xx xx Vol. xx, No. xx
200x xx HIGH ENERGY PHYSICS AND NUCLEAR PHYSICS xxx., 200x

Using the anisotropic-flow harmonics v_2 and v_4 to test dynamical descriptions of ultrarelativistic heavy-ion collisions

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Abstract We present various predictions for the anisotropic collective flow of particles in heavy-ion collisions, in particular scaling laws of the second and fourth harmonics v_2 and v_4, derived within ideal fluid dynamics. We also discuss qualitatively the deviation from the ideal behaviour expected in an out-of-equilibrium scenario.

Key words Relativistic heavy-ion collisions, anisotropic flow, fluid dynamics

In a non-central nucleus–nucleus collision, the impact parameter selects a preferred direction in the plane transverse to the beam, breaking the azimuthal symmetry. The single-particle transverse-momentum distribution (averaged over many events) is not isotropic, but presents a harmonic modulation, referred to as anisotropic flow[1] and quantified in terms of transverse-momentum- and rapidity-dependent Fourier coefficients $v_n(p_T, y)$. Since this anisotropy results from the reinteractions between outgoing particles, it carries information on the strength and the frequency of these interactions[2]. In particular, if they are frequent enough, i.e., if the mean free path λ is much smaller than the size R of the system (Knudsen number $Kn \equiv \lambda/R \ll 1$), then the latter expands according to the laws of fluid dynamics[3]. Modulo some assumptions, one can make analytical predictions for the behaviour of the distribution of outgoing particles, including its azimuthal anisotropies v_n, which are presented in section 1. In the opposite case when there are not enough collisions to ensure thermal equilibrium before anisotropic flow develops ($Kn \gtrsim 1$), qualitative arguments are yet possible, that lead to further predictions, see section 2.

1 Predictions of ideal fluid dynamics

Consider first the case in which reinteractions between particles are frequent enough to ensure the equilibration of the system at an early stage. There follows a hydrodynamic-like expansion in the vacuum, dominated by the many collisions between particles. When the system becomes dilute, however, its expansion is that of a free-streaming gas. Provided the transition between both regimes is sharp enough — which is a reasonable approximation when computing particle spectra if collective expansion dominates over thermal motion — the momentum distribution of the outgoing particles is given by the Cooper–Frye formula[4]

$$E\frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{p}} = C\int_\Sigma \exp\left(-\frac{p^\mu u_\mu(x)}{T}\right) p^\mu\,\mathrm{d}\sigma_\mu, \qquad (1)$$

where p^μ is the particle momentum, u_μ the 4-velocity of the fluid, Σ the freeze-out hypersurface, and C a constant. In the context of heavy-ion collisions, the validity of this formula amounts to having a small enough freeze-out temperature T.

In the small-T limit, the integral (1) can be performed by means of a saddle-point approximation[5]. One has to distinguish two kinds of particles.

Received xx January 2007

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1. *Slow particles* have a velocity $\mathbf{p}/m$ that equals the fluid velocity $\mathbf{u}$ at some point; this is where the particle energy in the fluid frame $p^\mu u_\mu$ reaches its minimum m. If T is small enough*, the dominant contribution to the integral in equation (1) comes from the neighborhood of this point, yielding

$$E\frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{p}} = c(m) f\left(\frac{\mathbf{p}_T}{m}, y\right), \tag{2}$$

where $\mathbf{p}_T$ and y are the particle transverse momentum and (longitudinal) rapidity. The function f is universal for all particle types, while $c(m)$ is a constant which depends on the particle species (actually, on the particle mass only). Thus, different particle types have similar momentum spectra, up to an overall factor, provided they are plotted as a function of p_T/m, as long as the latter is smaller than the maximum fluid velocity $u_{\max}$.

The mass-dependent constant $c(m)$ drops in the definition of the anisotropic-flow harmonic coefficients $v_n(p_T/m, y)$, which are thus the same for all slow particles, for any harmonic n. In particular, all particles at a given transverse *velocity* p_T/m have the same elliptic flow v_2, giving rise to the mass-scaling property that heavier particles have smaller elliptic-flow values, as qualitatively measured at the SPS[6] and at RHIC[7, 8].

2. *Fast particles*, on the other hand, have a velocity larger than the maximum fluid velocity. One can show[5] that they are mostly emitted by the regions of the fluid with a velocity $\mathbf{u}$ parallel to that of the particle and close to its maximum value. These conditions minimize $p^\mu u_\mu$ in equation (1), leading to a particle spectrum

$$E\frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{p}} \propto \exp\left(\frac{p_T u_{\max}(y,\phi) - m_T\sqrt{1+u_{\max}(y,\phi)^2}}{T}\right), \tag{3}$$

where m_T is the usual particle transverse mass, while $u_{\max}(y,\phi)$ denotes the maximal transverse velocity of the fluid along the direction of the particle transverse momentum. Neglecting for simplicity the ϕ dependence of $u_{\max}$, this yields a m_T-spectrum which does not scale with m_T, in qualitative agreement with experimental observations[9].

