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6.5 Debye mass

For the daisy diagrams we need $\Pi^{ab}_{\mu\nu}(k^0)$ at $k^0=0$ and $|\vec{k}| \ll T$. In fact, we may take the limit $\vec{k}\to 0$

In this limit

$$\Pi_3^{ab\mu\nu} = -\frac{g^2}{2} C_A \delta^{ab} \SumInt_q \frac{1}{q^4}\left\{(4d-2)q^\mu q^\nu + 2\eta^{\mu\nu} q^2\right\}$$

rotational invariance $\Rightarrow$

$$\SumInt_q \frac{q^\mu q^\nu}{q^4} = a\,\eta^{\mu\nu} + b\, u^\mu u^\nu \qquad \left(u = \begin{pmatrix}1\\ \vec{0}\end{pmatrix}\right)$$

contract with $\eta_{\mu\nu}$ $\Rightarrow$ $\quad (d+1)a + b = \SumInt_q \frac{1}{q^2} \qquad (*)$

00-component: $\quad a + b = \SumInt_q \frac{q_0^2}{q^4} \qquad (**)$

We had

$$\SumInt_q \frac{1}{-q^2} = \frac{T^2}{12}$$

Apply $T^2 \frac{d^2}{dT^2} = \frac{T}{2}\frac{d}{dT}$ on both sides $\Rightarrow$

$$\frac{T}{2}\frac{d}{dT}\left(T\sum_n \int \frac{d^dq}{(2\pi)^d} \frac{1}{(2\pi n T)^2 + \vec{q}^{\,2}}\right) = T\sum_n \int \frac{d^dq}{(2\pi)^d}\left\{\frac{1}{2}\frac{1}{-q^2} - \frac{-q_0^2}{q^4}\right\}$$

$$\Rightarrow \quad \frac{T^2}{24} + \SumInt_q \frac{q_0^2}{q^4} = \frac{T^2}{12} \quad \Rightarrow \quad \boxed{\SumInt_q \frac{q_0^2}{q^4} = \frac{T^2}{24}}$$

$$(*) - (**) \Rightarrow \quad d\cdot a = \SumInt_q \left(\frac{1}{q^2} - \frac{q_0^2}{q^4}\right) = \frac{T^2}{12}\left(-1 - \frac{1}{2}\right) = -\frac{T^2}{8}$$

$$a = -\frac{T^2}{24}, \quad b = -a + \SumInt_q \frac{q_0^2}{q^4} = T^2\left(\frac{1}{24} + \frac{1}{24}\right) = \frac{T^2}{12}$$

$$\SumInt_q \frac{q^\mu q^\nu}{q^4} = \frac{T^2}{24}\left(-\eta^{\mu\nu} + 2u^\mu u^\nu\right)$$

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$$\Pi_3^{ab\mu\nu} = -\frac{1}{2} g^2 T^2 C_A \delta^{ab} \underbrace{\left\{ 10 \, \frac{1}{24}\left(-\eta^{\mu\nu} + 2u^\mu u^\nu\right) - \frac{1}{12} 2\eta^{\mu\nu} \right\}}_{= \frac{1}{12}\left\{(-5-2)\eta^{\mu\nu} + 10 u^\mu u^\nu\right\}}$$

$$= -\frac{g^2}{24} T^2 C_A \delta^{ab} \left(-7\eta^{\mu\nu} + 10 u^\mu u^\nu\right)$$

$$\Pi_4^{ab\mu\nu} = -g^2 C_A \delta^{ab} \eta^{\mu\nu} d \underbrace{\Delta(0)}_{= \sum\!\!\!\!\!\!\int_q \frac{1}{-q^2} = \frac{T^2}{12}} = -\frac{g^2}{24} T^2 C_A \delta^{ab} \, 6\eta^{\mu\nu}$$

$$\Pi_{gh}^{ab\mu\nu} = g^2 C_A \delta^{ab} \sum\!\!\!\!\!\!\int_q \frac{q^\mu q^\nu}{q^4} = -\frac{g^2}{24} T^2 C_A \delta^{ab} \left(\eta^{\mu\nu} - 2u^\mu u^\nu\right)$$

$$\Pi_{fermion}^{ab\mu\nu} = -\frac{g^2}{3} T^2 \underbrace{tr(T^a T^b)}_{= \frac{1}{2}\delta^{ab}} N_f \, u^\mu u^\nu = -\frac{g^2}{24} T^2 \, 4N_f \, u^\mu u^\nu$$

