n-1)\langle \vec{k}_i \cdot \vec{k}_j\rangle = 0$, which suggests that $\langle \cos\phi\rangle \approx -1/(n-1)$ and that a distribution $d\sigma^n/d\phi$ might be expected to peak at $\phi = \pi$, the peak becoming less pronounced as n increases.

$$\frac{d\sigma^n}{d\phi} \equiv \sum_{i\neq j} \frac{d\sigma^n}{d\phi_{ij}}$$

Multiparticle distributions & cumulants

- M-particle probability distribution $f(\mathbf{p}_{i_1},\ldots,\mathbf{p}_{i_M})$:

probability that particles $\{i_1, i_2, \ldots, i_M\}$ have momenta $\mathbf{p}_{i_1}, \mathbf{p}_{i_2}, \ldots, \mathbf{p}_{i_M}$ irrespective of the momenta of the $N-M$ other particles.

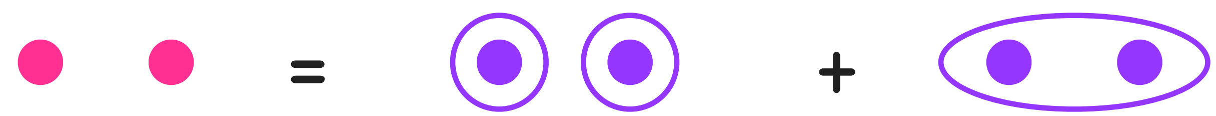
☞ normalized to unity: $f(\{\mathbf{p}_{i_k}\}) = \mathcal{O}(1),\ \forall M$

Independent particles: $f(\mathbf{p}_1,\mathbf{p}_2,\ldots,\mathbf{p}_N) = f(\mathbf{p}_1)\, f(\mathbf{p}_2)\cdots f(\mathbf{p}_N)$

Multiparticle distributions & cumulants

- M-particle cumulant of the probability distribution $f_c(\mathbf{p}_{i_1},\ldots,\mathbf{p}_{i_M})$: connected part of the probability distribution, responsible for the "correlations" (= deviations from statistical independence)

$$f(\mathbf{p}_1,\mathbf{p}_2) = f_c(\mathbf{p}_1)\, f_c(\mathbf{p}_2) + f_c(\mathbf{p}_1,\mathbf{p}_2)$$



(note: $f(\mathbf{p}) = f_c(\mathbf{p})$...)

At the three-particle level:

In the following, I shall also use "reduced cumulants"

$$\bar{f}_c(\mathbf{p}_1,\ldots,\mathbf{p}_M) \equiv \frac{f_c(\mathbf{p}_1,\ldots,\mathbf{p}_M)}{f(\mathbf{p}_1)\cdots f(\mathbf{p}_M)}$$

Total momentum conservation and M-particle distribution

In the presence of the constraint from total momentum conservation, the M-particle probability distribution reads:

$$f(\mathbf{p}_1,\ldots,\mathbf{p}_M) \equiv \frac{\left(\displaystyle\prod_{j=1}^{M} F(\mathbf{p}_j)\right)\displaystyle\int \delta^D(\mathbf{p}_1+\cdots+\mathbf{p}_N) \prod_{j=M+1}^{N} \left[F(\mathbf{p}_j)\,\mathrm{d}^D\mathbf{p}_j\right]}{\displaystyle\int \delta^D(\mathbf{p}_1+\cdots+\mathbf{p}_N) \prod_{j=1}^{N} \left[F(\mathbf{p}_j)\,\mathrm{d}^D\mathbf{p}_j\right]}$$

... which can be computed in the large-N limit, together with the corresponding cumulants.

N.B., Eur. Phys. J. C **30** (2003) 381

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... which can be computed in the large-N limit, together with the corresponding cumulants.

N.B., Eur. Phys. J. C **30** (2003) 381

Total momentum conservation and M-particle distribution

In the presence of the constraint from total momentum conservation, the M-particle probability distribution reads:

single-particle distribution
in the absence of constraint

dimension
of space

$$f(\mathbf{p}_1,\ldots,\mathbf{p}_M) \equiv \frac{\left(\displaystyle\prod_{j=1}^{M} F(\mathbf{p}_j)\right)\displaystyle\int \delta^D(\mathbf{p}_1+\cdots+\mathbf{p}_N)\prod_{j=M+1}^{N}\left[F(\mathbf{p}_j)\,\mathrm{d}^D\mathbf{p}_j\right]}{\displaystyle\int \delta^D(\mathbf{p}_1+\cdots+\mathbf{p}_N)\prod_{j=1}^{N}\left[F(\mathbf{p}_j)\,\mathrm{d}^D\mathbf{p}_j\right]}$$

M-independent denominator

... which can be computed in the large-N limit, together with the corresponding cumulants.

