8.7 <u>The HTL resummed fermion propagator</u>

When computing Σ in the HTL approximation, the $\vec{k}$ - integral

$$\int \frac{d^3k}{(2\pi)^3} \frac{1}{E_k} \left(f_B(E_k) + f_F(E_k) \right)$$

is saturated at $\vec{k} \sim T$ since this is the only scale in the integral.
Therefore it is called <u>hard</u> thermal loop.

$$\Sigma_{HTL}(p) = \frac{e^2T^2}{8} \int \frac{d\Omega}{4\pi} \frac{\not{v}}{v\cdot p} \quad , v = \begin{pmatrix} 1 \\ \vec{v} \end{pmatrix}, \quad |\vec{v}| = 1$$

$\vec{v}$ is the velocity of the particle in the loop, its direction is given by Ω.
$\Sigma_{HTL}(p)$ has discontinuity only for $|p^0| < |\vec{p}|$.

HTL resummed propagator: $\quad S_{HTL}(p) = \dfrac{-1}{\not{p} - \Sigma_{HTL}}$

NB: Since we only have rotational and not Lorentz invariance, $\Sigma_{HTL}(p)$ depends on p_0 and $|\vec{p}|$ separately which can make computations much more difficult than with mass resummation.

Special case: <u>$\vec{p} = 0$</u>

$$\Sigma = \frac{e^2T^2}{8} \int \frac{d\Omega}{4\pi} \frac{\gamma^0 - \vec{v}\cdot\vec{\gamma}}{p_0} = \frac{e^2T^2}{8p^0}\gamma^0$$

Then S_{HTL} has a pole at $p^0 - \dfrac{e^2T^2}{8p^0} = 0 \iff$

$$p_0 = \pm m_{pl} \quad , \quad m_{pl}^2 = \frac{e^2T^2}{8} \quad \underline{\text{plasmino}}$$

This describes a spatially homogeneous oscillation now consider $|p^\mu| \gg eT$:

S_{HTL} has poles close to the light cone $p_0 \simeq \pm|\vec{p}|$, with small corrections which vanish for $e \to 0$.
We can always write

$$\Sigma_{HTL} = a \not{p} + b\gamma^0$$

$$\mathrm{tr}(\not{p}\,\Sigma_{HTL}) = 4\left(a p^2 + b p^0\right) \overset{p^2 \sim e^2}{\simeq} 4 b p_0 \quad \Rightarrow$$

$$b \simeq \frac{1}{4p_0}\,\frac{e^2T^2}{8}\int\frac{d\Omega}{4\pi}\,\frac{1}{v\cdot p}\,\underbrace{\mathrm{tr}(\not{p}\cdot v)}_{=4v\cdot p} = \frac{e^2T^2}{8p_0}$$

We can neglect $a\not{p}$ because this is a small correction to $\not{p}$.

$$S_{HTL} \simeq \frac{-1}{\not{p} - b\gamma^0} = -\frac{\not{p} - b\gamma^0}{(p^0-b)^2 - \vec{p}^{\,2}}$$

poles at $p_0 = \pm|\vec{p}| + b \simeq \pm\left(|\vec{p}| + \frac{e^2T^2}{8|\vec{p}|}\right)$

We want to write this as

$$p_0 = \pm\sqrt{\vec{p}^{\,2} + m_\infty^2} = \pm\left(|\vec{p}| + \frac{m_\infty^2}{2|\vec{p}|}\right) \quad \Rightarrow$$

$$\boxed{m_\infty^2 = \frac{e^2T^2}{4}}$$ asymptotic mass

NB: $m_\infty^2 = 2m_{pl}^2$. There are different types of thermal masses!

8.8 HTL polarization tensor

In imaginary time we considered $\Pi^{\mu\nu}(p)$ for $p^0=0$, $|\vec{p}| \ll T$. In this HTL approximation we found

(*) $$\Pi^{ab}_{\mu\nu}(0,\vec{p}) = -m_D^2\,\delta^{ab}\,u_\mu u_\nu \qquad , \quad u = \begin{pmatrix}1\\ \vec{0}\end{pmatrix}$$

which is valid in QED and QCD.

the Debye mass

$$m_D^2 = g^2 T^2 \left(\frac{C_A}{3} + \frac{N_F}{6}\right)$$

Only A_0 is screened:

$$\Delta_{00}(0,\vec{p}) = \frac{1}{\vec{p}^2 + m_D^2}$$

$$\Delta_{00}(0,\vec{x}) = \frac{e^{-m_D|\vec{x}|}}{4\pi|\vec{x}|}$$

electrostatic screening.

Now consider $|\vec{p}| \ll T$, and $|p^0| \ll T$ with p^0 near the real axis. The computation of $\Pi(p)$ is very similar to Σ_{HTL}, so I just quote the result.

