Problem C13.1 The Higgs doublet φ and the lepton doublets ℓ_α transform under SU(2) gauge transformations as

$$\varphi \to V\varphi, \qquad \ell_\alpha \to V\ell_\alpha$$

with $V \in$ SU(2), so that $\varphi^\dagger \ell_\alpha$ is invariant under SU(2) gauge transformations. Show that

$$\widetilde{\varphi} := \varepsilon\varphi^*$$

with

$$\varepsilon := \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

transforms like φ,

$$\widetilde{\varphi} \to V\widetilde{\varphi}$$

(meaning that the complex conjugate $\bar{2}$ of the fundamental representation is equivalent to the fundamental representation 2). This implies that $\widetilde{\varphi}^\dagger \ell_\alpha$ is invariant under SU(2) gauge transformations as well.

Problem H13.1 Compute explicitly the photon polarization tensor $\Pi^{\mu\nu}(p)$ at 1-loop order in the HTL approximation, i.e., assuming $|p^\mu| \ll T$, to check the result that was quoted in the lecture.

(a) Start with the expression on page 3 of Sec. 5.3. Do the Matsubara sum, e.g., by using mathematica. It is useful to partial fraction the result into terms with single poles.

(b) First compute the spatial components Π^{mn}. Expand in $p/(\text{loop momentum})$, and keep only the leading order. Since you know from Sec. 5.3 that $\Pi^{mn}(p)$ in the HTL approximation vanishes at $p^0 = 0$, you can drop terms which do not depend p.

(c) Use $p_\mu \Pi^{\mu\nu}(p) = 0$ first to compute Π^{m0} and then Π^{00}.