N.B., Eur. Phys. J. C **30** (2003) 381

First cumulants

$$\bar{f}_c(\mathbf{p}_1, \mathbf{p}_2) = -\frac{D\, \mathbf{p}_1 \cdot \mathbf{p}_2}{N \langle \mathbf{p}^2 \rangle}$$

Back-to-back correlation, larger for particles with larger momenta

$$\begin{aligned} \bar{f}_c(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) &= -\frac{D}{N^2 \langle \mathbf{p}^2 \rangle} (\mathbf{p}_1 \cdot \mathbf{p}_2 + \mathbf{p}_1 \cdot \mathbf{p}_3 + \mathbf{p}_2 \cdot \mathbf{p}_3) \\ &\quad + \frac{D^2}{N^2 \langle \mathbf{p}^2 \rangle^2} \left[(\mathbf{p}_1 \cdot \mathbf{p}_2)(\mathbf{p}_1 \cdot \mathbf{p}_3) + (\mathbf{p}_1 \cdot \mathbf{p}_2)(\mathbf{p}_2 \cdot \mathbf{p}_3) \right. \\ &\qquad \left. + (\mathbf{p}_1 \cdot \mathbf{p}_3)(\mathbf{p}_2 \cdot \mathbf{p}_3) \right] \end{aligned}$$

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Want to relax the "isotropic emission" assumption? (take $D = 3$)

$$\bar{f}_c(\mathbf{p}_1,\mathbf{p}_2) = -\frac{p_{1,x}p_{2,x}}{N\langle p_x^2\rangle} - \frac{p_{1,y}p_{2,y}}{N\langle p_y^2\rangle} - \frac{p_{1,z}p_{2,z}}{N\langle p_z^2\rangle}$$

x,y,z principal axes of the $\langle\mathbf{p}\otimes\mathbf{p}\rangle$ tensor

N.B. 2003; Z.Chajęcki & M.A.Lisa, Braz. J. Phys **37** (2007) 1057

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$\langle p_z^2\rangle \gg \langle p_{x,y}^2\rangle$
\+
studies at midrapidity

x,y,z principal axes of the $\langle\mathbf{p}\otimes\mathbf{p}\rangle$ tensor

N.B. 2003; Z.Chajęcki & M.A.Lisa, Braz. J. Phys **37** (2007) 1057

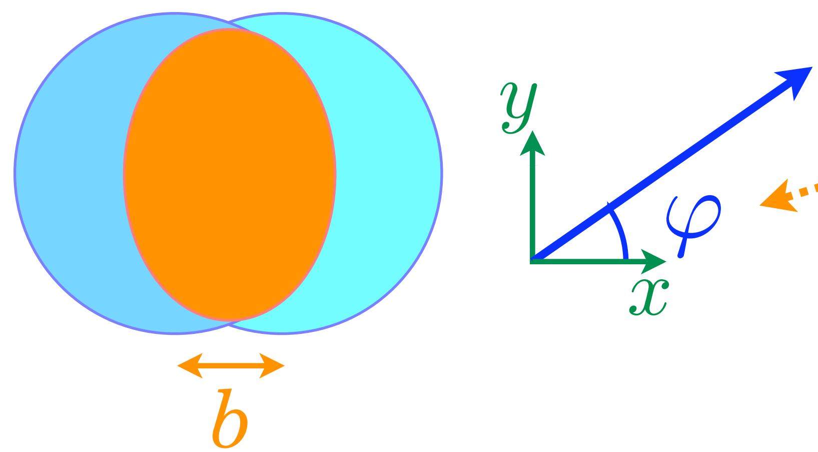
Second ingredient:

anisotropic flow

In a non-central ($\vec{b} \neq \vec{0}$) Au-Au collision the transverse emission of particles is anisotropic: $\langle p_x^2 \rangle \neq \langle p_y^2 \rangle$.

Anisotropic particle emission

In a non-central ($\vec{b} \neq \vec{0}$) Au-Au collision...



with respect to the reaction plane

The particle azimuthal distribution is a 2π-periodic function:

$$\frac{\mathrm{d}N}{\mathrm{d}\varphi} \propto f(\varphi) = \frac{1}{2\pi}\sum_{n=-\infty}^{+\infty} v_n \mathrm{e}^{\mathrm{i}n\varphi} = \frac{1}{2\pi}\left[\overset{v_0}{1} + 2\sum_{n=1}^{\infty} v_n \cos n\varphi\right]$$

where $v_n = \langle\cos(n\varphi)\rangle$ is real (in the absence of parity violation).

This is the generic case!

Central collisions exist only in the minds of theorists.

Anisotropic particle emission

Let $\bar{v}_2 \equiv \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle}$.

$$\bar{v}_2 \neq v_2 = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle$$

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+

total momentum conservation

$$\begin{aligned}
\bar{f}_c(\mathbf{p}_1, \mathbf{p}_2) &= -\frac{p_{1,x} p_{2,x}}{N \langle p_x^2 \rangle} - \frac{p_{1,y} p_{2,y}}{N \langle p_y^2 \rangle} \\
&= -\frac{2 p_{T\,1} p_{T\,2}}{N \langle p_T^2 \rangle \left(1 - \bar{v}_2^2\right)} \left[\cos(\varphi_2 - \varphi_1) - \bar{v}_2 \cos(\varphi_1 + \varphi_2) \right]
\end{aligned}$$

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\end{aligned}$$

☞ **two-particle probability distribution**

$$f(\mathbf{p}_1, \mathbf{p}_2) = f(\mathbf{p}_1)\, f(\mathbf{p}_2) \left[1 + \bar{f}_c(\mathbf{p}_1, \mathbf{p}_2) \right]$$

Of trigger and associated particles

Two-particle distribution ↔ two-particle probability distribution

$$f(\mathbf{p}_1,\mathbf{p}_2) = f(\mathbf{p}_1)\, f(\mathbf{p}_2)\big[1 + \bar{f}_c(\mathbf{p}_1,\mathbf{p}_2)\big]$$

Choose a "trigger particle (1)" in an event.

