Problem C3.1 In quantum theory the ground state of a harmonic oscillator has non-zero energy. In the lecture, the partition function was computed without taking into account the ground state energy. Now re-compute the partition function for massless scalar particles including the ground state energy. What is the additional contribution to the pressure? When is it okay to drop it?

Problem H3.1 Recall the two-point correlator from exercise 2.1. Carry out the summation over the bosonic Matsubara frequencies by using contour integration and show that

$$\langle\varphi(-i\tau,\mathbf{x})\varphi(0)\rangle = \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}} \frac{\cosh\left[\left(\frac{\beta}{2}-\tau\right)E\right]}{2E\sinh\left[\frac{\beta E}{2}\right]}\bigg|_{E=\sqrt{\mathbf{k}^2+m^2}}.$$

Hint: To make sure that you do not get a contribution if you move the integration contour to $\omega = -\infty$ you need to slightly modify the formula that was obtained in lecture. Remember that instead of $[1/2 + f_{\mathrm{B}}(\omega)]$ we could have written$[c + f_{\mathrm{B}}(\omega)]$ with arbitrary constant c, because both expressions have the same poles and the same residues.

Problem H3.2 In the lecture we computed

$$\frac{d^2 P_0}{d(m^2)^2} \qquad (*)$$

using perturbation theory with

$$\mathcal{L}_{\mathrm{int}} = -\frac{1}{2}\delta m^2 \varphi^2$$

at first oder in the 'interaction' and then differentiating with respect to m^2. Check our result by going to second order in the 'interaction', but now without any differentiation. This way you should also obtain $(*)$. [You don't need to do the integrals again.]