Quantum field theory	**Sheet 6**	May 21, 2024

Problem C6.1 The generating functional for a free scalar field is given by the path integral

$$Z[J] = N \int \mathcal{D}\varphi \exp\{iS'[\varphi]\}$$

with

$$S'[\varphi] = \int d^4x \left[-\frac{1}{2}\varphi(\partial^2 + m^2)\varphi + J\varphi \right]$$

and with the normalization factor N such that $Z[0] = 1$. Solve the equation of motion following from varying S' and insert the solution into $\exp\{iS'[\varphi]\}$. Compare your result with the result for $Z[J]$ from the lecture.

Problem C6.2 The neutral pion can decay into two photons:

$$\pi^0 \to \gamma + \gamma$$

Write the decay rate in the pion rest frame in terms of the invariant matrix element $\mathcal{M}$. You can assume that $\mathcal{M}$ is Lorentz-invariant.
Hint: The differential rate of a particle with 4-momentum p to decay into N particles with 4-momenta p_a can be written as

$$d\Gamma = \frac{1}{2p^0}(2\pi)^4 \delta\left(p - \sum_a p_a\right)\left[\prod_a \frac{d^3p_a}{(2\pi)^3 2p_a^0}\right]|\mathcal{M}|^2$$

Problem H6.1 Consider one Hermitean scalar field in 4 space-time dimensions with the following Lagrangian:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}m^2\varphi^2 - \frac{1}{6}g\varphi^3$$

(a) Consider the 1-point function $\langle\varphi(x)\rangle$ (the vacuum expectation value of φ) at order g. Start from the generating functional

$$Z[J] = \frac{\int \mathcal{D}\varphi \exp\left(iS + i\int d^4x J\varphi\right)}{\int \mathcal{D}\varphi \exp(iS)}$$

and expand in the interaction term. Evaluate the resulting expression via Wick's theorem in terms of Feynman propagators $\Delta_F(x - x')$. Draw the corresponding Feynman diagram.

(b) Now determine the contribution to the time ordered 2-point function $G^{(2)}(x_1, x_2) = \langle T\varphi(x_1)\varphi(x_2)\rangle$ for the interacting theory at order g^2. Again expand in the interaction term. Evaluate the resulting expression via Wick's theorem in terms of Feynman propagators $\Delta_F(x - x')$. Draw the corresponding Feynman diagrams.

(c) Compute the Fourier transform

$$\widetilde{G}^{(2)}(p_1, p_2) = \int d^4x_1 \int d^4x_2 e^{i(p_1\cdot x_1 + p_2\cdot x_2)} G^{(2)}(x_1, x_2)$$

of the 2-point function. Thereby write the position space Feynman propagators in terms of their Fourier transforms $\Delta_F(p)$, and perform all integrals, except the ones over the loop momenta.