7.3 Phase transitions

We have already seen that at very high T, $T \gg m_W$ electroweak symmetry is unbroken. We are now in the position to treat this in more detail.
But first start with a simple scalar model

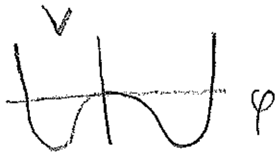
Spontaneous Symmetry breaking (SSB)

Let φ be a real scalar field with

$$\mathcal{L} = \frac{1}{2}(\partial\varphi)^2 - V(\varphi),$$

$$V(\varphi) = -\frac{\mu^2}{2}\varphi^2 + \frac{\lambda}{4}\varphi^4 \qquad \text{symmetric under } \varphi \to -\varphi$$

where $\mu > 0$



There are two possible ground states (vacua), corresponding to the two minima of V

$$V' = (-\mu^2 + \lambda\varphi^2)\varphi = 0 \iff \varphi = v$$

with

$$v = \pm \frac{\mu}{\sqrt{\lambda}}$$

Thus the ground state breaks the symmetry of S which is called SSB.

NB: in QM, which is $(0+1)$ dimensional QFT, the ground state $|0\rangle$ wave function is symmetric and $\langle 0|\varphi|0\rangle = 0$. This because the particle can tunnel from v to $-v$. However, in $(3+1)$ dimensions the tunneling rate vanishes, if the minima are exactly degenerate.

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To study the behavior at high T, we can use dimensional reduction,

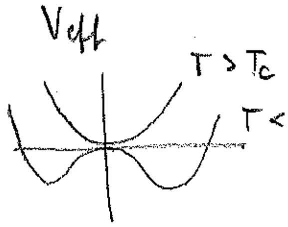
$$Z = \int \mathcal{D}\varphi_4 e^{iS} = \int \mathcal{D}\varphi_3 \, e^{S_{eff}}$$

$$S_{eff} = \beta \int d^3x \left\{ -\frac{1}{2} (\nabla\varphi)^2 - V_{eff}(\varphi) + \cdots \right\}$$

with $V_{eff} = \frac{1}{2} \underbrace{\left(-\mu^2 + \frac{\lambda}{4} T^2\right)}_{=: \, m^2_{eff}} \varphi^2 + \frac{\lambda}{4} \varphi^4$

where the second term comes from 1-loop matching.

For $T = T_c := \frac{2\mu}{\sqrt{\lambda}}$, m^2_{eff} changes sign.



This indicates that the expectation value $\langle\varphi\rangle$ changes continuously from $\langle\varphi\rangle \neq 0$ for $T < T_c$ to $\langle\varphi\rangle = 0$ for $T > T_c$, meaning that there is a second order phase transition.

However, perturbation theory does not work near $T = T_c$ because the loop expansion parameter

$$\frac{\lambda T}{m_{eff}}$$

diverges for $T \to T_c$. Using other methods (renormalization group, lattice) one can show that the phase transition is indeed second order.

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Electroweak symmetry breaking

SM gauge group: $\underset{\uparrow \atop \text{QCD}}{SU(3)} \times \underbrace{SU(2) \times U_Y(1)}_{\text{electroweak}}$

$SU(2)$: $d_A = 2^2 - 1 = 3$ gauge fields W_μ^a, coupling g

$U(1)$: hypercharge 1 gauge field B_μ, coupling g'

Higgs field H: $SU(2)$ doublet, hypercharge $= \frac{1}{2}$

Higgs potential

$$V = -\mu^2 H^+ H + \lambda (H^+ H)^2 \qquad (\mu > 0)$$

At $T = 0$: $\langle H \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$ $\qquad v = \frac{\mu}{\sqrt{\lambda}}$

