ELEMENTARY PARTICLE PHYSICS

WS 2016/2017: Exercise sheet 4

12. Show that the Maxwell equations $\partial^2 A^\mu = j^\mu$ in the gauge $\partial_\mu A^\mu = 0$ can be derived from the Lagrange density

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - j_\mu A^\mu$$

where $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ is the electromagnetic field strength tensor, by means of the Euler Lagrange equations

$$\partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu A^\nu)} - \frac{\partial \mathcal{L}}{\partial A^\nu} = 0.$$

13. In the lecture it was shown that $(\Lambda^{-1})_\mu^{\cdot\rho} = \Lambda^\rho_{\cdot\mu}$. Get familiar with this relation by looking explicitely at

    a) $(\Lambda^{-1})^0_{\cdot 1}, (\Lambda^{-1})^1_{\cdot 0}$ for boosts in the x-direction

    b) $(\Lambda^{-1})^1_{\cdot 2}, (\Lambda^{-1})^2_{\cdot 1}$ for rotations around the z-axis.

14. Show the following relations for the Pauli Matrices σ^i

    (i) $(\sigma^i)^2 = 1$

    (ii) $\{\sigma^i, \sigma^j\} = 2\delta^{ij}$

    (iii) $[\sigma^i, \sigma^j] = 2i \sum_{k=1}^3 \epsilon^{ijk} \sigma^k$

15. a) Show that $\gamma'^\mu = \Lambda^\mu_{\cdot\rho}\gamma^\rho$ satisfies the Clifford algebra $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$.
    b) Derive that $\mathrm{tr}\{\gamma^\mu\gamma^\nu\} = 4g^{\mu\nu}$ also follows from the Clifford algebra.
    c) Using that $\gamma^\nu\gamma^\nu/g_{\nu\nu}$ is the unit matrix in four dimensions for any fixed $\nu = 0, 1, 2, 3$ and the Clifford algebra,
    show that $\mathrm{tr}\{\gamma^\mu\} = 2g^{\mu\nu}\mathrm{tr}\{\gamma^\nu\}/g_{\nu\nu} - \mathrm{tr}\{\gamma^\mu\}$ for any $\mu = 0, 1, 2, 3$, from which one can easily derive that $\mathrm{tr}\{\gamma^\mu\} = 0$.
    d) Starting from the following (Dirac) form of the γ matrices,

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \gamma^k = \begin{pmatrix} 0 & \sigma^k \\ -\sigma^k & 0 \end{pmatrix}$$

show that $\gamma^{\mu\dagger} = \gamma^0\gamma^\mu\gamma^0$.