Yield of associated particles (2) ↔ conditional probability distribution

$$f(\mathbf{p}_{T\,2} \,|\, \mathbf{p}_{T\,1}) = \frac{f(\mathbf{p}_{T\,1},\mathbf{p}_{T\,2})}{f(\mathbf{p}_{T\,1})} \qquad \Big(\neq f(\mathbf{p}_{T\,2})\Big)$$

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Here $f(\mathbf{p}_{T\,2}|\,\mathbf{p}_{T\,1}) = f(\mathbf{p}_{T\,2})\left[1 - \dfrac{2p_{T\,1}p_{T\,2}}{N\langle p_T^2\rangle}(\cos(\varphi_1-\varphi_2) - \bar{v}_2\cos(\varphi_1+\varphi_2))\right]$

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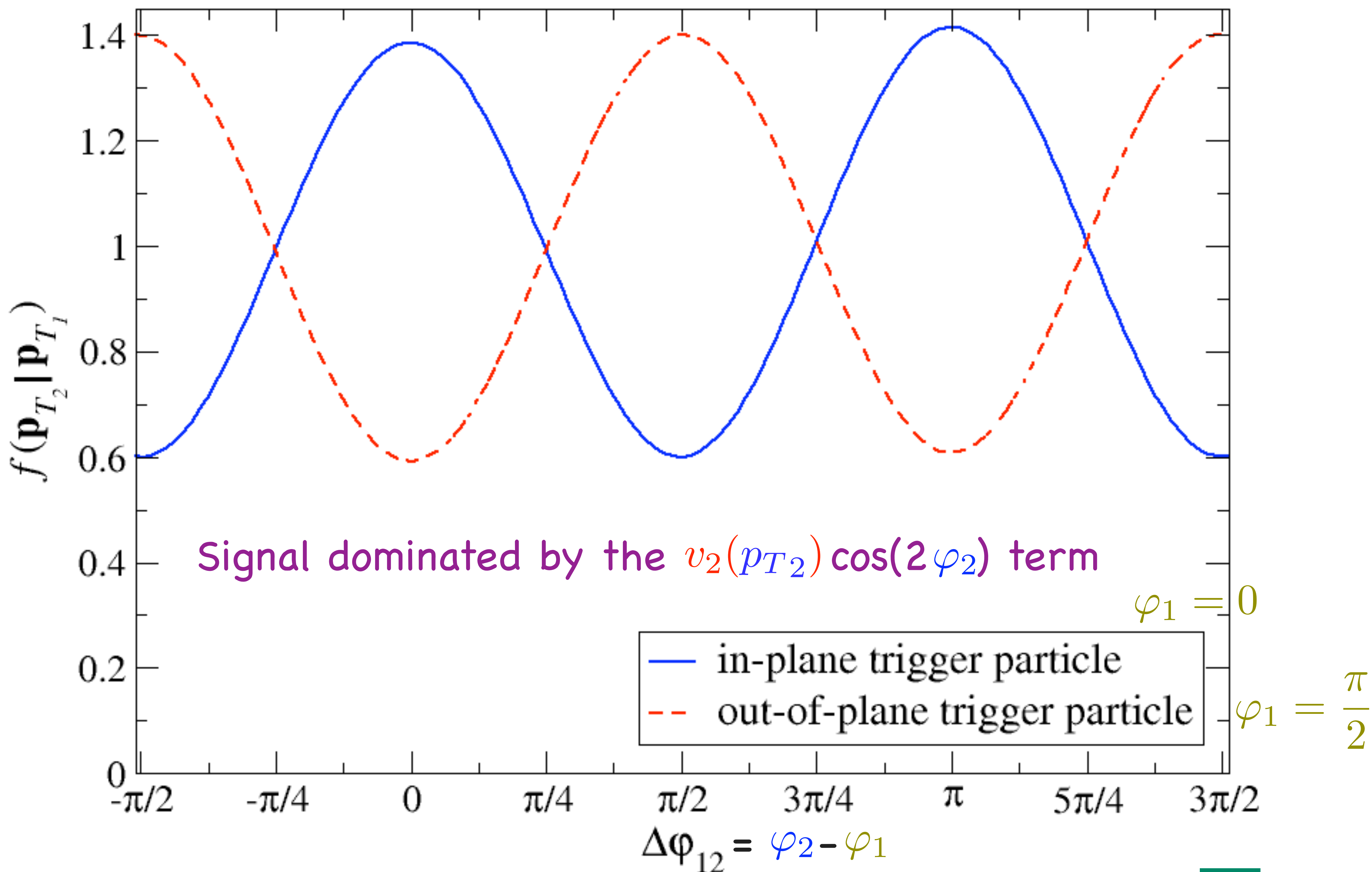
Here $f(\mathbf{p}_{T2} | \, \mathbf{p}_{T1}) = f(\mathbf{p}_{T2}) \left[1 - \frac{2 p_{T1} p_{T2}}{N \langle p_T^2 \rangle} \left(\cos(\varphi_1 - \varphi_2) - \bar{v}_2 \cos(\varphi_1 + \varphi_2) \right) \right]$

