7.10 Non-abelian HTLs

In QED we found that the effective action which includes $\Pi_{\mu\nu}^{HTL}$ can be written as

(*) $$S_{eff} = S + \Gamma$$

where $$\frac{\delta \Gamma}{\delta A^\mu(x)} = -J_\mu(x),$$

$$J_\mu(x) = m_D^2 \int \frac{d\Omega}{4\pi} v_\mu W(x, \vec{v})$$

(**) $$v \cdot \partial W = \vec{v} \cdot \vec{E}$$

In non-abelian gauge theories $\Pi_{\mu\nu}^{ab\,HTL} = \delta^{ab} \Pi_{\mu\nu}^{HTL}$ with the appropriate Debye mass.

Therefore there should also be something like (*) for NA gauge fields.

However, in NA theories the current is not gauge invariant, but it lives in the adjoint representation,

$$\boxed{J_\mu^a(x) = -\frac{\delta \Gamma}{\delta A^{\mu a}(x)}}$$

we could also have an adj. rep. W:

$$\boxed{J_\mu^a = m_D^2 \int \frac{d\Omega}{4\pi} v_\mu W^a(x, v).}$$

But then (**) would not be gauge covariant. This can be easily fixed, however by $\partial \to D_{adj}$.

$$\boxed{v\, D\, W = \vec{v} \cdot \vec{E}}$$

These eqs. could be called non-abelian Vlasov eqs.

Let us formally solve the eq. for W:

$$W = \frac{1}{v\cdot D}\,\vec{v}\cdot\vec{E} \quad \Rightarrow$$

$$-\frac{\delta\Gamma}{\delta A^\mu} = J_\mu = m_D^2\int\frac{d\Omega}{4\pi}\,v_\mu\,\frac{1}{v\cdot D}\,\vec{v}\cdot\vec{E}$$

Since $D = \partial - igA$, this means that $\frac{\delta\Gamma}{\delta A}$ and thus Γ contains arbitrarily high powers of A. Thus there is not only a HTL polarization tensor, but also infinitely many HTL vertices.

For example, if we expand

$$\frac{1}{v\cdot D} = [\,v\cdot\partial + g v\cdot A\,]^{-1} = [1 + \frac{1}{v\cdot\partial}\,g v\cdot A]^{-1}\frac{1}{v\cdot\partial}$$

$$= \frac{1}{v\cdot\partial} + \frac{1}{v\cdot\partial}\,g v\cdot A\,\frac{1}{v\cdot\partial} + \dots$$

The first term gives $\Pi_{\mu\nu}^{HTL}$, and the second gives rise to a HTL 3point function.

N.B.: $A^{ab}_{\text{adjoint}} \sim f^{abc}A^c_\mu$, it is clear that the HTL gauge field vertices are something non-abelian.

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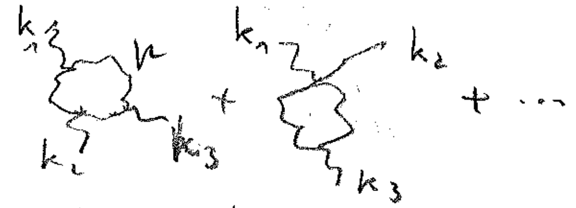
Our arguments leading to the NA Vlasov eqs. are suggestive, but they are of course not a proof. There are however explicit calculations of



which show that for $|k_i| \ll p \sim T$ the leading contribution agrees with

$$\frac{\delta^3 \Gamma}{\delta A^3}$$ from the non-abelian HTLs.

What is the physics behind this?

Hard particles experience a force by the soft fields, i.e. a momentum transfer.

