Supplement: Chiral fermions

$\gamma^5 := i\gamma^0\gamma^1\gamma^2\gamma^3$

properties: $(\gamma^5)^\dagger = \gamma^5$
$(\gamma^5)^2 = 1$
$\{\gamma^5, \gamma^\mu\} = 0$

Weyl rep: $\gamma^5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

$P_{L,R} := \frac{1}{2}(1 \mp \gamma^5)$ are projectors:

$P_L^2 = P_L$, $P_R^2 = P_R$, $P_L P_R = P_R P_L = 0$, $P_L + P_R = 1$

Weyl rep: $P_L = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, $P_R = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

A Dirac spinor ψ can be decomposed as

$$\psi = \psi_L + \psi_R \quad \text{with}$$

$\psi_L := P_L \psi$, $\psi_R := P_R \psi$

Weyl rep: $\psi_L = \begin{pmatrix} \chi \\ 0 \end{pmatrix}$, $\psi_R = \begin{pmatrix} 0 \\ \eta \end{pmatrix}$

in the Lagrangian we have

$\bar\psi i \not{D} \psi = \bar\psi_L i \not{D} \psi_L + \bar\psi_R i \not{D} \psi_R$

and

$m\bar\psi\psi = m(\bar\psi_L \psi_R + \bar\psi_R \psi_L)$

$\Rightarrow$ for $m=0$ the eqs. of motion for ψ_L and ψ_R decouple

Then

ψ_L describes left handed particles and right-handed antiparticles

ψ_R " right " left "