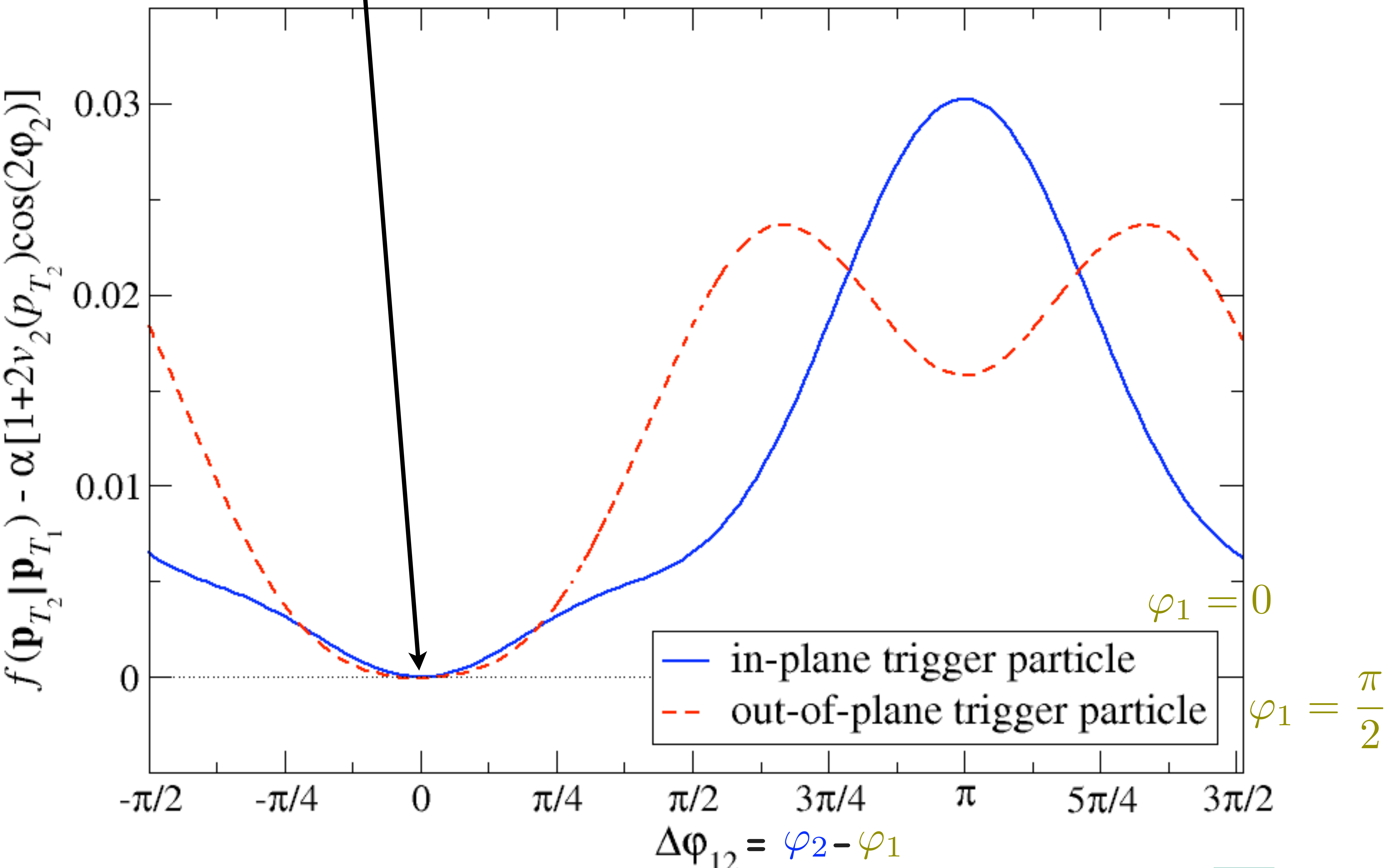
☞ can be plotted with N = 8000, $\langle p_T^2 \rangle$ = (500 MeV/c)^2, p_{T1} = 6 GeV/c, p_{T2} = 4 GeV/c, $v_2(p_{T2})$ = 0.2 and $\bar{v}_2$ = 0.1.

