Problem C7.1 Use the properies of the projection tensors listed in the lecture notes to show that the full gauge field propagator can be written as

$$G^{\mu\nu}(k) = P_t^{\mu\nu}(k)G_t(k) + P_l^{\mu\nu}(k)G_l(k) + \xi\frac{k^\mu k^\nu}{k^4}$$

where

$$G_i(k) = \frac{1}{-k^2 + \Pi_i(k)}$$

for $i = t, l$.

Problem C7.2 Use the Jacobi identity (see lecture) to check that the adjoint-representation generators T_A with $(T_A^a)^{bc} = -if^{abc}$ satisfy the Lie-algebra commutation relations

$$[T_A^a, T_A^b] = if^{abc}T_A^c$$

Problem H7.1 In the lecture the order e^2 contribution to the pressure in QED was computed in Feynman gauge $\xi = 1$. Repeat the calculation in a general covariant gauge to check that the result is ξ-independent.

Problem H7.2 Convince yourself that the determinant of an $n \times n$ matrix M can be written as an integral over $2n$ Graßmann variables $\overline{c}_1, \ldots, \overline{c}_n, c_1, \ldots c_n$ as

$$\mathrm{det}M = \int d\overline{c}\, dc\, e^{-\overline{c}Mc}.$$

If you are content with this, you may restrict yourself with the cases $n = 1$ and $n = 2$.