

# Early time development of azimuthal anisotropies from small to large Knudsen numbers

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# Onset of azimuthal anisotropies

Is the development at early times of azimuthal anisotropies – in particular, of anisotropic transverse flow – universal across different classes of models for the fireball?

👉 May be relevant for anisotropic flow in smaller systems

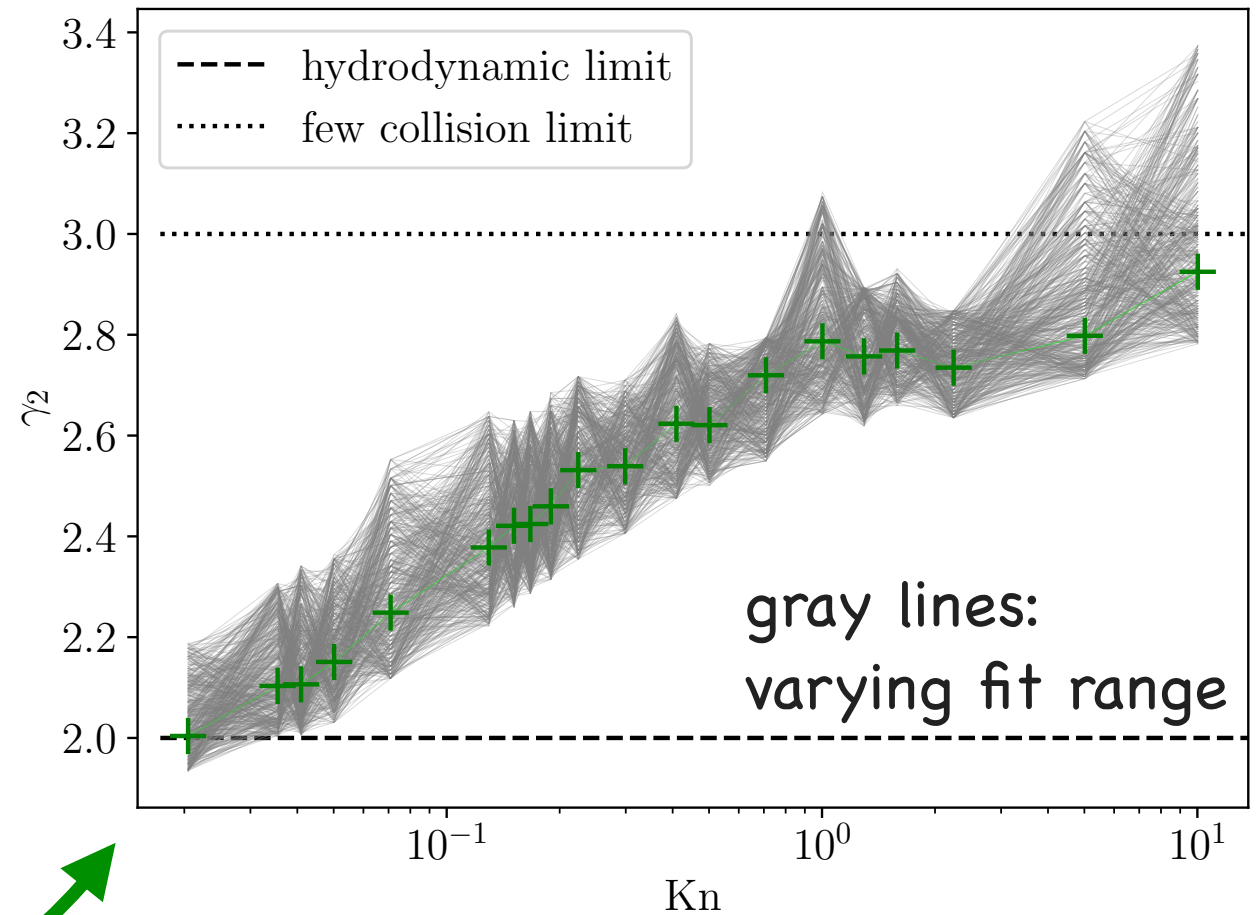
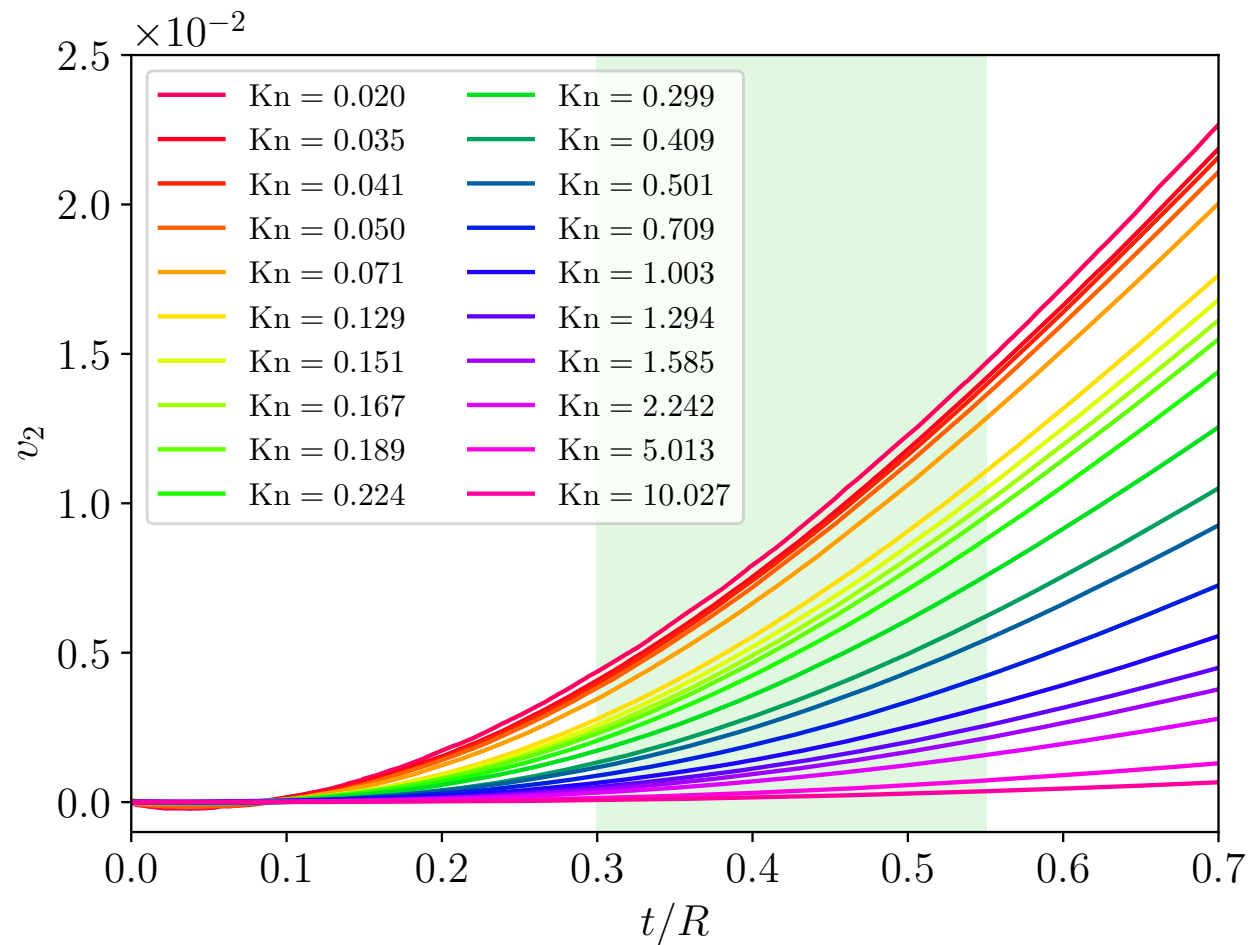
Investigations in kinetic transport theory (2D, massless particles, fixed semi-realistic initial geometry):

- Analytical calculations in the few-rescatterings regime (large Knudsen number  $Kn$ ).
- Transport code simulations with tunable cross section: from few to many rescatterings (“hydro limit”: small  $Kn$ ) per particle.