N.B., J. Phys. Conf. Ser. **110** (2008) 032005

Associated yield

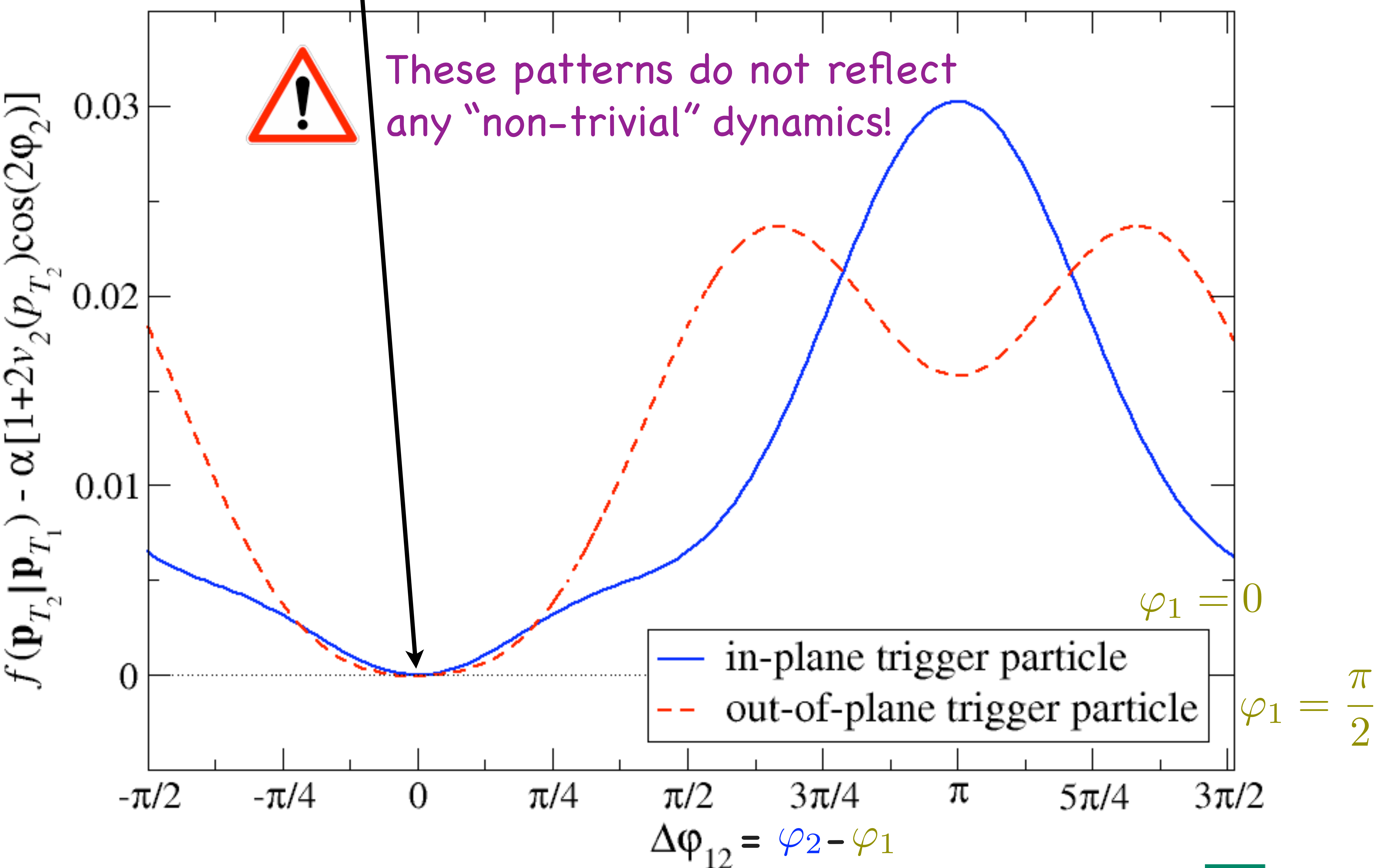


ZYAM-subtracted associated yield



ZYAM-subtracted associated yield

These patterns do not reflect any "non-trivial" dynamics!



From the associated yield to the correlation term (cumulant)

$$f(\mathbf{p}_{T2}|\,\mathbf{p}_{T1}) = \frac{f(\mathbf{p}_{T1},\mathbf{p}_{T2})}{f(\mathbf{p}_{T1})} = f(\mathbf{p}_{T2})\left[1 + \bar{f}_c(\mathbf{p}_{T1},\mathbf{p}_{T2})\right]$$

☞ subtracting $\alpha f(\mathbf{p}_{T2})$ from $f(\mathbf{p}_{T2}|\,\mathbf{p}_{T1})$ does not entirely suppress the "trivial" (= single-particle) influence of the emission anisotropy.

DIVIDING (by $f(\mathbf{p}_{T2})$) seems to be better suited for that purpose.

(In real life, things are not so simple, see later.)

