ELEMENTARY PARTICLE PHYSICS

WS 2016/2017: Exercise sheet 5

16. Compute how $(\bar{\psi}\gamma_\mu\psi)(\vec{0},t)$ and $(\bar{\psi}\psi)(\vec{0},t)$ behave under a rotation around the z-axis.

17. Using the (classical) solutions $u_s(p), v_s(p)$ of the Dirac equation as given in the lecture, compute

    (i) $u_s^\dagger(p)u_t(p)$ and $v_s^\dagger(p)v_t(p)$ resp.

    (ii) $\bar{u}_s(p)u_t(p)$ and $\bar{v}_s(p)v_t(p)$ resp.

    (iii) $\sum_{s=\pm} u_\alpha(p,s)\bar{u}_\beta(p,s)$ and $\sum_{s=\pm} v_\alpha(p,s)\bar{v}_\beta(p,s)$ resp.

18. Show that

$$\begin{aligned} [\hat{H}, \hat{a}^\dagger(\vec{p})] &= E_p\hat{a}^\dagger(\vec{p}) \\ [\hat{H}, \hat{a}(\vec{p})] &= -E_p\hat{a}(\vec{p}) \end{aligned} \tag{1}$$

with the Hamilton operator $\hat{H}$ and the commutation relations for $\hat{a}(\vec{p})$ and $\hat{a}^\dagger(\vec{p})$ as introduced in the lecture.

19. Show that

$$j_\mu(x) = i\left[\Phi^*(x)(\partial_\mu\Phi(x)) - \Phi(x)(\partial_\mu\Phi^*(x))\right]$$

is a conserved current for the Klein-Gordon theory. Show further that

$$i\int d^3\vec{x}\left[\Phi^*(x)(\partial_0\Phi(x)) - \Phi(x)(\partial_0\Phi^*(x))\right]$$

is the corresponding conserved charge.