[arXiv:2109.15218](https://arxiv.org/abs/2109.15218) + [arXiv:2201.13294](https://arxiv.org/abs/2201.13294)

# Early time development of $v_2$

Transport simulations:



Ansatz  $v_2(t) \propto t^{\gamma_2}$

fit

Many  $\neq$  few rescatterings!

$R$ : typical initial system size

Rule of thumb: mean number of rescatterings per particle  $\approx 0.5 Kn^{-1}$

# Early evolution of the single-particle phase space density

Using the Boltzmann eq.  $p^\mu \partial_\mu f(t, \vec{x}, \vec{p}) = \mathcal{C}[f]$ , the Taylor expansion of the phase space density at early times can be recast in the form

$$f(t, \vec{x}, \vec{p}) = f_{\text{f.s.}}(t, \vec{x}, \vec{p}) + t \frac{\mathcal{C}[f]|_0}{E} + \frac{t^2}{2} \left( -\frac{\vec{p}}{E^2} \cdot \vec{\nabla}_x \mathcal{C}[f] + \frac{1}{E} \partial_t \mathcal{C}[f] \right)_0$$

↑  
free-streaming with  
the same initial state

$$+ \frac{t^3}{3!} \left( \frac{(\vec{p} \cdot \vec{\nabla}_x)^2}{E^3} \mathcal{C}[f] - \frac{\vec{p}}{E^2} \cdot \vec{\nabla}_x \partial_t \mathcal{C}[f] + \frac{1}{E} \partial_t^2 \mathcal{C}[f] \right)_0 + \mathcal{O}(t^4)$$

where the subscript 0 means that the quantities are evaluated in the initial state.

In the terms beyond free streaming, the collision kernel  $\mathcal{C}[f]$  is linear in the cross section  $\sigma$ ,  $\partial_t \mathcal{C}[f]$  includes terms of order  $\mathcal{O}(\sigma^2)$ , and more generally  $\partial_t^k \mathcal{C}[f]$  contains contributions  $\mathcal{O}(\sigma^{k+1})$ .

