# ELEMENTARY PARTICLE PHYSICS

## WS 2016/2017: Exercise sheet 12

36. a) Show that the Lagrange density

$$\mathcal{L} = (\bar{u}, \bar{d}, \bar{s}) \left[ i\mathbb{1}_{3\times 3}\gamma^\mu \partial_\mu - \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix} \mathbb{1}_{4\times 4} \right] \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

is invariant under $SU(3)$ transformations

$$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix} \longrightarrow \exp\left\{ -i \sum_{a=1}^{8} \alpha^a \lambda^a \right\} \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

and accordingly for $(\bar{u}, \bar{d}, \bar{s})$, with the Gell-Mann matrices $\lambda^a$ and $\alpha^a$ real, if all three quarks have the same mass. Show further the invariance of $\mathcal{L}$ under transformations

$$\begin{pmatrix} u \\ d \\ s \end{pmatrix} \longrightarrow \exp\left\{ -i \sum_{a=1}^{8} \alpha^a \lambda^a \gamma_5 \right\} \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

if all quark masses vanish.

b) Rewrite the Lagrangian of part a) in terms of the chiral quark fields $q_{L,R} = P_{L,R}\, q$, see exercise 32, for the case of (mass) degenerate quarks $m_u = m_d = m_s =: m_q$. Show how the Lagrangian in this form transforms under separate transformations for the left and right-chiral fields $q_L \to \exp(-i\sum_a \alpha_L^a \lambda^a) q_L$ and $q_R \to \exp(-i\sum_a \alpha_R^a \lambda^a) q_R$. What do you obtain in the socalled chiral limit $m_q \to 0$ ?

37. a) The distribution functions of partons inside a proton, $u(x) := f_u(x)$, $\bar{u}(x) := f_{\bar{u}}(x)$ etc., obey socalled sum rules which follow from the conservation of quantum numbers. What rules for the parton distribution functions (PDFs) follow from the fact that $S = 0$, $I_3 = 1/2$ und $Q = +1$ for a proton ?

b) Assume isospin invariance, i.e. the distribution of u quarks inside a proton is the same as the distribution of d-quarks in a neutron, $u^p(x) =$

$d^n(x), d^p(x) = u^n(x), \bar{u}^p(x) = \bar{u}^n(x)$ etc.. Derive sum rules equivalent to those of a) for the neutron.

c) Express $F_2^{p,n}$ for proton and neutron in terms of PDFs and derive the bound

$$4 \geq \frac{F_2^n(x)}{F_2^p(x)} \geq \frac{1}{4}.$$

Hint: consider the monotony of the functions $f(y) = (a+y)/(b+y)$, $(a+4y)/(b+y)$ or $(a+y)/(b+4y)$ depending on $a$ and $b$.