Problem C4.1 Consider the Hermitean Klein-Gordon field. Explain why we have

$$U(\Lambda)^\dagger \varphi(x) U(\Lambda) = \varphi(\Lambda^{-1}x)$$

for a Lorentz transformation Λ. Use this relation to show that

$$U(\Lambda)^\dagger a^\dagger_{\vec{k}} U(\Lambda) = a^\dagger_{\overrightarrow{\Lambda^{-1}k}}$$

and hence

$$U(\Lambda)\left|\vec{k}\right\rangle = \left|\overrightarrow{\Lambda k}\right\rangle$$

as well as

$$U(\Lambda)\left|\vec{k}_1, \vec{k}_2\right\rangle = \left|\overrightarrow{\Lambda k_1}, \overrightarrow{\Lambda k_2}\right\rangle$$

Problem H4.1 Show that for the free complex Klein-Gordon field

(a)

$$\langle 0|T\varphi(x)\varphi^\dagger(0)|0\rangle = \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot x} \frac{i}{k^2 - m^2 + i\varepsilon}$$

(b)

$$\langle 0|T\varphi(x)\varphi(0)|0\rangle = 0$$

Problem H4.2 Consider a real scalar field in 6 space-time dimensions with the Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{g}{3}\varphi^3$$

Determine the mass dimensions of the field φ and of the coupling constant g.