Quantum field theory	**Sheet 11**	June 24, 2024

Problem C11.1

(a) By using your results from problem C10.1, show that the generators of the Lorentz group are related to the angular momentum by

$$J^{mn} = \epsilon^{mnl} J^l.$$

Use this relation to determine the matrix $D(\omega)$ for rotations of Dirac spinors in the Weyl representation. How do the upper two and the lower two components transform? Compare your result with the spin-1/2 representation of SO(3) (see section 5.1 of the lecture).

(b) Determine the explicit form of $D(\omega)$ for rotations by an angle θ around the x^3-axis.

Problem H11.1 Check the following relations for the coherent states of the fermionic harmonic oscillator:

(a)

$$\langle \overline{\psi}_1 | \psi_2 \rangle = \exp\left(-\frac{1}{2} \overline{\psi}_1 \psi_1 - \frac{1}{2} \overline{\psi}_2 \psi_2 + \overline{\psi}_1 \psi_2 \right),$$

(b)

$$|\psi\rangle\langle\overline{\psi}| = (1 - \overline{\psi}\psi)|0\rangle\langle 0| + \overline{\psi}|0\rangle\langle 1| + \psi|1\rangle\langle 0| - \overline{\psi}\psi|1\rangle\langle 1|,$$

(c) and finally

$$\int d\overline{\psi} d\psi |\psi\rangle\langle\overline{\psi}| = 1.$$

Problem H11.2 Determine the Fourier transform $S_F(p)$ of the Feynman propagator

$$[S_F(x)]_{\alpha\beta} := \langle 0 | T \psi_\alpha(x) \overline{\psi}_\beta(0) | 0 \rangle$$

for the Dirac field, using the operator expression for ψ and $\overline{\psi}$ in terms of creatinon and annihilation operators. For fermionic operators the time ordered product is defined by

$$T\psi(x)\overline{\psi}(x') := \Theta(t - t')\psi(x)\overline{\psi}(x') - \Theta(t' - t)\overline{\psi}(x')\psi(x).$$

Show that your result can be written as

$$S_F(p) = i \frac{\not{p} + m}{p^2 - m^2 + i\epsilon}.$$