Problem C13.1 Show that for the v-spinors the following completeness relation holds

$$\sum_{s=\pm} v_s(\mathbf{p})\overline{v}_s(\mathbf{p}) = \not{p} - m$$

Problem H13.1 Consider electron-muon scattering $e^-(p_1)\mu^-(p_2) \to e^-(p_3)\mu^-(p_4)$. For $E_1 \ll m_\mu$ this problem is quite similar to Rutherford scattering, the role of the heavy nucleus being played by the muon.

(a) Draw the Feynman diagrams which contribute at leading order.

(b) Write the invariant matrix element $\mathcal{M}$.

(c) Compute the spin-sum and -average of the the matrix element-squared $\langle|\mathcal{M}|^2\rangle$

(d) Evaluate the $p_i \cdot p_j$ and $t \equiv (p_1 - p_3)^2$ in the center of mass system assuming that E_1 is much smaller than the muon mass, and $|\mathbf{p}_1|$ is of the order of the electron mass. You should find

$$\left\langle|\mathcal{M}|^2\right\rangle = \frac{e^4 m_\mu^2}{v^4 E_1^2 \sin^4(\theta/2)}\left[1 - v^2\sin^2(\theta/2)\right]$$

where $v \equiv |\mathbf{p}_1|/E_1$ is the velocity of the incoming electron, and θ is the angle by which it gets reflected.

(e) Compute the differential cross section $d\sigma/d\cos\theta$ in the center of mass system. Compare your result with the classical formula for Rutherford scattering which can be found, e.g., in Wikipedia.