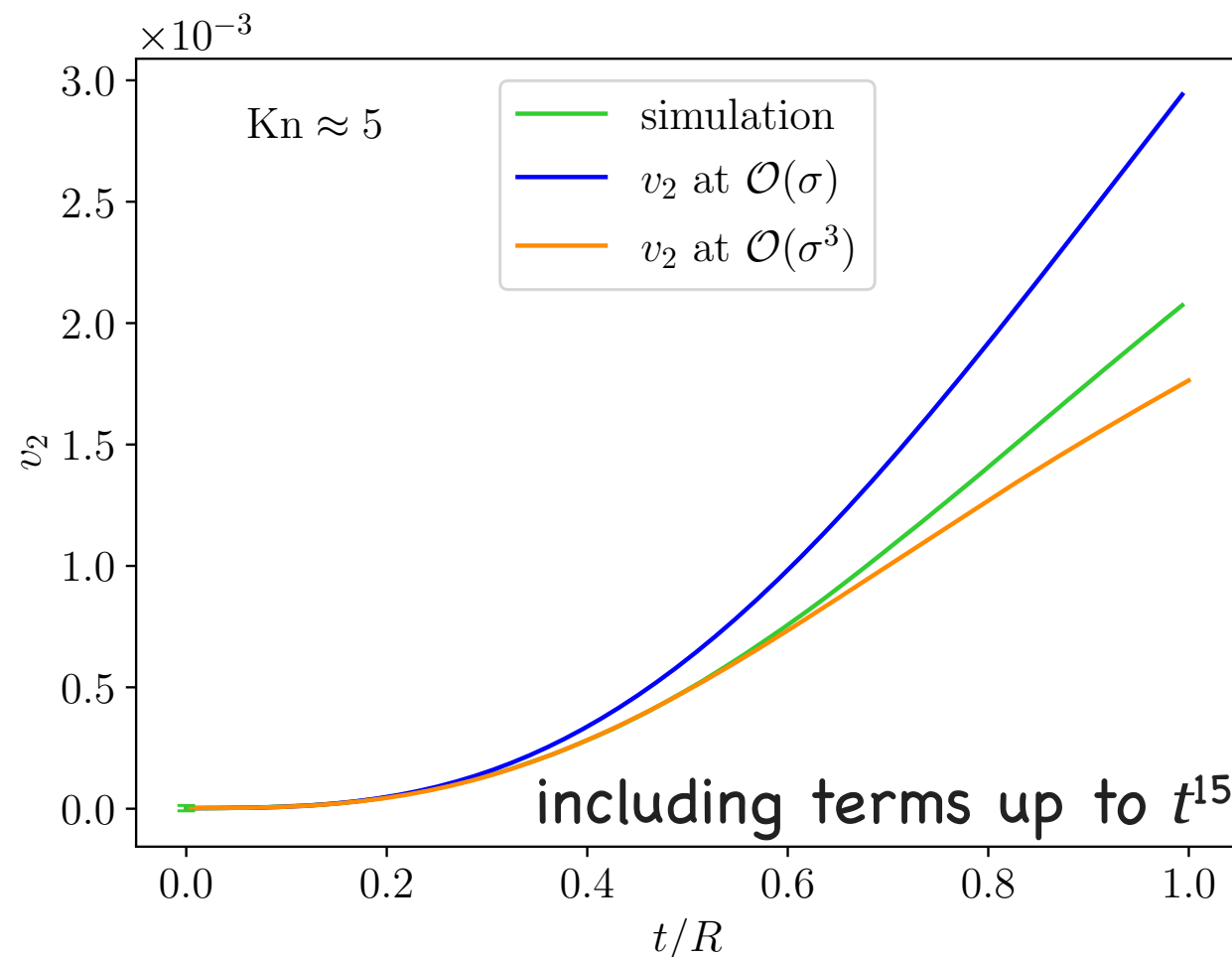
# Early time development of $v_2$

Invoking symmetry arguments (for an initial state without anisotropic flow), the early-time Taylor series yields

