ELEMENTARY PARTICLE PHYSICS

WS 2016/2017: Exercise sheet 9

29. The 4×4 matrix γ_5 is defined as $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$.

    a) Show that $\gamma_5^2 = 1$.

    b) Show that $\{\gamma_5, \gamma^\mu\} = 0$.

    c) Show that $P_{R/L} = (1 \pm \gamma_5)/2$ are projectors, i.e. $P_{R/L}^2 = P_{R/L}, P_R\, P_L = 0, P_R + P_L = 1$. Show then that $P_{R/L}\,\psi$ are eigenvectors of γ_5 with eigenvalues ± 1.

    d) Compute γ_5 in the Dirac representation of the γ matrices. Apply this to the u spinors as derived (in the same representation) in the lecture for the case of $m = 0$ and $\vec{p} = (0, 0, p)$ along the z direction. Compare with exercise 20.

30. a) Utilizing 29.b) proof that $\mathrm{tr}\{\gamma^\mu\} = 0 = \mathrm{tr}\{\gamma^\mu\gamma^\nu\gamma^\sigma\} = 0$.

    d) Compute $\mathrm{tr}\{\gamma_\mu\gamma_\nu\gamma_\sigma\gamma_\rho\}$, using the Clifford algebra, the properties of a trace and exercise 15.

    c) Show that $\mathrm{tr}\{\gamma^\mu\gamma_5\} = \mathrm{tr}\{\gamma^\mu\gamma^\nu\gamma_5\} = \mathrm{tr}\{\gamma^\mu\gamma^\nu\gamma^\rho\gamma_5\} = 0$ and that $\mathrm{tr}\{\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_5\} = 4i\epsilon^{\mu\nu\rho\sigma}$ with the totally antisymmetric (Levi-Civita) tensor $\epsilon^{0123} = +1$.