

We will encounter the spin sums

$$\sum_s u_s(\vec{p}) \bar{u}_s(\vec{p}) = \mathbb{D} \sum_s u_s(\vec{0}) \bar{u}_s(\vec{0}) \mathbb{D}^{-1}$$

$$\begin{aligned} \sum_s u_s(\vec{0}) \bar{u}_s(\vec{0}) &= m \sum_s \begin{pmatrix} \xi_s \\ \xi_{s+1} \end{pmatrix} (\xi_s^\dagger, \xi_{s+1}^\dagger) \delta^0 \\ &= m \sum_s \begin{pmatrix} \xi_s \xi_s^\dagger & \xi_s \xi_{s+1}^\dagger \\ \xi_{s+1} \xi_s^\dagger & \xi_{s+1} \xi_{s+1}^\dagger \end{pmatrix} = m \begin{pmatrix} \mathbb{1} & \mathbb{0} \\ \mathbb{0} & \mathbb{1} \end{pmatrix} = m(\gamma^0 + 1) \\ &= (k + m) \end{aligned}$$

We had $\mathbb{D}^{-1} \gamma^\mu \mathbb{D} = \Lambda^\mu_\nu \gamma^\nu \Rightarrow \mathbb{D}^{-1} \gamma_\mu \mathbb{D} = (\Lambda^{-1})^\nu_\mu \gamma_\nu$

$$\begin{aligned} \mathbb{D} k \mathbb{D}^{-1} &= k^\mu \mathbb{D} \gamma_\mu \mathbb{D}^{-1} = k^\mu \Lambda^\nu_\mu \gamma_\nu \\ &= p^\nu \gamma_\nu = \not{p} \Rightarrow \end{aligned}$$

$$\boxed{\sum_s u_s(\vec{p}) \bar{u}_s(\vec{p}) = \not{p} + m}$$

similarly for v_s

$$\begin{aligned} \sum_s v_s(\vec{0}) \bar{v}_s(\vec{0}) &= m \sum_s \begin{pmatrix} \eta_s \\ -\eta_{s+1} \end{pmatrix} (\eta_s^\dagger, -\eta_{s+1}^\dagger) \delta^0 \\ &= m \sum_s \begin{pmatrix} \eta_s \\ -\eta_{s+1} \end{pmatrix} (-\eta_s^\dagger, \eta_{s+1}^\dagger) = m \sum_s \begin{pmatrix} -\eta_s \eta_s^\dagger & \eta_s \eta_{s+1}^\dagger \\ \eta_{s+1} \eta_s^\dagger & -\eta_{s+1} \eta_{s+1}^\dagger \end{pmatrix} \\ &= m \begin{pmatrix} -\mathbb{1} & \mathbb{0} \\ \mathbb{0} & -\mathbb{1} \end{pmatrix} = m(\gamma^0 - 1) = \not{k} - m \Rightarrow \end{aligned}$$

$$\boxed{\sum_{s=1}^2 v_s(\vec{p}) \bar{v}_s(\vec{p}) = \not{p} - m}$$