The next step consists in deriving the anisotropic-flow harmonics for fast particles. For that, let us write the maximum fluid velocity as a Fourier series $u_{\max}(\phi) = u_{\max}(1 + 2V_2\cos(2\phi) + 2V_4\cos(4\phi) + \cdots)$, where for the sake of brevity we have dropped the dependence on y and neglected odd harmonics. Expanding the exponential in equation (3), one first obtains the $\cos(2\phi)$ term:

$$v_2(p_T, y) = \frac{V_2(y) u_{\max}(y)}{T}\left(p_T - \frac{m_T u_{\max}(y)}{\sqrt{1+u_{\max}(y)^2}}\right). \tag{4}$$

As for slow particles, this equation implies a mass ordering of the v_2 values of fast particles, although different particle species no longer have the same elliptic flow at a fixed transverse velocity. Inspecting then the term in $\cos(4\phi)$ and using the fact that V_4 is of order $(V_2)^2$ if $u_{\max}(\phi)$ is a smooth function, one finds that for p_T large enough and to leading order in the fluid-velocity anisotropy one has

$$v_4(p_T, y) = \frac{v_2(p_T, y)^2}{2} \tag{5}$$

for each individual particle type. Several caveats regarding this relation should be taken in consideration. To begin with, corrections to the expansion to leading order in the anisotropy can be sizable, as was exemplified in a numerical simulation of a mid-central Au-Au collision[5]. A second point to keep in mind is that the relation (5) is non-linear, so that any averaging — be it over transverse momentum, over rapidity, or over particle type — will spoil it:

$$\langle v_4(p_T, y)\rangle = \frac{1}{2}\langle v_2(p_T, y)^2\rangle > \frac{1}{2}\langle v_2(p_T, y)\rangle^2,$$

where $\langle\cdots\rangle$ denotes the specific average considered. Eventually, one should not forget that equation (5) only holds for p_T large enough, which might be an issue since one expects departures from equilibrium to be more pronounced for larger transverse momenta[10, 11].

2 Out-of-equilibrium scenario

The matter created in an ultrarelativistic heavy-ion collision expands into the vacuum, thus its collective movement is characterized by a Mach number

*More precisely, if $T \ll m u_{\max}^2/(1+u_{\max}^2)$, where $u_{\max}$ denotes the maximum fluid velocity.

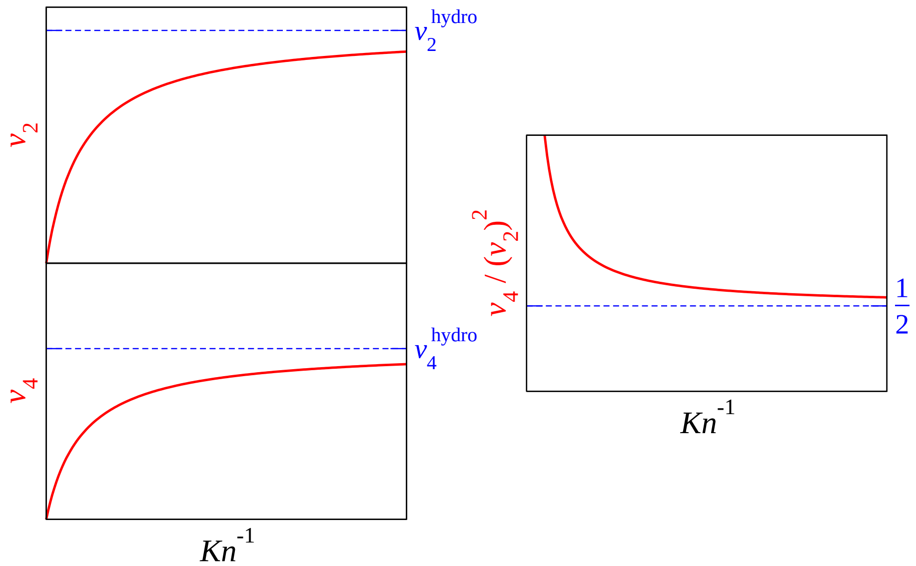

Figure 1: Sketches of the dependence with the number Kn^{-1} of collisions per particle of the anisotropic-flow harmonics v_2 (top left panel) and v_4 (bottom left panel) and of the resulting ratio $v_4/(v_2)^2$ (right panel).

of order unity[12]. This Mach number can actually be expressed as the product of two other dimensionless numbers, namely the Knudsen number Kn — which gives an estimate of the inverse of the number of collisions undergone by particles — and the Reynolds number Re — which quantifies the importance of viscous phenomena. Thus, a decrease in the number of reinteractions between particles (increasing Kn) amounts to growing viscous effects (decreasing Re).

