Thermal field theory	**Sheet 6**	November 15, 2023

Problem C6.1 Check that the gauge field propagator $\Delta^{\mu\nu}(k)$ given in the lecture is indeed the inverse of

$$\eta_{\mu\nu}k^2 - \frac{\xi - 1}{\xi}k_\mu k_\nu$$

Problem C6.2 Compute the trace

$$\mathrm{tr}\left(\gamma^\mu S(k)\right)$$

where

$$S(k) = \frac{-1}{\not{k} - m}$$

is the Dirac-field propagator in momentum space.

Problem H6.1

(a) The Dirac Lagrangian

$$\mathcal{L} = \overline{\psi}(\not{\partial} - m)\psi$$

is invariant under the phase transformations

$$\psi \to e^{-i\theta}\psi, \qquad \overline{\psi} \to e^{i\theta}\,\overline{\psi}$$

According to Noether's theorem such a continous symmetry implies a conservation law. Show that in this case the conserved current can be written as

$$J^\mu = -e\overline{\psi}\gamma^\mu\psi$$

(b) Now introduce a chemical potential for the charge

$$Q = \int d^3x J^0$$

by writing the partition function as

$$Z = \mathrm{tr}\left(e^{\beta(\mu Q - H)}\right)$$

Derive the fermion propagator for this case.

Hint: You can think of the chemical potential as a shift of the time derivative appearing in the Dirac Lagrangian. This should already tell you how the chemical potential changes the fermion propagator in momentum space.

(c) Use the fermion propagator to compute the expectation value of the charge density

$$\langle -e\overline{\psi}\gamma^0\psi\rangle$$

You don't need to perform the momentum integrals.