8.4 Production of sterile neutrinos

Standard Model of particle physics:

only left-handed (LH) neutrinos

neutrinos are massless

however: neutrinos have mass

possible explanation: extension of the SM with

right-handed neutrinos N_i

Yukawa interaction (for simplicity consider only 1 sterile flavor)

(*) $$\mathcal{L}_{int} = \sum_\alpha \overline{N}\, y_\alpha \tilde{\varphi}^\dagger l_\alpha + \text{H.c.}$$

Hermitian conjugate

flavor $\alpha \in \{e, \mu, \tau\}$

Levi-Civita

$\tilde{\varphi} = \epsilon \varphi^*$, φ = Higgs doublet

$l_\alpha = \begin{pmatrix} \nu_\alpha \\ e_\alpha \end{pmatrix}_L$ lepton doublet $(e_e = e, e_\mu = \mu, e_\tau = \tau)$

$P_L l_\alpha = l_\alpha$ left-handed (LH)

When $\langle\tilde{\varphi}\rangle \neq 0$, (*) gives rise to a Dirac mass term

$\tilde{\varphi}^\dagger l_\alpha$ gauge invariant $\Rightarrow$

N has no SM gauge interaction, sterile

Choose N to be a Majorana spinor, $N = N^c \equiv C\overline{N}^T$

charge conjugate

$[C^{-1}\gamma_\mu C = -\gamma_\mu^T]$

Since N is sterile, one may add

Majorana mass terms

$$\mathcal{L}_{Maj.} = -\tfrac{1}{2}\overline{N} M N$$

The neutrino Yukawa couplings y_α are typically very small. Then the N are never in thermal equilibrium, and only a few of them are present in the primordial plasma. They are long lived and thus one leading dark matter candidate.

We can apply our formula for the production rate to sterile neutrino production.

We assumed $\mathcal{L}_{int} = \bar{J} N$

Re-write (*) to bring it into this form.

Def $\quad I := \sum_\alpha y_\alpha \tilde{H}^\dagger l_\alpha$

$$\mathcal{L}_{int} = \bar{N} I + (\bar{N} I)^\dagger$$

For any two spinors

(i) $(\bar{\varphi}\psi)^\dagger = \psi^\dagger(\varphi^\dagger\gamma^0)^\dagger = \psi^\dagger \underbrace{\gamma^{0\dagger}}_{=\gamma^0}\varphi = \bar{\psi}\varphi$

For any two spinor fields:

(ii) $\bar{\varphi}\psi^c = \bar{\varphi}\,C\,\bar{\psi}^T = -\bar{\psi}(\bar{\varphi}C)^T = -\bar{\psi}\underbrace{C^T}_{=-C}\bar{\varphi}^T = \bar{\psi}\varphi^c$

Thus $\bar{N} I = \overline{N^c}\, I = \overline{I^c}\, N$ and

$$\mathcal{L}_{int} = (\bar{I} + \overline{I^c})N = \bar{J} N \quad \text{with}$$

$$J := I + I^c$$

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interaction picture

$$N(x) = \sum_{s\,\text{spin}} \int \frac{d^3p}{(2\pi)^3\, 2E} \left\{ e^{-ipx} a_{ps} u_{ps} + e^{ipx} a^{\dagger}_{ps} v_{ps} \right\}$$

For the production rate of fermions we had

$$2E(2\pi)^3 \frac{d\Gamma}{d^3p} = -\sum_s \int d^4x\, e^{ipx}\, v\, v^* \Delta^{<}_{\bar J \bar J^{\dagger}}(x)$$

where we included a sum over spin of the N.
We can take care of the spinor indices of v and $\bar J$ by writing

$$v\, v^* \Delta^{<}_{\bar J \bar J^{+}} \equiv v_a v_b^* \Delta^{<}_{\bar J_a \bar J_b^{+}} = \Delta^{<}_{\bar J v (\bar J v)^{+}}$$

$$\doteq -\left\langle \left(\bar I(0) + \overline{I^c}(0)\right) v + \left[\left(\bar I(x) + \overline{I^c}(x)\right) v\right]^{+} \right\rangle$$

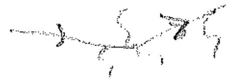
$$\overset{(i)}{=} -\left\langle \bar I(0)\, v\, \bar v\, I(x) + \overline{I^c}(0)\, v\, \bar v\, I^c(x) \right\rangle$$

The other two terms vanish because

$$\langle I\ \overline{I^c} \rangle = \left\langle \tilde\varphi^{\dagger} y_\alpha l_\alpha\, \overline{\left[\tilde\varphi^{\dagger} y_\beta l_\beta\right]^c} \right\rangle$$

$$\sim \langle l\, l \dots \rangle = 0$$

fermion arrow lines don't end



[lepton number is conserved in SM]

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$$\bar{v}\, I^c \overset{(ii)}{=} - \bar{I}\, v^c$$

because here v is a c-number spinor

u : particle spinor

v : antiparticle spinor

$$v^c = u$$

$$\Delta^<_{\bar{J} v, (\bar{J} v)^\dagger}(x) = - \langle \bar{I}(0)\, v \bar{v}\, I(x) + \bar{u}\, I(0)\, \bar{I}(x)\, u \rangle$$

Now sum over spins:

$$\sum_s v_s \bar{v}_s = \not{p} - M \,, \quad \sum_s u_s \bar{u}_s = \not{p} + M$$

$$\Delta^<_{\bar{J} v, (\bar{J} v)^\dagger}(x) = \mathrm{tr}\Big\{ (\not{p} - M)\, \Delta^<_{I \bar{I}}(x) - (\not{p} + M)\, \Delta^>_{I \bar{I}}(-x) \Big\}$$

$$P_L I = I \,, \quad \bar{I} P_R = I^\dagger \gamma^0 P_L = I^\dagger P_L \gamma^0 = (P_L I)^\dagger \gamma^0 \overset{!}{=} \bar{I} \Rightarrow$$

$$\Delta_{I \bar{I}} = P_L\, \Delta_{I \bar{I}}\, P_R$$

Therefore

$$\mathrm{tr}\left(M\, \Delta^<_{I \bar{I}} \right) = M\, \mathrm{tr}\left(P_L\, \Delta^<_{I \bar{I}}\, P_R \right)$$

$$= M\, \mathrm{tr}\Big(\underbrace{P_R P_L}_{=0}\, \Delta^<_{I \bar{I}} \Big) = 0$$

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Thus the production rate per unit volume Γ can be written in terms of SM correlators as

$$\boxed{2E\,(2\pi)^3\,\frac{d\Gamma}{d^3p} = -\mathrm{tr}\left\{ \not{p}\left[\Delta^{<}_{I\bar{I}}(p) - \Delta^{>}_{I\bar{I}}(-p) \right] \right\}}$$