Quantum field theory	**Sheet 9**	June 24, 2024

Problem C9.1 Re-consider the computation of the wave function renormalization of problem H8.1 in the MS scheme. You should obtain

$$Z = 1 - \frac{g^2}{12(4\pi)^3}\frac{1}{\varepsilon}.$$

Use this to compute (see lecture)

$$\gamma(g,\varepsilon) := \mu\frac{d}{d\mu}\ln Z^{1/2}(g,\varepsilon)$$

and then

$$\gamma(g) := \lim_{\varepsilon\to 0}\gamma(g,\varepsilon).$$

γ is called the **anomalous dimension** of φ. Can you guess why?

Problem H9.1 **β function in φ^3 theory.** Consider φ^3 theory in $d=6$ space-time dimensions with the Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}m_B^2\varphi^2 - \frac{1}{6}g_B\varphi^3.$$

At leading order the amputated three-point function $G^{(3)}_{\mathrm{amp}}(k_1,k_2,k_3)$ is given by $-ig_B$. It is corrected by the loop contribution at order g_B^3 which determines the running of the renormalized coupling. This will be worked out in this exercise, using the MS scheme.

(a) Compute the three-point function to order g_B^3. Draw the corresponding Feynman diagram. Evaluate the loop correction in $d = 6-2\varepsilon$ dimensions making use of Feynman parameters and Wick rotation. You should arrive at

$$G^{(3)}_{\mathrm{amp}} = -i\,g_B - i\,g_B^3\,\Gamma(\varepsilon)I,$$

where I is proportional to an integral over Feynman parameters, which you not need to evaluate.

(b) The renormalized amputated 3-point function is

$$G^{(3)}_{\mathrm{amp,R}} = Z^{3/2}\mu^{-3\varepsilon}G^{(3)}_{\mathrm{amp}}$$

Write

$$g_B = \mu^\varepsilon\left(g + \frac{ag^3}{\varepsilon}\right)$$

and determine the numerical coefficient a such that the divergent terms of $G^{(3)}_{\mathrm{amp,R}}$ cancel. You should find

$$a = -\frac{3}{8}\frac{1}{(4\pi)^3}$$

(c) Determine the beta-function $\beta(g,\varepsilon)$, and then $\lim_{\varepsilon\to 0}\beta(g,\varepsilon) =: \beta(g)$. You should find

$$\beta(g) = -c\,g^3,$$

with some numerical coefficient $c > 0$.

(d) Solve the differential equation that governs the scale dependence of the renormalized coupling,

$$\mu'\frac{dg}{d\mu'} = \beta(g(\mu')),$$

with the initial condition $g(\mu) = g$. How does $g(\mu')$ behave for $\mu'\to\infty$? Determine the position of the pole.