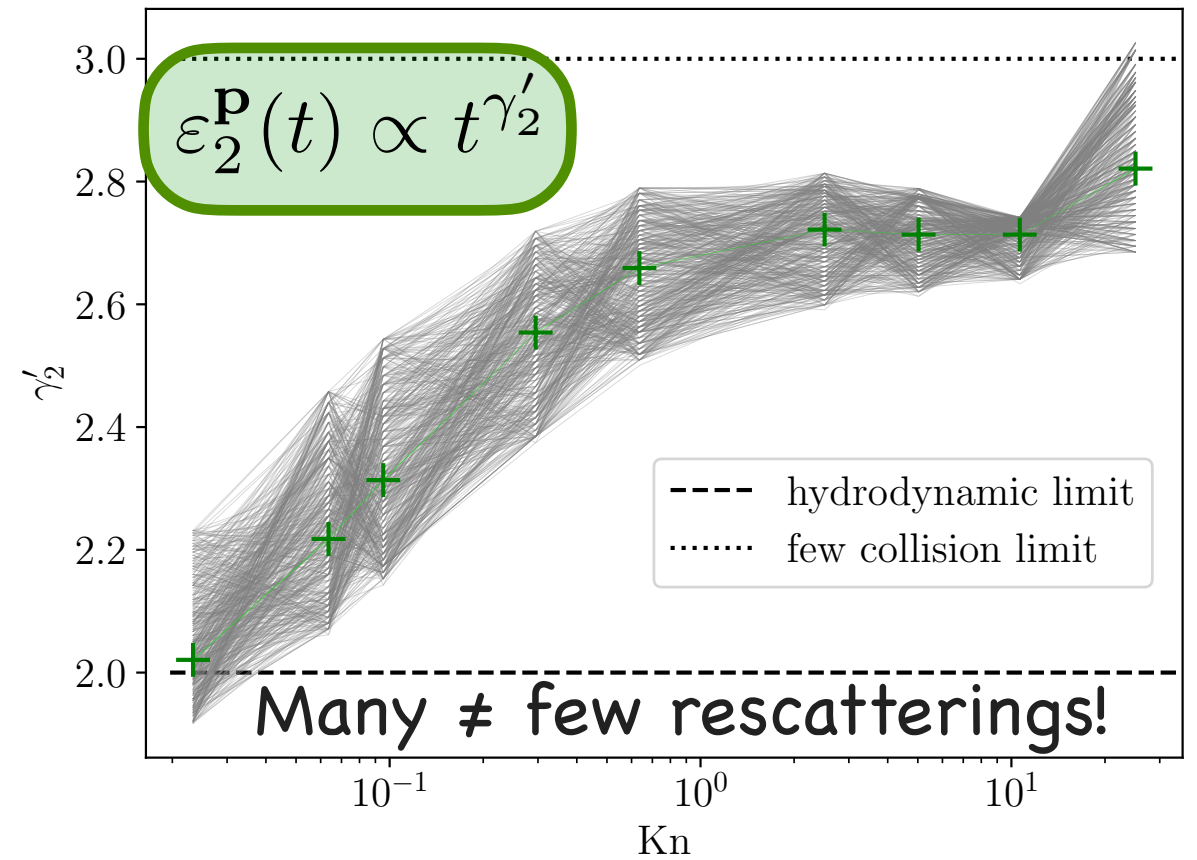
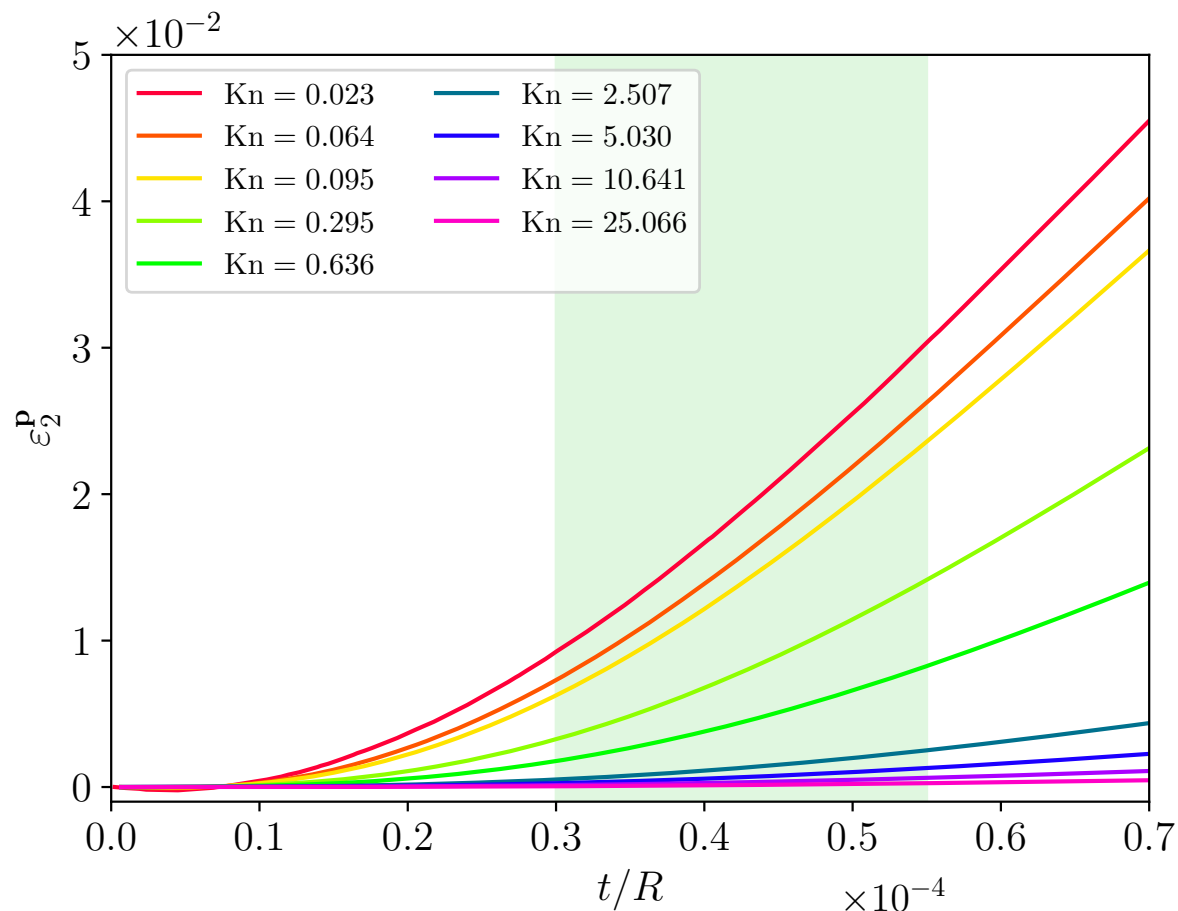
$$v_2(t) = \mathcal{O}(\sigma)t^3 + \mathcal{O}(\sigma^2)t^4 + [\mathcal{O}(\sigma) + \mathcal{O}(\sigma^3)]t^5 + \dots$$

