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## 7.2 Effective 3-dimensional theories

Effective theories are a powerful tool to deal with multiple-scale problems.

In weakly coupled ($g \ll 1$) theories at finite $T$ we encountered several scales:

1) $T$ "hard"

2) $m_{th} \sim gT$ in gauge theory, $m_{th} \sim \sqrt{\lambda}\, T$ in
"soft"

In sect. 7.1 we found a third one:

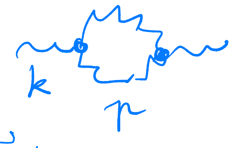
3) $g^2 T$, sometimes called "ultrasoft".

which appears in the 3-dimensional Yang-Mills theory containing only the spatial gauge fields with $k^0 = 0$.

It leads to screening of magnetostatic interactions. Therefore $g^2 T$ is sometimes called "magnetic mass".

N.B. in the imaginary time formalism a non-zero frequency is always hard.

We have seen that the hard field modes affect the softer ones, e.g. through thermal mass generation.

In sub-diagrams like 

$k$ soft, $p$ hard

one can expand in $\vec{k}/p$

The leading term is $O(|\vec{k}|^0)$ and gives the mass term which can be described by the term $\frac{1}{2} m_D^2 (A_0^a)^2$ in the Lagrangian.

The next term would be of order $\vec{k}^2$ giving a contribution to $\Pi^{\mu\nu}$ proportional to $\vec{k}^2$.

$\vec{k}^2$ also appear in the free inverse propagator, which is due to the term $\frac{1}{2}(\nabla A_0^a)^2$ in the Lagrangian. Therefore the $O(\vec{k}^2)$ piece of $\Pi^{\mu\nu}$ can be described by modifying the coefficient of $(\nabla A_0^a)^2$ in $\mathcal{L}$.

Such effects can be captured by so called <u>effective field theories</u> (EFTs). In our case the EFT is a 3-d QFT.

For a <u>scalar field with</u> (

$$\mathcal{L} = \tfrac{1}{2}(\partial\varphi)^2 - \frac{m^2}{2}\varphi^2 - \frac{\lambda}{4}\varphi^4$$

with renormalized mass $\ll T$ one can write

$$Z = \int \mathcal{D}\varphi_4 \, e^{iS[\varphi_4]} = \int \mathcal{D}\varphi_3 \, e^{S_3[\varphi_3]}$$

$\underset{4\text{-}d}{\varphi_4 = \varphi_4(t,\vec{x})}$, $\quad \underset{3\text{-}d}{\varphi_3 = \varphi_3(\vec{x})}$ fields

$S_3$ is an effective action which is the result of 'integrating out' high momentum ($k^0 \neq 0, |\vec{k}| \gg m_3$) modes, where $m_3$ is a characteristic 'soft' scale like the mass.

$$S_3 = \beta \int d^3x \, \mathcal{L}_3$$

$\mathcal{L}_3$ is a polynomial in $\varphi$ and spatial derivatives of $\varphi$. In principle, all terms which are consistent with the symmetry can appear.

$$\mathcal{L}_3 = f + \frac{1}{2}(\nabla\varphi)^2 - \frac{m_3^2}{2}\varphi^2 - \frac{\lambda_3}{4}\varphi^4 - \frac{g_3}{6}\varphi^6 + \dots$$

$[g_3] = -2$. Since it comes from expanding in $k/p$ we have $\quad g_3 \sim \frac{1}{T^2}$

The coefficients in $\mathcal{L}_3$ can be determined by <u>matching</u>.
One computes correlation functions from $S_4$ and $S_3$ and demand that they match at momenta $\ll T$.
$f$ is obtained by matching the pressure $P$.
Does it mean that we have to compute everything (which would make this procedure pointless)?
No: To determine $f$ it is good enough to compute $P$ with an IR cutoff $\Lambda$ with $T \gg \Lambda \gg m_3$ because for $|\vec{k}| \ll \Lambda$ the 3-d and 4-d theories give the same

