Problem C12.1 Prove the following relations for the traces of Dirac matrix by using the Clifford algebra.

(a) $\mathrm{tr}(\gamma^\mu \gamma^\nu) = 4\eta^{\mu\nu}$

(b) $\mathrm{tr}(\gamma^\alpha \gamma^\beta \gamma^\sigma \gamma^\rho) = 4(\eta^{\alpha\beta}\eta^{\sigma\rho} - \eta^{\alpha\sigma}\eta^{\beta\rho} + \eta^{\alpha\rho}\eta^{\beta\sigma})$

Hint: First use the Clifford algebra to move γ^α to the right. Then use the cyclic property of the trace.

Problem H12.1 Consider the 4-point function

$$\langle 0|T\psi(x_1)\overline{\psi}(x_2)\psi(x_3)\overline{\psi}(x_4)|0\rangle$$

of free Dirac fields. Use the generating functional $Z[\eta, \overline{\eta}]$ to show that it can be written as

$$S_F(x_1 - x_2)S_F(x_3 - x_4) - S_F(x_1 - x_4)S_F(x_3 - x_2)$$

How does Wick's theorem for fermions differ qualitatively from the one for bosons?

Problem H12.2 Consider a Dirac fermion ψ and a a real scalar field φ. The Lagrangian is

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\mathrm{int}}$$

with

$$\mathcal{L}_0 = \overline{\psi}(i\partial\!\!\!/ - m)\psi + \frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}M^2\varphi^2.$$

The two fields interact through the so-called **Yukawa interaction**

$$\mathcal{L}_{\mathrm{int}} = -h\varphi\overline{\psi}\psi.$$

The constant h is called **Yukawa coupling**. As usual, perturbation theory is obtained by expanding $\exp\{i\int d^4x\mathcal{L}_{\mathrm{int}}\}$ in the path integral.
Consider the connected scalar field propagator at order h^2.

(a) Draw the corresponding Feynman diagram. [A fermion propagator is drawn with an arrow-line —◂— with the arrow pointing from $\overline{\psi}$ to ψ.]

(b) Compute the scalar self-energy $\Pi(p)$ (in momentum space). Write your result as a Feynman-parameter integral which you do not need to compute.