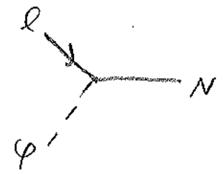
# 8.6 Fermion self-energy

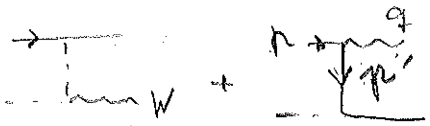
Motivation: consider again production of sterile neutrinos

(*) We found a contribution from $l\varphi \to N$




proportional to $M^2$

(**) Consider the process $l\varphi \to N W$




$p'^2 = (p-q)^2 < 0 \Leftrightarrow |p'_0| < |\vec{p}'|$
space-like

These contain a SM gauge coupling $g$

Now the rate is proportional to $g^2 T^2$

When $gT \gtrsim M$, (**) becomes as important than (*).

The squared amplitude contains $\frac{1}{p'^2}$ giving rise to an IR divergence.

When computing the pressure, when we stay in imaginary time, IR divergences only appear for bosons.

For scalars, they are screened when the HTL thermal mass is included in the propagator.

We will see that for Minkowski $p'$ ($p'_0 \in \mathbb{R}$) there is also a HTL in the fermion self-energy term which screens the IR singularity.

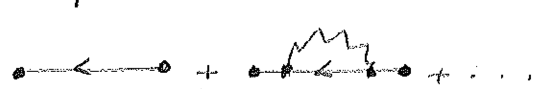
Consider electron self-energy in QED (the l-self-energy is obtained by multiplying with $P_L$)

Full e-propagator

$$S(p) = \frac{-1}{\not p - \Sigma(p)}$$ selfenergy

expand:

$$S = -\left[\not p\left(1-\frac{1}{\not p}\Sigma\right)\right]^{-1} = -\frac{1}{1-\frac{1}{\not p}\Sigma}\frac{1}{\not p}$$

(*) $$= -\frac{1}{\not p} - \frac{1}{\not p}\Sigma\frac{1}{\not p} + \dots =$$ 

path integral

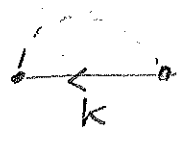
$$S(x) = \frac{1}{Z}\int \underbrace{e^{iS}}_{=e^{iS_0}\left(1+iS_{int}+\frac{1}{2}(iS_{int})^2+\dots\right)} \psi(x)\,\bar\psi(0)$$

$$iS_{int} = i\int d^4x\, \bar\psi e \not A \psi$$

$$\Sigma \overset{(*)}{=} -$$  $$= -e^2 \sum\!\!\!\!\!\!\int_k \gamma_\mu \frac{-1}{\not k}\gamma_\nu \Delta^{\mu\nu}(k-p)$$

Feynman gauge: $\Delta^{\mu\nu} = \eta^{\mu\nu}\frac{1}{(k-p)^2}$

$$\gamma^\mu \frac{1}{\not k}\gamma_\mu = \frac{1}{k^2}\underbrace{\gamma^\mu \not k \gamma_\mu}_{= 2k^\mu\gamma_\mu - \not k \overbrace{\gamma^\mu\gamma_\mu}^{=\eta^\mu{}_\mu = d+1} = -(d-1)\not k}$$

This is almost the same Matsubara sum as for the sterile ν production rate, there we had 

$$\Sigma(p) = -(d-1)\, e^2 \int \frac{d^d k}{(2\pi)^d}\; T \sum_{k^0} \frac{\not{k}}{k^2 (k-p)^2}$$

cf. 7.5

$$= -\frac{1}{2E_k}\,\frac{1}{2E_{k-p}} \Bigg\{ \left[ \frac{1}{-p^0 - E_{k-p} + E_k} - \frac{1}{-p^0 + E_{k-p} + E_k} \right] \left[\tfrac{1}{2} - f_F(E_k)\right] \left[ E_k \gamma^0 - \vec{k}\cdot\vec{\gamma} \right]$$

$$+ \left[ -\frac{1}{-p^0 - E_{k-p} + E_k} + \frac{1}{-p^0 - E_{k-p} - E_k} \right] \left[\tfrac{1}{2} + f_B(E_{k-p})\right] \left[ (p^0 + E_{k-p})\gamma^0 - \vec{k}\cdot\vec{\gamma} \right]$$

$$+ \left[ \frac{1}{-p_0 + E_{k-p} + E_k} - \frac{1}{-p_0 + E_{k-p} - E_k} \right] \left[\tfrac{1}{2} + f_B(-E_{k-p})\right] \left[ (p^0 - E_{k-p})\gamma^0 - \vec{k}\cdot\vec{\gamma} \right]$$