Description & measurement

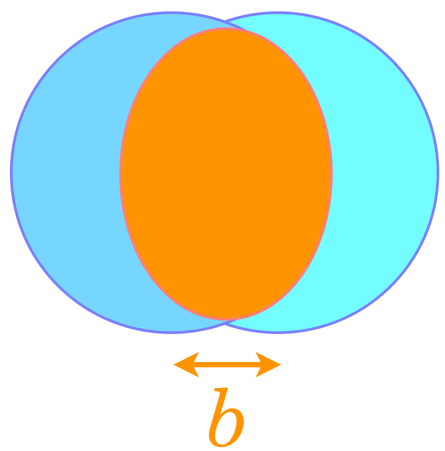
of two-particle correlations

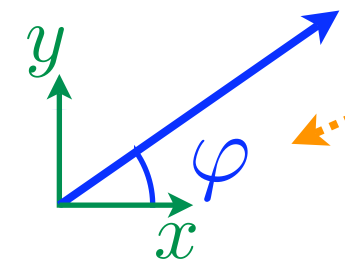
A two-step process:

- no model dependence at the level of the two-particle distribution;
- isolating the correlation term: some modeling required

Anisotropic particle emission

All collisions are non-central: single-particle yields depend on azimuth





with respect to the reaction plane

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Anisotropic two-particle distribution

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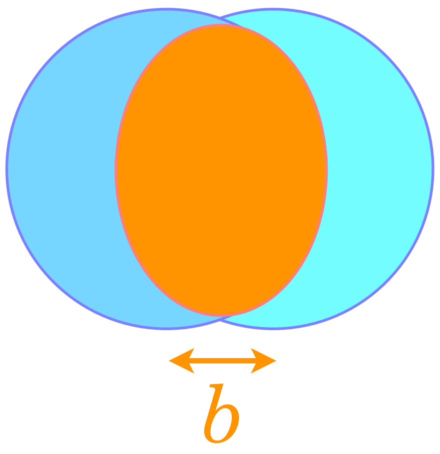

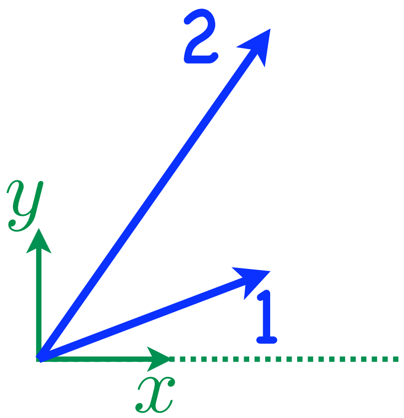

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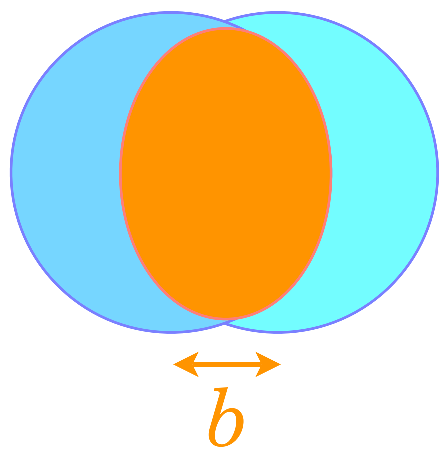
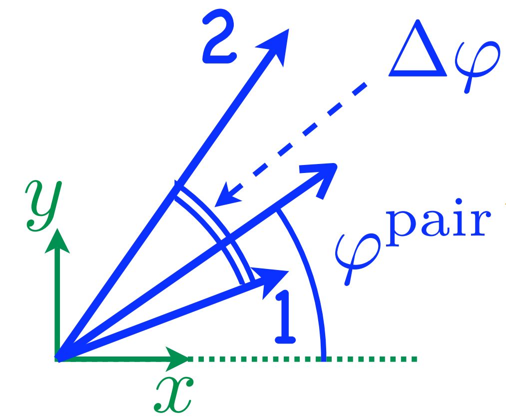


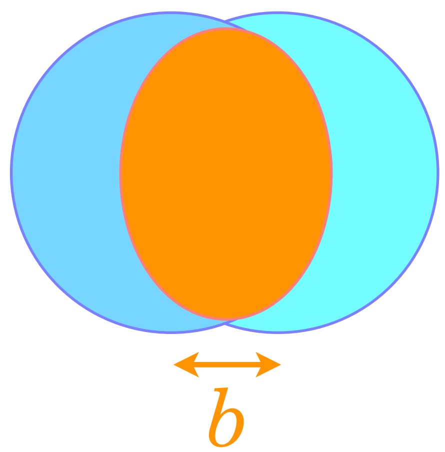


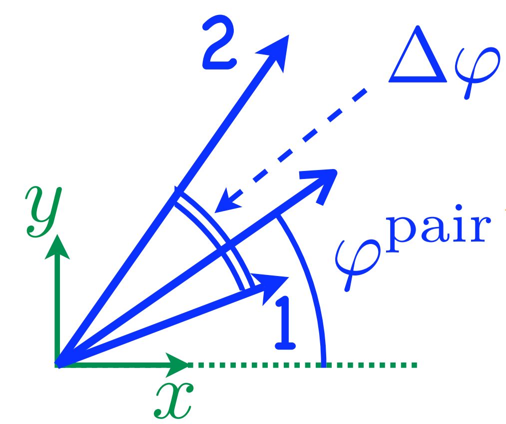
with respect to the reaction plane

Let $\Delta\varphi \equiv \varphi_2 - \varphi_1$ and $\varphi^{\rm pair} \equiv x\varphi_1 + (1-x)\varphi_2$ ($0 \le x \le 1$)

