Problem C7.1 In the lecture it was claimed that disconnected parts of the 4-point function do not contribute to the S-matrix. To see this in an example, take one diagram contributing to the 4-point function at lowest order ($O(\lambda_B^0)$). Fourier transform it and insert the result into the LSZ formula.

Problem H7.1 Consider one Hermitian scalar field in 4 space-time dimensions with the following Lagrangian:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}m^2\varphi^2 - \frac{1}{6}g\varphi^3$$

(a) Determine the connected 4-point function $G_c^{(4)}(x_1, \ldots x_4)$ at leading order (which one is that?) in the coupling constant g. Draw all contributing Feynman diagrams. Write $G_c^{(4)}(x_1, \ldots x_4)$ in terms of $\Delta_F(x)$.

(b) Compute the Fourier transform $\widetilde{G}_c^{(4)}(p_1, \ldots p_4)$. Determine the S-matrix element $\langle \mathbf{p}_3\, \mathbf{p}_4 | S - 1 | \mathbf{p}_1\, \mathbf{p}_2 \rangle$ and write it in terms of the Lorentz-invariant Mandestam-variables $s = (p_1 + p_2)^2$, $t = (p_1 - p_3)^2$, and $u = (p_1 - p_4)^2$.