But in the non-abelian case, even an interactions without momentum transfer change the (color-) charge of a particle

gluons.

quarks

Perturbation theory with HTLs

HTL resummed propagators:

$$\Delta_{t,e} = \frac{1}{-k^2 + \Pi_{t,e}}$$

for $k \sim gT$. $k^2 \sim g^2 T^2$, $\Pi \sim m_D^2 \sim g^2 T^2$

that is, k^2 and Π are of similar size and one and one has to include Π in Δ.

estimate HTL 3-point function:

$$\Gamma^{(3)}_{HTL} = \text{[diagram]} \sim g\, m_D^2 \frac{k^0}{(v\cdot k)^2} \sim \frac{g m_D^2}{k}$$

For $k \sim gT$: $\Gamma^{(3)}_{HTL} \sim g^2 T$ which is of the same size as the tree level vertex, so in PT both have to be included.

$m_D^2 \sim g k \sim g^2 T$

What happens for even <u>smaller momenta $\ll gT$?</u>

$\Pi_{e,t} \sim m_D^2$ when $k^0 \sim |\vec{k}|$.

Then $\Pi_{e,t} \gg k^2$ and they completely dominate over the 'tree level', propagator are "small"

$\mapsto$ field fluctuations are small

From the calculation of the pressure we know that magnetic-scale gluons ($k \sim g^2 T$) are large in the sense that $\partial \sim gA$, so that there is no expansion parameter and the magnetic scale physics is non-perturbative.

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Estimate the size of magnetic-scale fields:

$g^2 T \sim g A(x) \Leftrightarrow A(x) \sim gT$, so that

(*) $\langle A(x) A(0) \rangle \sim g^2 T^2$

On the other hand,

$$\langle A(x) A(0) \rangle = \Delta^>(x) = \int \frac{d^4k}{(2\pi)^4} [1 + f_B(k^0)] \rho(k)$$

For $|k^0| \ll T$: $f_B = \frac{1}{e^{k_0/T} - 1} \simeq \frac{T}{k^0} \gg 1$

$\langle A(x) A(0) \rangle \sim T \int d^3k \; \Delta(k)$

When $\Pi \sim g^2 T^2$, $\vec{k} \sim g^2 T$

$\langle A(x) A(0) \rangle \sim T (g^2 T)^3 \frac{1}{g^2 T^2} \sim g^4 T^2$

much smaller than (*). How can that be?

We know that $\Pi_t(k) = 0$ when k_0 (no screening of magnetostatic fields). $\Rightarrow$ $\Pi_t(k) \ll g^2 T^2$ when $|k_0| \ll |\vec{k}|$

Thus the characteristic frequency of magnetic-scale gauge fields is much smaller than $g^2 T$.

To see how small, compute Π_t for $|k_0| \ll |\vec{k}|$:

$$\Pi_t = \frac{m_D^2}{2} \left[\frac{k_0^2}{\vec{k}^2} - \frac{k_0 k^{\prime 2}}{\vec{k}^2} \int \frac{d\Omega}{4\pi} \frac{1}{v \cdot k} \right] \simeq \frac{m_D^2}{2} k_0 \underbrace{\int \frac{d\Omega}{4\pi} \frac{1}{v \cdot k}}$$

$$\simeq \frac{2\pi}{4\pi} \int_{-1}^{1} d\cos\theta \frac{1}{k^0 - |\vec{k}| \cos\theta} \underset{\text{for } \operatorname{Im} k^0 > 0}{\simeq} \frac{1}{2} k^0 \int_{-1}^{1} dc \frac{1}{i0^+ - |\vec{k}| c}$$

$$\simeq \frac{1}{2}(-i\pi) \int_{-1}^{1} dc \; \delta(|\vec{k}| c) = -\frac{i\pi}{2} \frac{1}{|\vec{k}|} \Rightarrow$$

$$\boxed{\Pi_t(k) \simeq -i \frac{\pi}{4} m_D^2 \frac{k_0}{|\vec{k}|} \quad \text{or } |k_0| \ll |\vec{k}|, \operatorname{Im} k_0 > 0}$$

characteristic frequency: $k^2 \simeq -\vec{k}^2 \sim \Pi_t \Leftrightarrow$

$k^0 \sim \frac{|\vec{k}|^3}{m_D^2} \Leftrightarrow \boxed{k_0 \sim g^4 T}$

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