Anisotropic two-particle distribution

All collisions are non-central: two-particle yields depend on azimuth





with respect to the reaction plane

Let $\Delta\varphi \equiv \varphi_2 - \varphi_1$ and $\varphi^{\rm pair} \equiv x\varphi_1 + (1-x)\varphi_2$ ($0 \leq x \leq 1$)

At fixed $\Delta\varphi$ the two-particle azimuthal distribution is a 2π-periodic function of $\varphi^{\rm pair}$:

$$\frac{\mathrm{d}^2 N}{\mathrm{d}\varphi_1 \mathrm{d}\varphi_2} = \frac{\mathrm{d}^2 N}{\mathrm{d}\varphi^{\rm pair}\mathrm{d}\Delta\varphi} \propto \frac{1}{2\pi}\sum_{n=-\infty}^{+\infty} v_n^{\rm pair}(\Delta\varphi)\, \mathrm{e}^{in\varphi^{\rm pair}}$$

N.B. & J.-Y.Ollitrault, Phys. Rev. C **70** (2004) 064905

Anisotropic two-particle distribution

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Why should one characterize the 2-particle distribution $\propto f(\mathbf{p}_1,\mathbf{p}_2)$ or the 2-particle correlation (cumulant) $\bar{f}_c(\mathbf{p}_1,\mathbf{p}_2)$ with these $v_n^{\rm pair}(\Delta\varphi)$?

- because there exist model-independent methods to measure them accurately (the "usual" flow analysis methods);
- and because they are easier to compute in theoretical studies than the 2-particle distribution / cumulant itself.

Cf. the advantages of v_n over the "squeeze-out ratio" and similar quantities.

N.B. & J.-Y.Ollitrault, Phys. Rev. C **70** (2004) 064905

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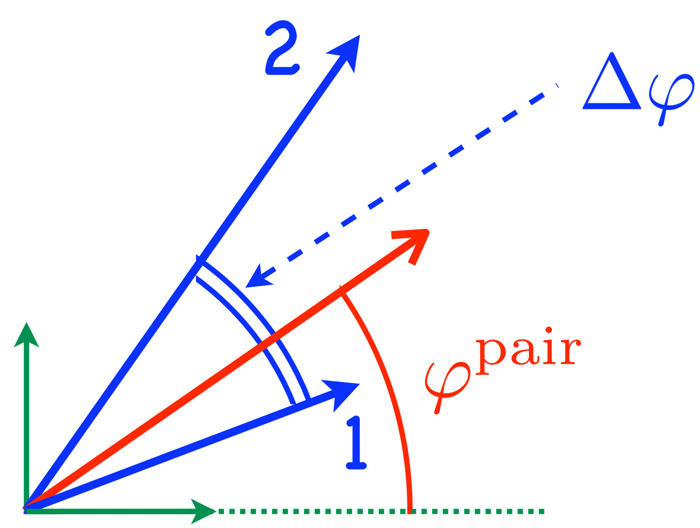
where $v_n^{\rm pair}(\Delta\varphi)$ is in general complex-valued!

There is no $\varphi^{\rm pair} \to -\varphi^{\rm pair}$ symmetry:



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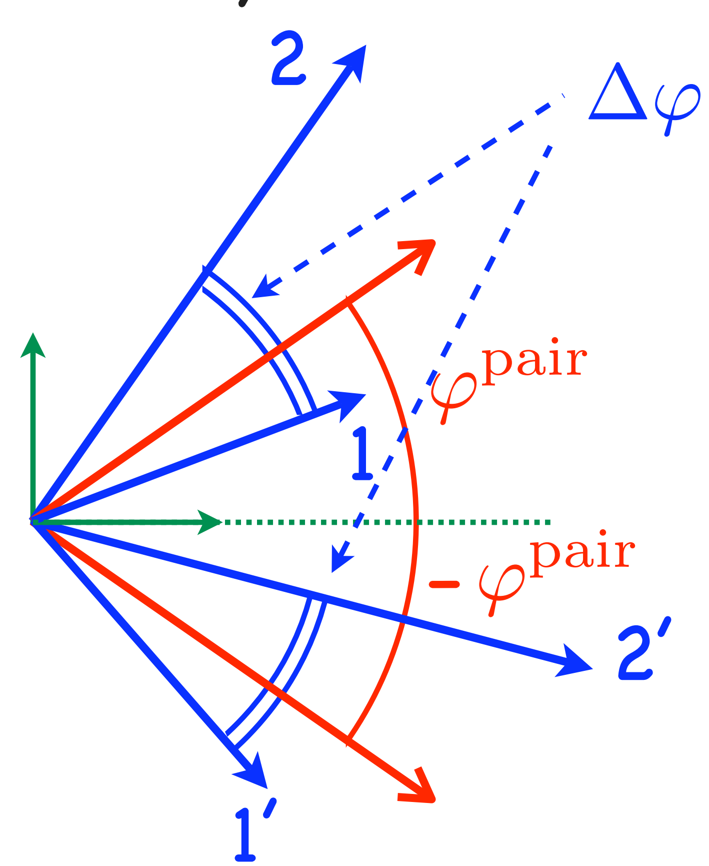

