ELEMENTARY PARTICLE PHYSICS

WS 2016/2017: Exercise sheet 6

20. The socalled helicity of a Dirac field is obtained from $h(\vec{p}) = \vec{e}(\vec{p}) \cdot \vec{\Sigma}$ where

$$\vec{e}(\vec{p}) = \frac{\vec{p}}{|\vec{p}|} \quad \text{and} \quad \Sigma^i = \begin{pmatrix} \sigma^i & 0 \\ 0 & \sigma^i \end{pmatrix}$$

    a) Show that $h^2(\vec{p}) = 1$. What are the eigenvalues of h ?

    b) Show that $P_\pm = (1 \pm h)/2$ are projectors, i.e. $P_\pm^2 = P_\pm, P_+P_- = 0, P_+ + P_- = 1$.

    c) With $\vec{p}$ along the z direction, $\vec{p} = (0, 0, p)$ show that $h(\vec{p})u(\vec{p}, s) = su(\vec{p}, s)$ with $u(\vec{p}, s)$ as given in the lecture.

21. Verify the equal time (anti-) commutation relations for fermion fields

$$\{\hat{\psi}(t, \vec{x}), \hat{\psi}(t, \vec{y})\} = 0 = \{\hat{\bar{\psi}}(t, \vec{x}), \hat{\bar{\psi}}(t, \vec{y})\}$$

$$\{\hat{\psi}(t, \vec{x}), \hat{\psi}^\dagger(t, \vec{y})\} = \delta(\vec{x} - \vec{y}).$$

22. Work out the expression for the charge operator

$$\hat{Q} = \int d^3x \hat{\bar{\psi}} \gamma^0 \hat{\psi}$$

    in terms of the creation and annihilation operators for fermions and anti-fermions.