11/23

6.4 Gluon polarization tensor: ghosts & fermions

Action for Faddeev-Popov ghosts:

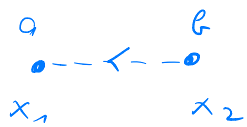
$$S_{ghost} = \int d^{d+1}x \; \bar{c}^a \partial^\mu D_\mu^{ab} c^b$$

$$D_\mu^{ab} = \delta^{ab}\partial_\mu - ig \underbrace{(T_A^c)^{ab}}_{= -if^{cab}} A_\mu^c$$ covariant derivative in adj. rep.

To simplify the calculation we integrate by parts so that the derivative does not act on A_μ^c:

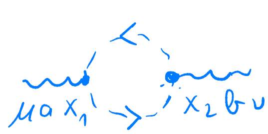
$$S_{ghost} = -\int d^{d+1}x \, (\partial^\mu \bar{c}^a)(\delta^{ab}\partial_\mu - g f^{cab} A_\mu^c) c^b$$

ghost propagator:



$$= \langle c^a(x_1)\bar{c}^b(x_2)\rangle \equiv \Delta_{gh}^{ab}(x_1 - x_2)$$

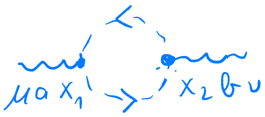
$$\Delta_{gh}^{ab}(q) = \delta^{ab} \frac{1}{-q^2}$$ (like massless scalar)



$$= g^2 \langle \partial_\mu \bar{c}^c f^{acd} c^d(x_1) \, \partial_\nu \bar{c}^e f^{bef} c^f(x_2) \rangle_0$$

Grassmann

$$= -g^2 \langle c^f(x_2) \partial_\mu \bar{c}^c(x_1) \rangle_0 \, f^{acd} \langle c^d(x_1) \partial_\nu \bar{c}^e(x_2) \rangle_0 \, f^{bef}$$



$$= -g^2 \sum\!\!\!\!\!\!\!\int_q e^{-iq(x_2-x_1)} \frac{iq_\mu}{-q^2} \sum\!\!\!\!\!\!\!\int_r e^{-ir(x_1-x_2)} \frac{ir_\nu}{-r^2}$$

$$\underbrace{f^{acd} f^{bdc}}_{= -f^{acd} f^{bcd} = -\delta^{ab} C_A}$$

$$= -\delta^{ab} C_A g^2 \sum\!\!\!\!\!\!\!\int_{qr} e^{i(q-r)(x_1-x_2)} \frac{q_\mu r_\nu}{q^2 r^2}$$

$$= -\Pi^{ab}_{gh,\mu\nu}(x_1 - x_2)$$

$$k + q - r = 0$$

$$\Pi^{ab}_{gh\,\mu\nu}(k) = \int_0^\beta d\tau \int d^dx\, e^{ikx}\, \Pi^{ab}_{gh\,\mu\nu}(x) =$$

$$= \delta^{ab} C_A\, g^2 \sum\!\!\!\!\!\!\!\int_q \frac{q_\mu (k+q)_\nu}{q^2 (q+k)^2}$$

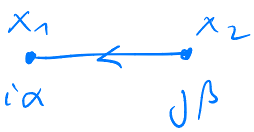
$$\sum\!\!\!\!\!\!\!\int_q \frac{q_\mu}{q^2(q+k)^2} \overset{q \to -q-k}{=} -\sum\!\!\!\!\!\!\!\int_q \frac{q_\mu + k_\mu}{q^2(q+k)^2} \Rightarrow \sum\!\!\!\!\!\!\!\int_q \frac{q_\mu}{q^2(q+k)^2} = -\frac{1}{2} k_\mu \sum\!\!\!\!\!\!\!\int_q \frac{1}{q^2(q+k)^2}$$

$$\Pi^{ab}_{gh\,\mu\nu}(k) = g^2 C_A \delta^{ab} \sum\!\!\!\!\!\!\!\int_q \frac{1}{q^2(q+k)^2} \left(q_\mu q_\nu - \frac{1}{2} k_\mu k_\nu \right)$$

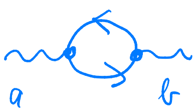
N_f Dirac fermions

$$\mathcal{L}_{fermion} = \sum_{\alpha\beta} \bar{\psi}_{i\alpha}\,\delta_{\alpha\beta}\left[i\left(\delta_{ij}\not{\partial} - i g T^a_{ij} \not{A}^a\right) - m_\alpha\right]\psi_{j\beta}$$

flavor indices group (color) indices



$$= \langle \psi_{i\alpha}(x_1)\,\bar{\psi}_{j\beta}(x_2)\rangle = \delta_{\alpha\beta}\,\delta_{ij}\,S(x)$$



$= \mathrm{tr}(T^a T^b)\ \left[\right.$  $\left.\right]_{QED}$

Fundamental representation of $SU(N)$: $\mathrm{tr}(T^a T^b) = \frac{1}{2}$