In practice, one can use dim. regularization to cut off momenta.
At LO in $\lambda$: $P = P_0$ ($P_0$ = ideal gas pressure)
In the eff. theory we can neglect $m_3$ for the matching, because we only include $|\vec{k}| \gg m_3$. [In particular, we don't need to resum anything] Massless integrals vanish in dim. reg.
Thus the 3-d theory gives $(Z = e^{\beta P V})$ $P = -f \Rightarrow$

$$f = -P_0 + \mathcal{O}(\lambda)$$

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$m_3^2$ is determined by matching 2-point functions;

4-d:  $= -\frac{\lambda}{4} T^2$

~~in the 3-d theory we treat $m_3^2 \varphi^2$ as a perturbation,~~

in the 3-d theory we may treat $m_3^2 \varphi_3^2$ as perturbation:

 $= - m_3^2$

matching the two gives $m_3^2 = \frac{1}{4} \lambda T^2$

$\lambda_3$ is obtained by matching 4-point functions, and so on.

## QCD

In the 3-d theory we only have $A_0^a$, $\vec{A}^a$ and no fermions

Symmetries: 3-d rotations scalar vector

3-d gauge transformation $U(\vec{x})$:

$\vec{A} \rightarrow U \vec{A} U^+ + \frac{i}{g} U \nabla U^+$ like a gauge field

$A_0 \rightarrow U A_0 U^+$ like adjoint scalar

NB: t-dependent U would mix $\omega_n = 0$ with $\omega_n \neq 0$ modes

Invariants for $\mathcal{L}$: adjoint

$F^{amn} F^a_{mn}$, $tr([\vec{D} A_0]^2)$ $tr(A_0^2)$, $\lambda \, tr(A_0^4)$, $\lambda' [tr A_0^2]^2$

These all have mass dimension $\leq 4$

$$F^a_{\mu\nu} F^{a\mu\nu} = F^a_{mn} F^a_{mn} - 2 F^a_{m0} F^a_{m0}$$

$$F_{m0} = \frac{i}{g} [D_m, D_0] = [D_m, A_0] \Rightarrow F^a_{m0} = (D_m A_0)^a,$$

$\mathrm{tr}(\vec{D} A_0)^2$ appears at tree level

$$\mathcal{L}_3 = \{ -\frac{1}{4} F^a_{mn} F^a_{mn} - \frac{1}{2}(\vec{D} A^a_0) \cdot (\vec{D} A^a_0) + \frac{m_3^2}{2} A^a_0 A^a_0 + \dots$$

$$g_3^2 = g_4^2 + \mathcal{O}(g_4^4)$$

$$m_3^2 = g^2 T^2 \left( \frac{N}{3} + \frac{N_f}{6} \right) + \mathcal{O}(g^4 T^2)$$

from 1-loop matching

This EFT is sometimes called EQCD

electrostatic

One can go one step further:
$m_3 \sim gT \gg g^2 T$ which is the characteristic scale of $\vec{A}$. One can integrate out $A_0$, which give the so called MQCD.

magnetostatic

This can also be done in perturbation theory with the expansion parameter $\frac{g^2 T}{m_3}$.

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at leading order this gives

$$\mathcal{L}_M = f_M - \frac{1}{4} F^a_{mn} F^a_{mn}$$

This is pure Yang-Mills theory in 3 dimensions.
It describes static magnetic fields.
We have already seen that MQCD

(i) is non-perturbative

(ii) has only 1 parameter, $g^2 T$

(iii) contributes $O(g^6 T^4)$ to the pressure, provided there is a mass gap.

It is widely believed that there is a mass gap, and lattice simulations show that this is the case.
This is called <u>magnetic screening</u>.
The $O(g^6 T^4)$ contribution from MQCD can be computed on the lattice, and there is a well defined perturbative expansion of $P$ with $g$ (modulo $\log(1/g)$) as expansion parameter.

Here one sees a big difference between QED and QCD.
MQED is a free theory at leading order, and there is no magnetic screening.