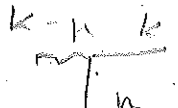
$$+ \left[ -\frac{1}{-p_0 - E_{k-p} - E_k} + \frac{1}{-p_0 + E_{k-p} - E_k} \right] \left[\tfrac{1}{2} - f_F(-E_k)\right] \left[ -E_k \gamma^0 - \vec{k}\cdot\vec{\gamma} \right] \Bigg\}$$

Now one can analytically continue.
The $\vec{k}$-integrand has two types of poles:

(i) the ones which contributed to the production rate at

$p^0 = \pm (E_{k-p} + E_k)$ , $|p^0| \geq |\vec{p}| \Rightarrow p$ time-like

(ii) $p^0 = \pm (E_{k-p} - E_k)$



$p^2 < 0$ "space-like"

integrating over $\vec{k}$ the poles are 'smeared' giving rise to cuts

# HTL approximation

Consider $|p_\mu| \ll T$. We will see that the integral is saturated at $|\vec{k}| \sim T$, so we can expand in $p_\mu / |\vec{k}|$.

$$E_{k-p} \simeq \left(\vec{k}^2 - 2\vec{k}\cdot\vec{p}\right)^{1/2} \simeq E_k\left(1 - \frac{\vec{k}\cdot\vec{p}}{\vec{k}^2}\right)$$

$$" \;\; = E_k - \hat{k}\cdot\vec{p} \qquad \left(\hat{k} := \frac{\vec{k}}{|\vec{k}|}\right)$$

$$\Sigma \simeq +(d-1)e^2 \int \frac{d^dk}{(2\pi)^d} \frac{1}{4\vec{k}^2} E_k$$

$$\Big\{ \frac{1}{-p_0 + \hat{k}\cdot\vec{p}} (\gamma^0 - \hat{k}\cdot\vec{\gamma}) \Big[ \underset{\sim}{\tfrac{1}{2}} - f_F(E_k) - \Big( \underset{\sim}{\tfrac{1}{2}} + f_B(E_k) \Big) \Big]$$

$$+ \frac{1}{-p_0 - \hat{k}\cdot\vec{p}} (-\gamma^0 - \hat{k}\cdot\vec{\gamma}) \Big[ + \Big( \underset{\sim}{\tfrac{1}{2}} + f_B(E_k) \Big) - \Big( \underset{\sim}{\tfrac{1}{2}} - f_F(E_k) \Big) \Big] \Big\}$$

The $\vec{k}$-integral is UV finite, and we can put $d=3$.

Substitute $\vec{k} \to -\vec{k}$ in the second line

$$\Sigma \simeq + e^2 \int \frac{d^3k}{(2\pi)^3} \frac{1}{E_k} \frac{\not{v}}{v\cdot p} \left[ f_F(E_k) + f_B(E_k) \right]$$

with

$$v = \begin{pmatrix} 1 \\ \hat{k} \end{pmatrix}$$

The integrations over $|\vec{k}| = E_k$ and $\Omega$ factorize

$$\int \frac{d^3k}{(2\pi)^3} \frac{1}{E_k} f_{B,F}(E_k) = \begin{cases} \frac{T^2}{12} \\ \frac{T^2}{24} \end{cases}$$

$$\boxed{\Sigma(p) = \frac{e^2}{8} T^2 \int \frac{d\Omega}{4\pi} \frac{\not{v}}{v\cdot p}}$$

HTL selfenergy

<u>For $p \lesssim eT$:</u> $\Sigma \sim e^2 T^2 \frac{1}{eT} \gtrsim p \Rightarrow$

$\Sigma$ cannot be treated as a perturbation, and must be resummed into the propagator.
This screens the IR singularity in the production rate.
NB: $\Sigma_{HTL}$ has a discontinuity and imaginary part only for space-like momenta.

Now consider time-like $p$. Special case: $\vec{p} = 0$

$$\Sigma(p) = \frac{1}{8} e^2 T^2 \frac{\gamma^0}{p^0}$$

Then the resummed propagator has a pole for

$$p^0 = \frac{e^2 T^2}{8} \frac{1}{p_0} \Leftrightarrow \quad p_0 = \pm \frac{eT}{2\sqrt{2}} \quad \underline{\text{plasmino}}$$

NB: $\Sigma_{HTL}$ has a discontinuity and imaginary part only for space-like momenta.