**Problem C10.1** Consider an infinitesimal rotation. For general representations one has

$$D(\vec{\theta}) = \mathbb{1} - i\theta^k J^k \qquad (*)$$

(a) A vector $\vec{x}$ transforms as (draw a picture)

$$\vec{x} \to \vec{x} + \delta\vec{x}, \qquad \delta\vec{x} = \vec{\theta} \times \vec{x} \qquad (**)$$

Recall that a general infinitesimal Lorentz transformation acts on a 4-vector like

$$x^\mu \to x^\mu + \omega^\mu{}_\nu x^\nu$$

Determine $\omega^\mu{}_\nu$ for the rotation.

(b) To write $(**)$ in the form $(*)$ find matrices $J^k$ such that

$$\delta\vec{x} = -i\theta^k J^k \vec{x}$$

**Problem C10.2** Check that the $\gamma^\mu$ in the Weyl representation satisfy the Clifford algebra.

**Problem H10.1**

(a) Show that the $S^{\mu\nu} = \frac{i}{4}[\gamma^\mu, \gamma^\nu]$ satisfy the relation

$$[S^{\mu\nu}, \gamma^\rho] = i\left(\eta^{\nu\rho}\gamma^\mu - \eta^{\mu\rho}\gamma^\nu\right).$$

(b) Use (a) and the Jacobi identity $[A,[B,C]] + [C,[A,B]] + [B,[C,A]] = 0$ to show that $S^{\mu\nu}$ is a representation of the Lorentz algebra.

**Problem H10.2**

(a) Show that the Dirac action

$$S = \int d^4x\, \overline{\psi}(i\partial\!\!\!/ - m)\psi$$

is invariant under the **parity** transformation

$$\psi(t, \vec{x}) \to \gamma^0 \psi(t, -\vec{x})$$

(b) Show that if $\psi(t, \vec{x})$ is a solution of the Dirac equation, then $\gamma^0\psi(t, -\vec{x})$ is a solution to the Dirac equation.