4/24

2.5 Finite symmetry transformations

We found for infinitesimal symmetry transformations

$$\delta_\varepsilon \varphi_a = i[Q, \varphi_a] \quad \text{with} \quad \dot{Q} = 0$$

We can also write the transformation as

$$\varphi_a \to \varphi_a + i\varepsilon(Q\varphi_a - \varphi_a Q) = (1 + i\varepsilon Q)\varphi_a(1 - i\varepsilon Q)$$

$$= U^\dagger \varphi_a U$$

with $U \equiv 1 - i\varepsilon Q$

Now consider $\varepsilon = \frac{\alpha}{N}$ with finite α and $N \in \mathbb{N}$.

Perform the transformation N times and take $N \to \infty$

$$\lim_{N\to\infty} \left(1 - i\frac{\alpha}{N}Q\right)^N = e^{-i\alpha Q}$$

Then $U(\alpha) \equiv e^{-i\alpha Q}$ describes a finite symmetry transformation. $\boxed{\varphi_a \to U^\dagger \varphi_a U}$

Example: $Q = H$ Hamiltonian

$U(t) = e^{-iHt}$ time translation

Q Hermitean $\Rightarrow$ U is <u>unitary</u> $U^\dagger U = 1$

This way we obtain a unitary representation of our symmetry transformations on our Hilbert space.