Anisotropic two-particle distribution

At fixed $\Delta\varphi$ the two-particle azimuthal distribution is a 2π-periodic function of $\varphi^{\rm pair}$:

$$\frac{\mathrm{d}^2 N}{\mathrm{d}\varphi_1 \mathrm{d}\varphi_2} = \frac{\mathrm{d}^2 N}{\mathrm{d}\varphi^{\rm pair}\mathrm{d}\Delta\varphi} \propto \frac{1}{2\pi}\sum_{n=-\infty}^{+\infty} v_n^{\rm pair}(\Delta\varphi)\, \mathrm{e}^{in\varphi^{\rm pair}}$$

where $v_n^{\rm pair}(\Delta\varphi)$ is in general complex-valued!

$$\frac{\mathrm{d}^2 N}{\mathrm{d}\varphi^{\rm pair}\mathrm{d}\Delta\varphi} \propto \frac{1}{2\pi}\left[1 + 2\sum_{n=1}^{+\infty}\left(v_{c,n}^{\rm pair}(\Delta\varphi)\cos n\varphi^{\rm pair} + v_{s,n}^{\rm pair}(\Delta\varphi)\sin n\varphi^{\rm pair}\right)\right]$$

where $v_{c,n}^{\rm pair}(\Delta\varphi) \equiv \left\langle \cos n\varphi^{\rm pair}\right\rangle$ and $v_{s,n}^{\rm pair}(\Delta\varphi) \equiv \left\langle \sin n\varphi^{\rm pair}\right\rangle$ are now real numbers, that characterize $f(\mathbf{p}_1, \mathbf{p}_2)$ in a model-independent way.

N.B. & J.-Y.Ollitrault, Phys. Rev. C **70** (2004) 064905

From the two-particle distribution to the two-particle cumulant

$$f(\mathbf{p}_1, \mathbf{p}_2) = f(\mathbf{p}_1)\, f(\mathbf{p}_2) \left[1 + \bar{f}_c(\mathbf{p}_1, \mathbf{p}_2)\right] \tag{1}$$

- First, extract the first Fourier coefficients v_n, $v_{c,n}^{\rm pair}$, $v_{s,n}^{\rm pair}$: well under control, uncertainty increases with n.
- These are model-independent numbers: publish them!

  (and let the theorists try to reproduce them with their favorite code).

From the two-particle distribution to the two-particle cumulant

$$f(\mathbf{p}_1,\mathbf{p}_2) = f(\mathbf{p}_1)\, f(\mathbf{p}_2)\left[1 + \bar{f}_c(\mathbf{p}_1,\mathbf{p}_2)\right] \qquad (1)$$

- First, extract the first Fourier coefficients v_n, $v_{c,n}^{\mathrm{pair}}$, $v_{s,n}^{\mathrm{pair}}$: well under control, uncertainty increases with n.
- Compute the single- and two-particle distributions $f(\mathbf{p}_1)$, $f(\mathbf{p}_2)$, $f(\mathbf{p}_1,\mathbf{p}_2)$ (or rather, a good approximation thereof).
- Perform the division.

Issue: Equation (1) holds at fixed reaction-plane orientation — doing it for various event-plane bins introduces unwanted terms.

From the two-particle distribution to the two-particle cumulant

For a fixed orientation of the reaction plane

$$\frac{f(\mathbf{p}_1,\mathbf{p}_2)}{f(\mathbf{p}_1)\,f(\mathbf{p}_2)} = 1 + \bar{f}_c(\mathbf{p}_1,\mathbf{p}_2)$$

In practice, one measures numbers of particles or particle pairs, that (might) have significant fluctuations from event to event:

$$\frac{\langle N_{\rm pairs}(\mathbf{p}_1,\mathbf{p}_2)\rangle}{\langle N_1(\mathbf{p}_1)\rangle\langle N_2((\mathbf{p}_2)\rangle} \neq \frac{f(\mathbf{p}_1,\mathbf{p}_2)}{f(\mathbf{p}_1)\,f(\mathbf{p}_2)}$$

so that some arbitrariness enters the determination of $\bar{f}_c(\mathbf{p}_1,\mathbf{p}_2)$ at this point. ("Usual" assumption: it vanishes at some point in phase space...)

Correlation studies

- The most appropriate way to remove the effect of the anisotropy in single-particle emission ("anisotropic flow") is to perform a division, rather than a subtraction.

- Model-independent numbers are preferable to model-dependent ones! (Even if they do not possess an intuitive interpretation.)

- Further work is needed, also on the theoretical side.