$\Rightarrow$

$$\lim_{\vec{k} \to 0} \Pi^{ab\mu\nu}(0, \vec{k}) = -\frac{g^2}{24} T^2 \delta^{ab} \left\{ C_A \left[(-7+6+1)\eta^{\mu\nu} + (10-2) u^\mu u^\nu \right] + N_f \, 4 u^\mu u^\nu \right\}$$

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$$\lim_{\vec{k}\to 0} \Pi^{ab}_{\mu\nu}(0,\vec{k}) = -g^2 T^2 \delta^{ab} \left(\frac{C_A}{3} + \frac{N_f}{6} \right) u_\mu u_\nu$$

$$\boxed{\lim_{\vec{k}\to 0} \Pi^{ab}_{00}(0,\vec{k}) = -\delta^{ab} m_D^2} \qquad (*)$$

$$\boxed{m_D^2 = g^2 T^2 \left(\frac{C_A}{3} + \frac{N_f}{6} \right)} \qquad \text{Debye mass}$$

$$\lim_{\vec{k}\to 0} \Pi^{ab}_{m\nu}(0,\vec{k}) = 0$$

Like in QED, only A_0 receives a thermal mass m_D.

The loop momenta q contributing to $(*)$ are of order T, while in the daisy diagrams $|\vec{k}|$ is of order $m_D \sim gT$.

Therefore $(*)$ is called a <u>hard thermal loop</u> (HTL), while $\vec{k}$ is called soft.

7 Dimensional reduction

We have seen that the IR-divergent and IR-sensitive contributions are due to momenta with $p^0 = 0$.
These come from the $\omega_n = 0$ Fourier modes of bosonic fields. These fields live in d dimensions, where $d \to 3$ in the end.
Thus the IR-sensitive contributions are determined by a 3-dimensional QFT.
Constructing this theory will allow us to make the IR resummation more systematic and transparent.
But we start with a simple and interesting argument:

7.1 The Linde problem

Consider the $\omega_n = 0$ sector of some thermal QFT with coupling constant g

Each additional loop comes with a factor g^2 and with a factor T from $T\sum_n \int d^dk \xrightarrow[\text{only } n=0]{} T\int d^dk$

[For scalars with interaction $\lambda\varphi^4$, g^2 corresponds to λ.]

Thus it looks like the loop expansion parameter is $g^2 T$. But $g^2 T$ is not dimensionless.

6.5T

Suppose the 3d QFT has a mass parameter m.

Then the expansion parameter is $\frac{g^2 T}{m}$.

For A_0^a in QCD this would $m_D \sim gT$, $\frac{g^2 T}{m_D} \sim g$

[for the scalar field: $m \sim \sqrt{\lambda} T$, $\frac{\lambda T}{m} \sim \sqrt{\lambda}$]

For A_0, and the scalar field the thermal mass is generated by "hard momenta $|\vec{p}| \sim T$, through the hard thermal loops (HTL).

But what if there is no HTL, which is the case for $\vec{A}$?

Then a mass or an IR cutoff could be generated by the 3-d theory itself.

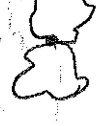
There the only parameter is $g^2 T$. Thus we must have

$$m \sim g^2 T$$

and the expansion parameter would be $\frac{g^2 T}{m} \sim 1$

Such contributions would not be perturbatively calculable.

How big would be their contribution to the pressure?

$P_{non-pert.} \sim$  $\sim g^2 T^2 m^2 \sim g^2 T^2 (g^2 T)^2 \Rightarrow$

to get the dimensions right

2 loops

$$\boxed{P_{non-perturbative} \sim g^6 T^4}$$