👉 systematic (but tedious...) approach!

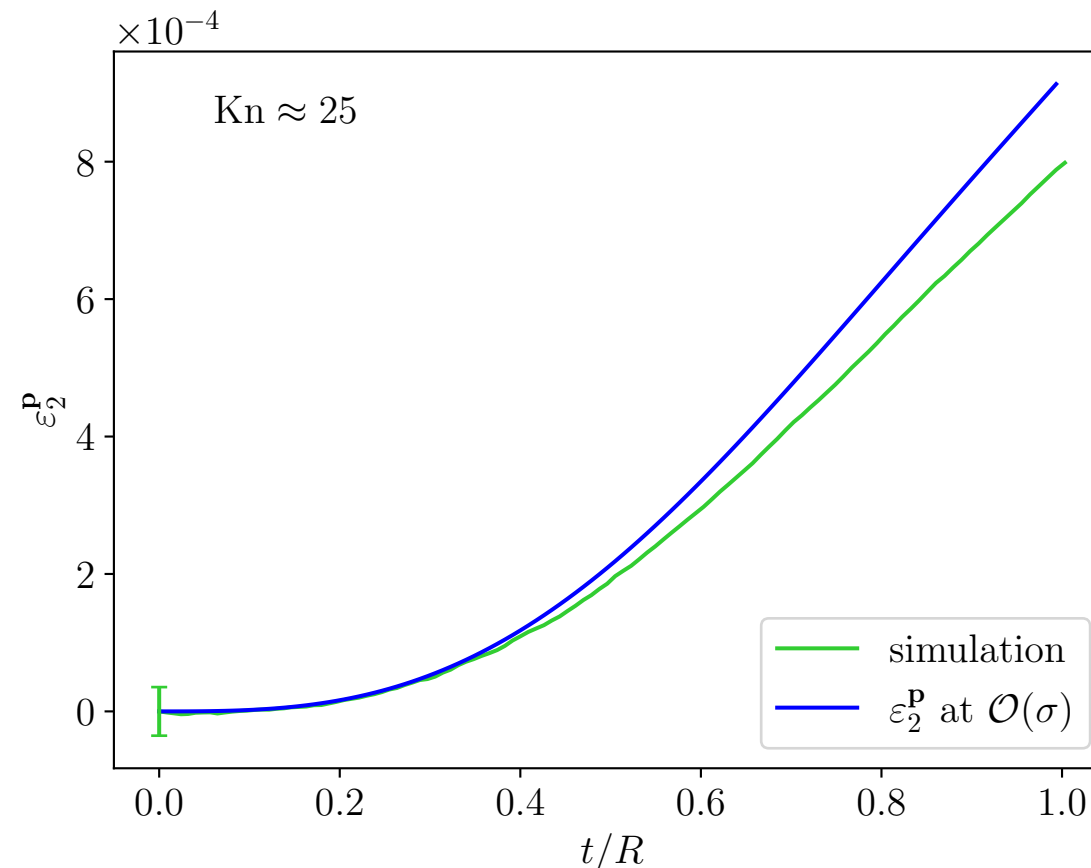
$v_2(t) \propto t^3$   
at early times!