$\Pi^{ab}_{\mu\nu} = \Pi_{\mu\nu}\,\delta^{ab}$ with

$$\boxed{\Pi_{\mu\nu}(p) = m_D^2 \left[-u_\mu u_\nu + p^0 \int \frac{d\Omega}{4\pi}\,\frac{v_\mu v_\nu}{v\cdot p}\right]}$$

It is the same in QED and non-abelian theories with the appropriate m_D.

For $p^0=0$ this reproduces (*).

In QED: $\Pi_{\mu\nu}(x) = \langle J_\mu(x) J_\nu(0)\rangle$

Check current conservation:

$$p^\mu \Pi_{\mu\nu}(p) = m_D^2 \left[-p^0 u_\nu + p^0 \underbrace{\int \frac{d\Omega}{4\pi}\,\frac{v\cdot p\; v_\nu}{v\cdot p}}_{=\int \frac{d\Omega}{4\pi} v_\nu = \delta_{\nu 0} = u_\nu}\right] = 0$$

ok

We have seen that for $p^\mu \Pi_{\mu\nu} = 0$ one can write

$$\Pi^{\mu\nu} = P_t^{\mu\nu} \Pi_t + P_\ell^{\mu\nu} \Pi_\ell$$

where $P_{t,\ell}^{\mu\nu}$, both satisfy $p_\mu P_{t,\ell}^{\mu\nu} = 0$

$P_t^{mn} = \delta^{mn} - \hat{p}^m \hat{p}^n$, $P_t^{\mu 0} = 0$ Spatially transverse

$P_\ell^{\mu\nu} = \frac{p^\mu p^\nu}{p^2} - \eta^{\mu\nu} - P_t^{\mu\nu}$ has longitudinal spatial components.

$$\left.\begin{array}{l} \Pi^{00} = m_D^2 \left[-1 + p_0 \int \frac{d\Omega}{4\pi} \frac{1}{v \cdot p} \right] \\ P_\ell^{00} = \frac{p_0^2}{p^2} - 1 = \frac{p_0^2 - (p_0^2 - \vec{p}^2)}{p^2} = \frac{\vec{p}^2}{p^2} \end{array}\right\} \Rightarrow$$

$$\boxed{\Pi_\ell = m_D^2 \frac{p^2}{\vec{p}^2} \left[-1 + p_0 \int \frac{d\Omega}{4\pi} \frac{1}{v \cdot p} \right]}$$

$P_t^{mm} = 2$, $P_\ell^{mm} = \frac{\vec{p}^2}{p^2} + 3 - 2 = \frac{p_0^2}{p^2}$

$\Pi^{mm} = m_D^2 p_0 \int \frac{d\Omega}{4\pi} \frac{1}{v \cdot p}$ and $\Pi^{mm} = 2 \Pi_t + \frac{p_0^2}{p^2} \Pi_\ell \Longrightarrow$

$$\Pi_t = \frac{1}{2} \left(\Pi^{mm} - \underbrace{\frac{p_0^2}{p^2} \Pi_\ell}_{= \frac{p_0^2}{\vec{p}^2}} \right)$$

$$= \frac{m_D^2}{2} \left\{ p_0 \int \frac{d\Omega}{4\pi} \frac{1}{v \cdot p} - \frac{p_0^2}{p^2} \frac{p^2}{\vec{p}^2} \left[-1 + p_0 \int \frac{d\Omega}{4\pi} \frac{1}{v \cdot p} \right] \right.$$

$$= \frac{m_D^2}{2} \left\{ \underbrace{\left(1 - \frac{p_0^2}{\vec{p}^2} \right)}_{= \frac{-p^2}{\vec{p}^2}} p_0 \int \frac{d\Omega}{4\pi} \frac{1}{v \cdot p} + \frac{p_0^2}{\vec{p}^2} \right\}$$

$$\boxed{\Pi_t(k) = \frac{m_D^2}{2} \left\{ - \frac{p^2}{\vec{p}^2} p_0 \int \frac{d\Omega}{4\pi} \frac{1}{v \cdot p} + \frac{p_0^2}{\vec{p}^2} \right\}}$$

HTL resummed propagator

free propagator

$$\Delta_0^{\mu\nu} = -\frac{1}{k^2}\left[-\eta^{\mu\nu} + (1-\xi)\frac{k^\mu k^\nu}{k^2}\right]$$

$$= -\frac{1}{k^2}\left[P_l^{\mu\nu} + P_t^{\mu\nu} - \xi\frac{k^\mu k^\nu}{k^2}\right]$$

$\Rightarrow$ HTL resummed propagator

$$\Delta^{\mu\nu} = P_l^{\mu\nu}\Delta_l + P_t^{\mu\nu}\Delta_t - \xi\frac{k^\mu k^\nu}{k^2}$$

with

$$\Delta_{l,t} = \frac{-1}{k^2 - \Pi_{l,t}}$$

Also magnetic fields are screened for $k^0 \neq 0$.

Dynamical screening.

8.7