(in an appropriate gauge)

$$\mathcal{L}_{Higgs} = (D_\mu H)^+ (D^\mu H) - V$$

$$D_\mu = \partial_\mu - i g W_\mu^a \frac{\sigma^a}{2} - i g' \frac{1}{2} B_\mu$$

gauge field masses (for simplicity put $g' = 0$)

$$\mathcal{L}_{W\text{-mass}} = g^2 (W_\mu \langle H \rangle)^+ W^\mu \langle H \rangle = g^2 W_\mu^a W_\mu^b \underbrace{\langle H \rangle^+ \frac{\sigma^a}{2} \frac{\sigma^b}{2} \langle H \rangle}$$

$$= \frac{1}{4} \langle H^+ \rangle \underbrace{\frac{1}{2}\{\sigma^a, \sigma^b\}}_{= \delta^{ab}} \langle H \rangle$$

$$= \frac{g^2 v^2}{8} W_\mu^a W^{\mu a} \qquad \Longrightarrow \qquad m_W^2 = \frac{g^2 v^2}{4}$$

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In addition, H has Yukawa interactions with all fermions
For our discussion, the most important one is with the top:

$$\mathcal{L}_{\text{top-Yukawa}} = -h_t \bar{t}_R \tilde{H}^+ Q_3 + \text{H.c.}$$

top-Yukawa coupling, Hermitean conjugate

t_R right handed top quark $(1+\gamma^5)t_R = t_R$, $SU(2)$ singlet

$$Q_3 = \begin{pmatrix} t_L \\ b_L \end{pmatrix}$$ 3rd family, $SU(2)$ doublet

bottom quark, left-handed

$$\tilde{H} = \varepsilon H^*$$

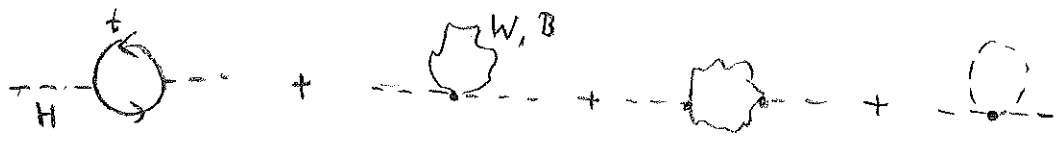
Levi-Civita

$$\langle \tilde{H} \rangle = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix} \rightarrow \text{top quark mass term}$$

$$\mathcal{L}_{\text{top-mass}} = \frac{v}{\sqrt{2}} h_t (\bar{t}_R t_L + \bar{t}_L t_R) \Longrightarrow$$

$$m_{\text{top}} = \frac{v h_t}{\sqrt{2}}$$

dimensional reduction $\rightarrow$ thermal mass for H



$$m_{\text{eff}}^2 = -\mu^2 + \left(3h_t^2 + \frac{9}{4}g^2 + \frac{3}{4}g'^2 + 6\lambda\right)\frac{T^2}{12}$$

again $m_{\text{eff}}^2 = 0$ could hint to a PT

For given $h := \sqrt{2H^\dagger H}$, $\vec{W}$ has the mass $\frac{gh}{2}$

If $\frac{gh}{2} \gg m_{\text{eff}}$, one can integrate out $\vec{W}$.

Massive gauge bosons have 3 possible polarizations.
One has a Debye mass in addition to $gh/2$ and will not contribute below.
The pressure of an ideal gas of $3 \cdot 2 \cdot d_A = 6$ massive bosons is

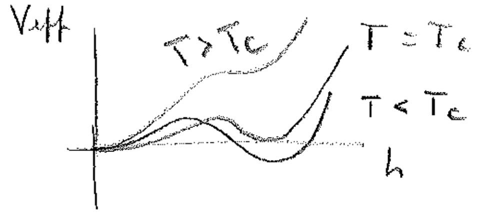
$$P = 6 \left\{ \frac{\pi^2}{90} T^4 - \frac{m^2 T^2}{24} + \frac{m^3 T}{12\pi} + \mathcal{O}(m^4) \right\}$$

This is already included in meff

this comes from the 3-d theory.

$\Rightarrow$ integrating out W in the 3-d theory gives
$[\exp(\beta P V) = \exp(-\beta V_{\text{eff}} V)]$

$$V_{\text{eff}} = \frac{1}{2} m_{\text{eff}}^2 h^2 - \frac{g^3 T}{16\pi} h^3 + \frac{\lambda}{4} h^4$$



first order phase transition

problem: perturbation theory does not work well for $m_H = 125$ GeV

lattice simulations $\Rightarrow$ no phase transition