Problem C10.1 In the lecture the adjoint representation of a Lie group was defined through its generators. A finite transformation acting on A^a can be written as

$$A \to UAU^\dagger$$

where $U = \exp(-i\theta^a T^a)$, and $A = A^a T^a$ with generators T^a in the fundamental representation. To check this, consider infinitesimal θ, meaning that you expand to first order in the θ^a, and compare the above transformation with the transformation $\exp(-i\theta^a T_A^a)^{bc} A^c$.

Problem H10.1

(a) For

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ h \end{pmatrix}$$

the Higgs potential reads

$$V = -\frac{\mu^2}{2} h^2 + \frac{\lambda}{4} h^4$$

Express μ and λ through the vacuum expectation value $\langle h \rangle \equiv v$ and the Higgs mass m_H.

(b) Compute explicitly the contribution to the thermal Higgs mass originating from the self-coupling λ. Hint: Express the Higgs doublet H in the interaction term $\lambda (H^\dagger H)^2$ via four real scalar fields as

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi_1 + i\varphi_2 \\ \varphi_3 + i\varphi_4 \end{pmatrix}$$

and compute the self-energy for φ_i.

(c) Consider the effective Higgs potential in the dimensionally reduced 3D theory from the lecture:

$$V_{\text{eff}} = \frac{m_{\text{eff}}^2}{2} h^2 - \frac{g^3 T}{16\pi} h^3 + \frac{\lambda}{4} h^4$$

Choose the temperature such that $m_{\text{eff}} = 0$. What value do you find for the temperature? Use $g = 2/3$ and $g' = 1/3$ for the gauge couplings, and $m_{\text{top}} = 175$ GeV, $m_H = 125$ GeV, $v = 246$ GeV. Determine the mass squared m^2 as the second derivative of this potential at its minimum. Requiring that the loop expansion parameter $\lambda T/m$ is smaller than 1, what is the upper limit on the value of the Higgs mass?