# An alternative measure of elliptic flow



$$c_{2P} \equiv \frac{\langle T^{xx} - T^{yy} \rangle}{\langle T^{xx} + T^{yy} \rangle}$$

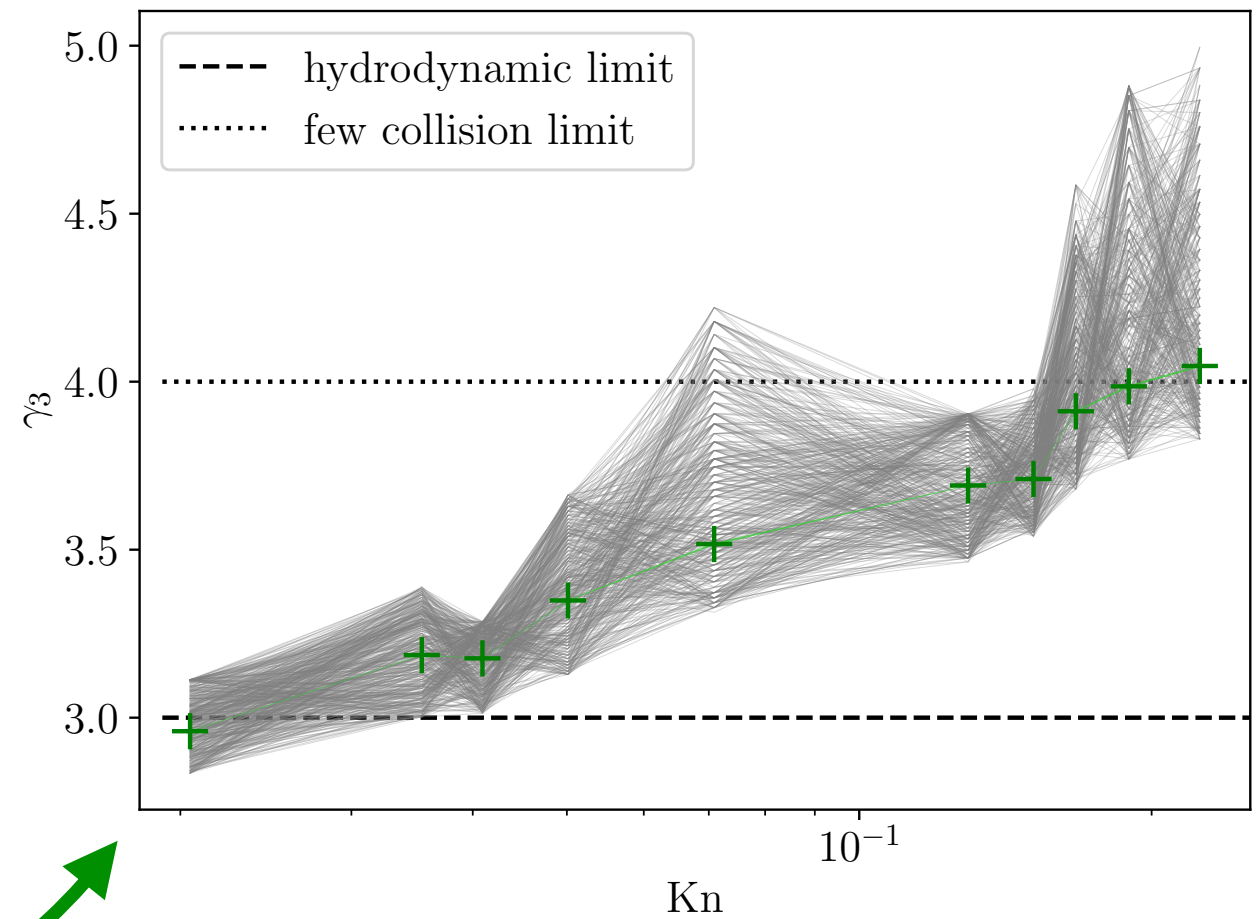