Although a detailed quantitative study of the out-of-equilibrium evolution of the matter created in nucleus–nucleus collisions would require a numerical transport approach, yet one can make qualitative predictions for the behaviour of anisotropic flow[13]. Thus, if there is no reinteraction between the outgoing particles to leave an imprint of the initial anisotropy in position space, then the momentum distribution of the particles is isotropic (if it was such initially): no anisotropic flow develops, $v_n(Kn^{-1}=0)=0$.

Consider next the case where outgoing particles undergo few collisions. Intuitively, one builds up more flow by increasing the number of collisions, and this seems to be confirmed in transport computations, in which the amount of elliptic flow increases with the interaction cross-section[2]. In parallel, the fourth harmonic v_4 also increases with Kn^{-1}. As a consequence, one can expect a *decrease* of the ratio $v_4/(v_2)^2$ when the number of collisions per particle increases[14] (see figure 1).

Eventually, when the number of collisions per particle is large, the system equilibrates; increasing Kn^{-1} further no longer affects anisotropic flow, which saturates to its “hydrodynamical” value[14].

Using the general behaviour outlined above, that in a non-equilibrated system anisotropic flow increases with the number of collisions and is smaller than if it develops while the system is in equilibrium — in which case it no longer depends on Kn —, we can make further predictions. First, while for v_2 or v_4 the (saturating) value reached in the hydrodynamical regime is a maximum, for the ratio $v_4/(v_2)^2$ it is a minimum. More precisely, we know from section 1 that for fast identified particles, at given transverse momentum and rapidity, this minimum equals $\frac{1}{2}$ (up to large-eccentricity corrections). Therefore, in a non-equilibrated scenario one expects $v_4(p_T,y)/v_2(p_T,y)^2 > \frac{1}{2}$ (see figure 1), in agreement with experimental findings at RHIC[15].

Next, if the system is not equilibrated when anisotropic flow develops, v_2 depends on the particle density, which is directly proportional to the inverse of the Knudsen number[13]. This is arguably what one sees at RHIC on the dependence of elliptic flow on rapidity[16] and on the centrality (after accounting for the difference in the initial geometry) and energy of the collision, using different colliding nuclei[17].

3 Discussion

Different scenarios for the degree of equilibration of the matter created in ultrarelativistic nucleus–nucleus collisions during the ~ 5 first fm/c when anisotropic flow mostly develops (at least, close to midrapidity) lead to varying predictions for the behaviour of the harmonics v_n. It might prove hazardous to argue in favor of one scenario on the basis of the size of anisotropic flow only — in the equilibrium case, the size of v_2 is directly proportional to the sound velocity[13], so that one can accomodate weaker anisotropies by assuming a smaller velocity. The scaling laws predicted in the various approaches, however, are more robust.

In that spirit, we have discussed several scaling behaviours of the anisotropic-flow harmonics. In the case of a truly equilibrated medium described by ideal fluid dynamics, $v_n(p_T/m, y)$ is universal for all species of slow particles (those whose velocity is smaller than the maximum fluid velocity), reflecting the fact that all particles with a given velocity $\mathbf{p}/m$ preserve the anisotropy of the fluid cell from which they originate. For fast particles, elliptic flow obeys equation (4) and the ratio $v_4(p_T, y)/v_2(p_T, y)^2$ equals $\frac{1}{2}$[5].

If, on the contrary, anisotropic flow has developed while the system was not in equilibrium, the latter ratio should be larger than this lower bound. Additionally, the amount of flow (especially, of v_2, which is the largest, hence the more easily measured) should vary with the number of collisions undergone by the particles. This is reflected in any variation with the system size, in particular, by any dependence of the measured v_2 (divided by the initial spatial eccentricity, to cancel out geometrical effects) on a characteristic length, since such a dependence violates the scale invariance of perfect fluid dynamics.

Data from Au–Au and Cu–Cu collisions at RHIC seem to favor the latter scenario. If indeed anisotropic flow develops at RHIC in a non-equilibrated medium, then one can anticipate that in Pb–Pb collisions at LHC it will probe a system closer to equilibrium. This would manifest itself in particular in an increased elliptic flow v_2, as well as a smaller ratio $v_4/(v_2)^2$ at given transverse momentum and longitudinal rapidity.

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