**Problem C3.1** Do rotations commute with Lorentz boosts? Hint: Start with a particle at rest, and consider a rotation around the $x$-axis and a boost in $y$-direction.

**Problem H3.1** Consider the real Klein-Gordon field. Let the vacuum state be normalized to one, $\langle 0|0\rangle = 1$.

(a) Compute the scalar product

$$\langle \mathbf{k}_1 |\, \mathbf{k}_2 \rangle$$

of the 1-particle state vectors $|\mathbf{k}\rangle = a^\dagger_{\mathbf{k}}|0\rangle$

(b) Now consider the wave packet

$$|f\rangle := \int \frac{d^3k}{(2\pi)^3 2k^0}\, f(\mathbf{k})\, a^\dagger_{\mathbf{k}}|0\rangle$$

Which condition does the complex-valued function $f$ have to satisfy, so that $\langle f|f\rangle = 1$?

(c) Compute the scalar product of 2-particle states

$$\langle \mathbf{k}_1, \mathbf{k}_2 |\, \mathbf{k}_3, \mathbf{k}_4 \rangle$$

**Problem H3.2** Consider a field theory with two complex-valued scalar fields with identical masses. The theory is described by the Lagrangian density

$$\mathcal{L} = \partial_\mu \varphi^\dagger \partial^\mu \varphi \;\; - m^2 \varphi^\dagger \varphi \;, \qquad \text{with } \varphi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}.$$

(a) Show that this Lagrangian is invariant under the transformations

$$\varphi \to \varphi' = e^{-i\epsilon_a T_a}\varphi$$

where $a = 0, \ldots, 3$. The parameters $\epsilon_a$ are real. The $T_a$ are the Hermitean $2 \times 2$ matrices $T_0 = \mathbf{1}_{2\times 2}$, $T_a = \sigma_a/2$ for $a = 1, 2, 3$ with the Pauli matrices $\sigma_a$.

(b) Show that the Noether current $J_a$ associated with the symmetry generated by $T_a$ is given by

$$J_a^\mu = i\varphi^\dagger \overset{\leftrightarrow}{\partial^\mu} T_a \varphi.$$

(c) Show that the charges satisfy

$$[Q_a, Q_b] = i\varepsilon_{abc} Q_c \qquad \text{and} \qquad [Q_a, Q_0] = 0$$

for $a, b, c \in \{1, 2, 3\}$, where $\varepsilon_{abc}$ is the totally antisymmetric Levi-Civita symbol.