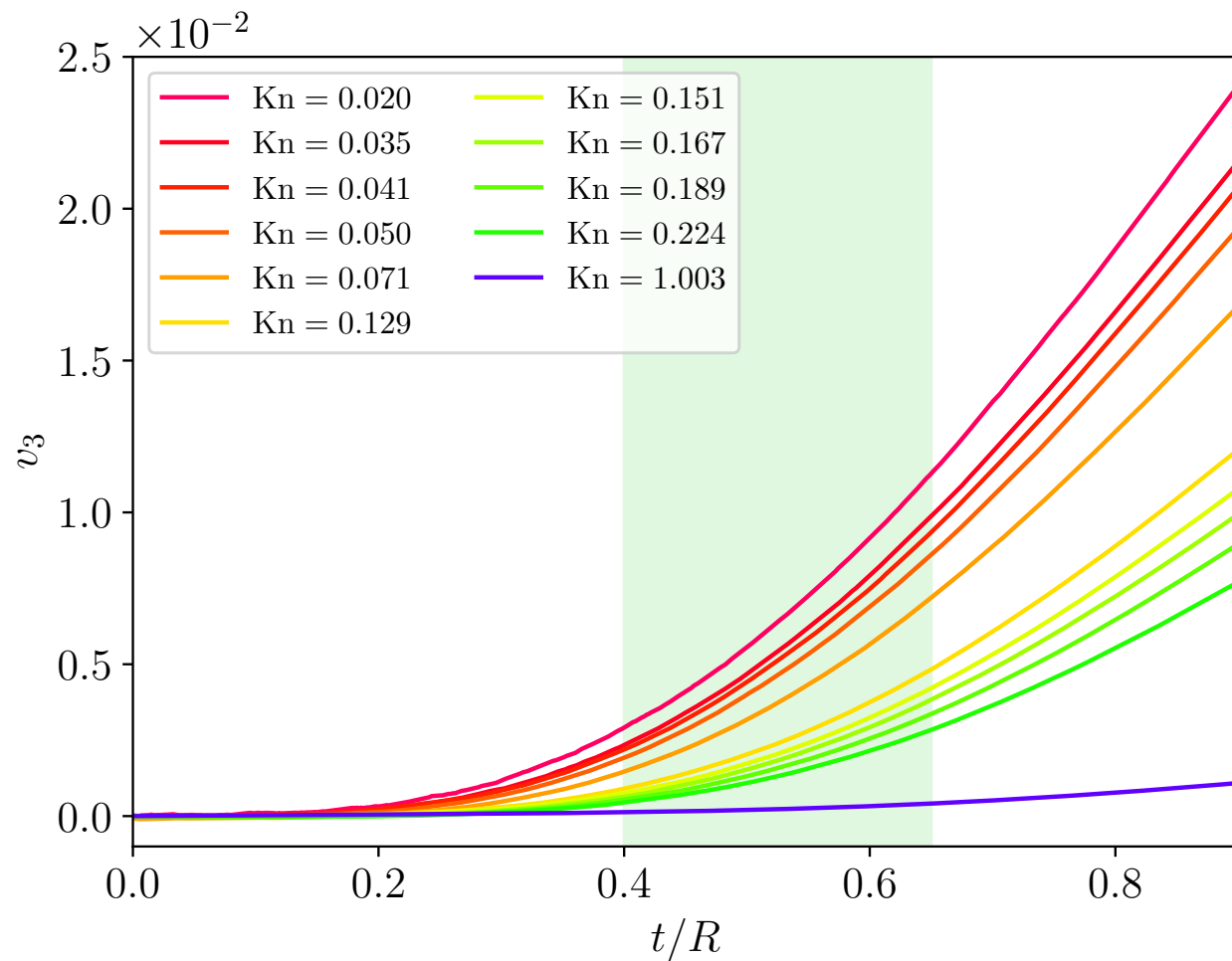


Bonus slide



# Early time development of $v_3$

Transport simulations:



Ansatz

$$v_3(t) \propto t^{\gamma_3}$$

fit

Many  $\neq$  few rescatterings!